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O.M.R. Serial No. | | | | | | | | |

Question Booklet Number

M.A./M.Sc. (SEM.-IV)(NEP) (SUPPLE.) EXAMINATION, 2024-25
MATHEMATICS
(Calculus of Variations)

Paper Code							
B	0	3	1	0	0	9	T

Question Booklet Series
A

Time : 1 : 30 Hours **Max. Marks : 75**

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.
4. Four alternative answers are mentioned for each question as - A, B, C & D in the booklet. The candidate has to choose the correct / answer and mark the same in the OMR Answer-Sheet as per the direction :

(Remaining instructions on last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, उसे तुरन्त बदल लें।
4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर- A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR उत्तर-पत्रक में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

(शेष निर्देश अन्तिम पृष्ठ पर)

1. The solution to Euler's equation is referred to :

(A) Maximals

(B) Minimals

(C) Extremals

(D) Admissible solution

2. A necessary condition for $\mathrm{I}[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ to be an extremum is :

(A) $\frac{\partial F}{\partial x} - \frac{d}{dy}\left(\frac{\partial F}{\partial x'}\right) = 0$

(B) $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

(C) $\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$

(D) $\frac{\partial F}{\partial y} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y'}\right) = 0$

3. Integral $\mathrm{I}[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$, if F is independent of y', then Euler-Lagrange's equation is :

(A) $\frac{\partial F}{\partial y} = 0$

(B) $\frac{\partial F}{\partial y'} = 0$

(C) $\frac{\partial F}{\partial y} = \text{constant}$

(D) $\frac{\partial F}{\partial y} = \frac{\partial F}{\partial y'} - \frac{\partial F}{\partial x}$

4. Functional $\mathrm{I}[y(x)] = \int_{x_1}^{x_2} F(x, y, y')dx,$, if F is independent of y, then Euler-Lagrange's equation is :

(A) $\frac{\partial F}{\partial y} = \text{constant}$

(B) $\frac{\partial F}{\partial y'} = 0$

(C) $\frac{\partial F}{\partial y'} = \text{constant}$

(D) $\frac{\partial F}{\partial y} = \frac{\partial F}{\partial y'}$

5. If $F(x, y, y')$ is independent of x, then Euler's equation for the functional $\int_{x_1}^{x_2} F(x, y, y')dx$ reduces to:

(A) $Fy - \frac{d}{dx}\{F_{y'}\} = 0$

(B) $F - y' F_{y'} = 0$

(C) $F - y' F_{y'} = c$

(D) $F + y' F_{y'} = c$

6. The solution of Brachistochrone problem is :

(A) a parabola

(B) a family of circles

(C) a catenary

(D) an inverted cycloid

7. A necessary condition that the integral $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ will be stationary is :

(A) $\delta I =$ constant

(B) $\delta I \neq$ constant

(C) $\delta I = 0$

(D) $\delta I \neq 0$

8. The shortest distance between two points in a plane is :

(A) a curve

(B) catenary

(C) straight line

(D) equiangular spiral

9. Geodesics on a surface is a curve along which the distance between any two points of the surface?

(A) Extreme value

(B) Maximum

(C) Average value

(D) Minimum

10. If the area of the surface of revolution of a curve $y = y(x)$ is $2\pi \int_{x_1}^{x_2} y\sqrt{1+y'^2}\,dx$ and is minimum, then the curve is :

(A) family of straight line

(B) family of circles

(C) a catenary

(D) a cycloid

11. Which one of the following is linear functional?

(A) $I[y(x)] = \int_{x_1}^{x_2} (y' + 2y^2)\,dx$

(B) $I[y(x)] = \int_{x_1}^{x_2} (y'^2 + 2y^2)\,dx$

(C) $I[y(x)] = \int_{x_1}^{x_2} (y'^2 + 2y)\,dx$

(D) $I[y(x)] = \int_{x_1}^{x_2} (y' + 2y)\,dx$

12. The extremal of the functional $v[y(x)] = \int_0^1 \left[(y')^2 + 12xy\right] dx, y=(0)=0,$ $y(1)=1$ represents :

(A) family of circles

(B) nodal cubic curve

(C) family of straight line

(D) catenary

13. The condition that $\frac{\partial^2 F}{\partial y' \partial y'} >< 0$ for the functional $\mathrm{I}\left[y(x)\right]=\int_{x_1}^{x_2} F(x,y,y')\,dx$ is:

(A) Only necessary

(B) Only sufficient

(C) Necessary and sufficient both

(D) Neither necessary nor sufficient

14. If a particle slide from one point to another point in the shortest time under the action of gravity, then the path is :

(A) catenary

(B) straight line

(C) cycloid

(D) family and circles

15. Which curves can give an extremum of the functional $\mathrm{I}\left[y(x)\right]=\int_0^1\left[(y')^2+12xy\right]dx$, $y(0)=0$, $y(1)=1$?

(A) $y(x) = x^3$

(B) $y(x) = x^3 - x$

(C) $y(x) = x^3 + 2x$

(D) $y(x) = x^3 + x$

16. An extremal to the functional $v\left[y(x)\right]=\int_a^b \sqrt{1+(y')^2}\,dx$ is

(A) a catenary

(B) the straight line connecting $(a, y(a))$ and $(b, y(b))$

(C) The family of circles

(D) an inverted cycloid

17. For $v\left[y(x)\right]=\int_a^b F(x,y,y')\,dx$, v is called :

(A) function

(B) arc length

(C) range

(D) functional

18. Euler's equation for the functional :

$$\int_{x_1}^{x_2}\left[p(x)y'^2+2q(x)yy'+r(x)y^2\right]dx$$

(A) First order linear differential equation

(B) Second order non-linear differential equation

(C) Second order linear differential equation

(D) None of the above

19. The curve joining two points in a plane that yields a surface of revolution of minimum area when revolved about the x-axis is :

(A) Straight line

(B) Catenary

(C) Cycloid

(D) Family and paraboloid

20. The closed plane curve of given length that encloses maximum area is :

(A) Catenary

(B) Circle

(C) Parabola

(D) Ellipse

21. The geodesic is on right circular cylinder is :

(A) Great circle

(B) Circular helix

(C) Cone

(D) Straight line

22. The geodesic on a sphere of a radius 'a' are its :

(A) Straight line

(B) Cone

(C) Catenary

(D) Great circle

23. The other form of Euler-Lagrange's equation $\frac{\partial F}{\partial y}-\frac{d}{dx}\left\{\frac{\partial F}{\partial y'}\right\}=0$ is :

(A) $\frac{d}{dx}\left[F_y - y'\frac{\partial F}{\partial y'}\right]-\frac{\partial F}{\partial x}=0$

(B) $\frac{d}{dx}\left[F - y'\frac{\partial F}{\partial y}\right]-\frac{\partial F}{\partial x}=0$

(C) $\frac{d}{dx}\left[F - y'\frac{\partial F}{\partial y'}\right]-\frac{\partial F}{\partial x}=0$

(D) None of the above

24. Extremal of the functional $v[y(x)]=\int_1^e \left(xe^y - ye^x\right)dx$ with $y(1)=1$, $y(e)=e$:

(A) $y(x) = x - \log x$

(B) $y(x) = \log x - x$

(C) $y(x) = \log x + x$

(D) No extremal exist

25. Extremal of the functional $v[y(x)]=\int_0^1 \left(y^2 + y'x^2\right)dx$ with $y(0)=0$, $y(1)=1$:

(A) $y = x$

(B) $x^2 = y$

(C) $y = 2x$

(D) No extremal exist

26. Extremal for the variational problem $v[y(x)]=\int_0^{\log 2}\left(e^{-x}y'^2 - e^x y^2\right)dx$ satisfy the differential equation :

(A) $y''+y'+e^{2x}y=0$

(B) $y''-y'-e^{2x}y=0$

(C) $y''-y'+e^{2x}y=0$

(D) $y''-y'+e^{x}y=0$

27. Extremal of the functional $v[y(x)]=\int_a^b \frac{\sqrt{1+(y')^2}}{y}dx$ is :

(A) $(x+a)^2 = 4y$

(B) $(x-A)^2 + y^2 = B^2$

(C) $y = c\cosh\frac{x-c}{c_1}$

(D) $y = Ax + B$

28. Extremal for the variational problem $v[y(x)] = \int_{x_0}^{x_1} y'(1 + x^2 y')dx$ is solution of differential equation :

(A) $xy'' + 2y' = 0$

(B) $1 + 2x^2 y' = 0$

(C) $1 + 2x^2 y' = \text{c}$

(D) $xy'' + y' = 0$

29. Area of the surface of revolution obtained by revolving a curve $y = f(x)$ with end points of (x_1, y_1), (x_2, y_2) is :

(A) $\int_{x_1}^{x_2} 2\pi\sqrt{1+(y')^2}\,dx$

(B) $\int_{y_1}^{y_2} 2\pi x\sqrt{1+(y')^2}\,dx$

(C) $\int_{x_1}^{x_2} 2\pi y\sqrt{1+(y')^2}\,dx$

(D) None of the above

30. Extremal $y = y(x)$ for the variational problem $v[y(x)] = \int_0^1 1 + (y'')^2\,dx$ satisfies the O.D.E. is :

(A) Homogeneous linear differential equation of fourth order

(B) Homogeneous non-linear differential equation of fourth order

(C) Homogeneous linear differential equation of order more than fourth order

(D) None of the above

31. The extremals of the functional $v[y(x)] = \int_0^{\pi/4} \left[(y'')^2 - y^2 + x^2\right]dx$, $y(0) = 0, y'(0) = 1,\ y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ is :

(A) $y = \cos x$

(B) $y = \sin x$

(C) $y = \tan x$

(D) $y = x$

32. The functional $\int_0^{\frac{\pi}{2}} \left[(y')^2 - y^2 + 2xy\right]dx$ with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ can be extremized on the curve :

(A) $y = x + \cos x$

(B) $y = x - x \sin x$

(C) $y = x + \frac{\pi}{2}\sin x$

(D) $y = x - \frac{\pi}{2}\sin x$

33. How many extremals are possible for the extremals $v[y(x)] = \int_0^1 (y'' + y)\,dx$, $y(0) = 1$, $y(1) = 1$, $y'(0) = 1$, $y'(1) = 1$?

(A) Only one

(B) Exactly two

(C) Infinite many

(D) None of the above

34. Extremal of the functional $\mathrm{I}[y(x)] = \int_1^2 \frac{x^3}{y'^2}\,dx$, $y(1) = 0$ and $y(2) = 3$ is :

(A) $y = x^2 + 1$

(B) $y = x^2 - 1$

(C) $y = x^3 - 1$

(D) $y = x - 1$

35. Extremal of the functional $\mathrm{I}[y(x)] = \int_0^1 \left[(y')^2 + 4y^2\right]dx$, $y(0) = e^2$ and $y(1) = 1$ lies on the curve:

(A) $y = e^{2x}$

(B) $y = e^{x}$

(C) $y = e^{2-2x}$

(D) $y = e^{2+2x}$

36. The functional $\mathrm{I}[y(x)] = \int_0^{\frac{\pi}{2}} \left[(y')^2 - y^2\right]dx$; $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$ has :

(A) Unique extremal

(B) Two extremal

(C) No extremal

(D) Infinite extremal

37. The functional $\mathrm{I}[y(x)] = \int_0^1 \left[xy + y^2 - 2y^2 y'\right]dx$, $y(0)=1$, $y(1)=2$ has :

(A) One extremal

(B) No extremal

(C) Infinite extremal

(D) Two extremal

38. For the functional $\mathrm{I}[y(x)] = \int_a^b \left(y^2 + 2xyy'\right)dx$; $y(a) = y_a$ and $y(b) = y_b$, The number of extremal equals to :

(A) 0

(B) 1

(C) 2

(D) infinite

39. The Euler's equation for a functional of the form $\int_a^b F(x, y)\,dx$ is :

(A) $F_{y'} = c_1$

(B) $F_y - \frac{d}{dx}\left\{F_{y'}\right\} = c_1$

(C) $F_y = c_1$

(D) None of the above

40. The Euler equation for the functionals

$$\mathrm{I}\left[z(x,y)\right]=\iint_{D}\left[\left(\frac{\partial z}{\partial x}\right)^{2}+\left(\frac{\partial z}{\partial y}\right)^{2}\right]dxdy$$

satisfy :

(A) Wave equation

(B) Poisson's equation

(C) Laplace equation

(D) Heat equation

41. The necessary conditions for existence of extremals for functional $\mathrm{I}\left[u(x,y)\right]=\iint_{D}F\left(x,y,u,u_{x},u_{y}\right)dxdy$ is :

(A) $\dfrac{\partial F}{\partial u}+\dfrac{\partial}{\partial x}\left(\dfrac{\partial F}{\partial u_{x}}\right)-\dfrac{\partial}{\partial y}\left(\dfrac{\partial F}{\partial u_{y}}\right)=0$

(B) $\dfrac{\partial F}{\partial u}-\dfrac{\partial}{\partial x}\left(\dfrac{\partial F}{\partial u_{x}}\right)-\dfrac{\partial}{\partial y}\left(\dfrac{\partial F}{\partial u_{y}}\right)=0$

(C) $\dfrac{\partial F}{\partial u}-\dfrac{d}{dx}\left(\dfrac{\partial F}{\partial u_{x}}\right)-\dfrac{d}{dy}\left(\dfrac{\partial F}{\partial u_{y}}\right)=0$

(D) None of the above

42. The necessary condition for the functional $\mathrm{I}\left[y(x)\right]=\int_{x}^{x_{2}}F(x,y,y',y'')\,dx$ to be an extremum is :

(A) $F_{y}-\dfrac{d}{dx}\left\{F_{y'}\right\}=0$

(B) $F-y'Fy'=c$

(C) $F_{y}-\dfrac{d}{dx}\left\{F_{y'}\right\}-\dfrac{d^{2}}{dx^{2}}\left\{F_{y''}\right\}=0$

(D) $F_{y}-\dfrac{d}{dx}\left\{F_{y'}\right\}+\dfrac{d^{2}}{dx^{2}}\left\{F_{y''}\right\}=0$

43. The extremals of the functional $\mathrm{I}\left[y(x),z(x)\right]=\int_{0}^{\frac{\pi}{2}}\left(y'^{2}+z'^{2}+2yz\right)dx$ are the solution of simultaneous differential equation :

(A) $y''+z=0,\ z''+y=0$

(B) $y''-z=0,\ z''+y=0$

(C) $y''-z=0,\ z''-y=0$

(D) $y''+z=0,\ z''-y=0$

44. Euler's equation for the functional $\int_{1}^{2}\left(y'^{2}+z^{2}+z'^{2}\right)dx$ are given by :

(A) $y''+y=0,\ z''+z=0$

(B) $y''=0,\ z''-z=0$

(C) $y''-y=0,\ z''=0$

(D) None of the above

45. The extremal of the functional $\mathrm{I}\left[y(x)\right]=\int_{0}^{\pi}\left[(y')^{2}-y^{2}\right]dx$, $y(0)=1$, $y(\pi)=\alpha$ has :

(A) A unique extremal if $\alpha = 1$

(B) Infinite many extremal if $\alpha = 1$

(C) A unique extremal if $\alpha = -1$

(D) Infinite many extremal if $\alpha = -1$

46. The extremal of the functional $\int_0^2 \frac{y'^2}{x} dx$ with $y(0) = a$, $y(2) = b$ is parabola passing through origin, then a and b are :

(A) $a = 0, b = 1$

(B) $a = 1, b = 2$

(C) $a = -1, b = 2$

(D) $a = 0, b = 2$

47. If $F(x, y, y')$ is a function of y' alone then the extremal of the functional $\int_x^{x_2} F(x, y, y') dx$ is :

(A) Circle

(B) Straight line

(C) Nodal cubic curve

(D) Catenary

48. For the functional $\mathrm{I}[y(x)] = \int_a^b (x - y)^2 dx$ the extremal is a :

(A) Straight line

(B) Circle

(C) Catenary

(D) Parabola

49. Euler's equation for the functional $\mathrm{I}[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy$ is :

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(B) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

(C) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \cdot \frac{\partial z}{\partial x} = 0$

(D) None of the above

50. Extremal of the functional $\mathrm{I}[y(x)] = \int_{x_0}^{x_1} (2xy + y'''^2) dx$ lies on the:

(A) Four parameter family of curves

(B) Five parameter family of curves

(C) Six parameter family of curves

(D) Seven parameter family of curves

51. The simplification of the Euler-Lagrange equation is known as :

(A) Belframi identity

(B) Hamilton's identity

(C) Liouville's identity

(D) Legendre's condition

52. $F(x,y,y')$ is not functional because :

(A) Its range is infinite

(B) Its range is IR

(C) Its range is not IR

(D) None of the above

53. Solid figure of revolution which, for a given surface area, has maximum volume is :

(A) a circle

(B) a sphere

(C) an ellipse

(D) a parabola

54. In the equation $H = f + \lambda g,$ where $\int_1^2 Hdx$ to be en extremum, λ is called as :

(A) Isoperimetric constant

(B) Kernels

(C) Lagrange multiplier

(D) Green's function

55. The curve made by a cable of fixed length suspended from two points for minimum gravitational potential energy is :

(A) Circular

(B) Parabolic

(C) Hyperbolic

(D) Catenary

56. The problem where a curve of given perimeter and its encloses the maximum area is known as :

(A) Geodesics

(B) Brachistochrone

(C) Isoperimetric

(D) Weierstrass

57. The transversality condition which together with the relation $y(x_1) = \phi(x_1)$ is :

(A) $\left[F + (\phi' + y')\frac{\partial F}{\partial y'}\right]_{x=x_1} = 0$

(B) $\left[F + (\phi' - y')\frac{\partial F}{\partial y'}\right]_{x=x_1} = 0$

(C) $\left[F + (\phi' - y')\frac{\partial F}{\partial y}\right]_{x=x_1} = 0$

(D) None of the above

58. The function $E(x,y,y',p) = F(x,y,y') - F(x,y,p) - (y'-p)F(x,y,p)$ is known as :

(A) Weierstrass function

(B) Lagrange's function

(C) Jacobi condition

(D) Legende condition

59. The Jacobi equation is given by :

(A) $\left(F_{yy}-\frac{d}{dx}F_{yy'}\right)u-\frac{d}{dx}\left(F_{y'y'}\ u'\right)=0$

(B) $\left(F_{yy}-\frac{d}{dx}F_{yy'}\right)u+\frac{d}{dx}\left(F_{y'y'}\ u'\right)=0$

(C) $\left(F_{yy}-\frac{d}{dx}F_{y'y'}\right)u-\frac{d}{dx}\left(F_{yy'}\ u'\right)=0$

(D) None of the above

60. The Euler-Ostrogradsky equation for $\mathrm{I}\left[u(x,y)\right]=\iint_D\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]dxdy$ where the value of u are prescribed on the boundary c of the domain D :

(A) $\nabla^2 u=f(x,y)$

(B) $\nabla u=0$

(C) $\nabla^2 u=0$

(D) None of the above

61. Extremal of the isometric problem $\int_a^b (y')^2\,dx$ subject to $\int_a^b ydx=c$ is :

(A) $y=\lambda x^2+ax+b$

(B) $y=\lambda x^3+ax+b$

(C) $y=\lambda x+b$

(D) $y=\lambda x^3+ax^2+bx+c$

62. The extremal of the functional $\mathrm{I}\left[z(x,y)\right]=\iint_{\mathrm{D}}\left[\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+2zf(x,y)\right]dxdy$ where $f(x,y)$ is known function :

(A) $\nabla^2 z=0$

(B) $\nabla^2 z=2f(x,y)$

(C) $\nabla^2 z=f(x,y)$

(D) $\nabla^2 z=f(x,y)z$

63. The length of a curve expressed in the parametric from $x=x(t)$, $y=y(t)$ as 't' increases from t_1 to t_2 is given by :

(A) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

(B) $\pi\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

(C) $z\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

(D) None of the above

64. For what curve, the time taken along which the least when velocity at any point of it is $v=x$.

(A) a family of circle

(B) a family of straight line

(C) catenary

(D) cycloid

65. The shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$ is :

(A) $\frac{4}{\sqrt{5}}$

(B) $2\sqrt{2}-1$

(C) $\frac{19\sqrt{2}}{8}$

(D) $\frac{19}{8\sqrt{2}}$

66. Shortest distance between parabola $y^2 = 4x$ and the straight line $x + y = -5$ is :

(A) $2\sqrt{2}$

(B) $2\sqrt{2}-1$

(C) $3\sqrt{2}$

(D) $2\sqrt{3}$

67. The shortest distance between point A(1,0) and ellipse $4x^2 + 9y^2 = 36$ is :

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{4}{\sqrt{5}}$

68. The minimum distance between circle $x^2 + y^2 = 1$ and straight line $x + y = 4$ is :

(A) $2\sqrt{2}$

(B) $2\sqrt{2}-1$

(C) $2\sqrt{2}+2$

(D) $3\sqrt{2}-1$

69. The function of the form $\mathrm{I}\left[y(x)\right] = \int_{x_1}^{x_2} g(x,y)\sqrt{1+(y')^2}\,dx$ where $g(x, y)$ does not vanish at the movable boundary point x_2 then transversality condition reduce to :

(A) Legendre's condition

(B) Orthogonality condition

(C) Jacobi condition

(D) Weierstrass condition

70. The extremum of the functional $\mathrm{I}\left[y(x)\right] = \int_0^2 \left(e^{y'}+3\right)dx$, $y(0)=0$, $y(2)=1$:

(A) $y^2 = \frac{x}{2}$

(B) $y = \frac{x}{2}$

(C) $y = 2x$

(D) $y = x$

71. Use the equation for the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y', y'')dx$ to be an extremum is :

(A) Euler-Lagrange's equation

(B) Euler-Ostrogradasky equation

(C) Euler-Poisson equation

(D) Euler's equation

72. The extremal of the functional $I[y(x)] = \int_0^{\log 2} \left(e^{-x}y'^2 - e^x y^2\right)dx$ possesses solution if we substitute :

(A) $x = \log v,\ y = u$

(B) $x = \log u,\ y = v$

(C) $x = \log u,\ y = v^2$

(D) $x = \log u,\ y = u^2$

73. Function $y(x)$ for which $\int_0^1 \left(x^2 + y'^2\right)dx$ is stationary given that $\int_0^1 y^2 dx = 2$; $y(0) = 0$ is :

(A) $y = \sin m\pi x$

(B) $y = \pm 2\sin m\pi x$

(C) $y = \pm 3\sin m\pi x$

(D) $y = 4\sin m\pi x$

74. Extremal of the functional $I[y(x)] = \int_1^2 \frac{x^2}{(y')^2}dx,$ $y(1) = 0$ and $y(2) = 3$ is :

(A) $y = x^2 - 1$

(B) $y = x^3 - 1$

(C) $y = x - 1$

(D) $y = x^3 - x$

75. For the functional $I[y(x)] = \int_a^b (x - y)^2\, dx,$ the extremal is a :

(A) Straight line

(B) Parabola

(C) Circle

(D) Catenary

76. The functional $I[y(x)] = \int_0^1 (1+x)(y')^2\, dx,$ $y(0) = 0, y(1) = 1$ possess :

(A) Strong maxima

(B) Strong minima

(C) Weak minima but not a strong maxima

(D) Weak maxima but not a strong minima

77. In a conservative field a system moves from t_1 to t_2 in such a way that $\int_{t_1}^{t_2}(\mathrm{T}-\mathrm{v})d_t$ is an extremum, when :

(A) Maximum

(B) Minimum

(C) Neither maximum nor minima

(D) None of the above

78. The Hamilton's canonical equation of motion is :

(A) $\dot{q}_1 = \frac{\partial H}{\partial p_i}, \dot{p}_j = \frac{\partial H}{\partial q_i}$

(B) $\dot{q}_1 = \frac{\partial H}{\partial p_i}, \dot{p}_j = -\frac{\partial H}{\partial q_i}$

(C) $\dot{q}_1 = \frac{-\partial H}{\partial p_i}, \dot{p}_j = -\frac{\partial H}{\partial q_i}$

(D) None of the above

79. In a conservative field the Hamilton's principle is :

(A) $\delta\int_{t_1}^{t_2}\mathrm{L}dt = 0$

(B) $\delta\int_{t_1}^{t_2}\mathrm{L}dt \neq 0$

(C) Both (A) and (B)

(D) None of the above

80. Let q_1, q_2 ..... q_n denote generalised coordinate of material system and $\mathrm{L} = \mathrm{T} - \mathrm{V}$ is the Lagrange's function then the Lagrange's equation of motion are:

(A) $\frac{\partial \mathrm{L}}{\partial qr} - \frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{q}r}\right) = 0, (r = 1, 2, ...n)$

(B) $\frac{\partial \mathrm{L}}{\partial \dot{q}r} - \frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial qr}\right) = 0, (r = 1, 2, ...n)$

(C) $\frac{\partial \mathrm{L}}{\partial \dot{q}r} + \frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial qr}\right) = 0, (r = 1, 2, ...n)$

(D) None of the above

81. The mechanical system in which 't' the time, does not enter explicitly in $r_{\mathrm{v}} = r_{\mathrm{v}}(q_1, q_2 .... q_n, t)$ is called :

(A) Holonomic system

(B) Rheanomic system

(C) Scleronomic system

(D) None of the above

82. If H is the Hamiltonian and 'f' is any function depending upon positions, momenta and time, if [H, F] is the Poisson Bracket, then :

(A) $\frac{df}{dt} = \frac{\partial f}{\partial t} - [H, f]$

(B) $\frac{df}{dt} = \frac{\partial f}{\partial t} + [H, f]$

(C) $\frac{df}{dt} = -\frac{\partial f}{\partial t} + [H, f]$

(D) $\frac{df}{dt} = -\frac{\partial f}{\partial t} - [H, f]$

83. For a conservative field, the Lagrange's function (L) of the system defined as:

(A) L = K.E. - P.E.

(B) L = K.E. + P.E.

(C) $L = (K.E. + P.E.)^2$

(D) None of the above

84. For a conservative holonomic dynamical system, principle of least actions is :

(A) $\delta\int_{t_0}^{t_1}(2L)\,dt = 0$

(B) $\delta\int_{t_0}^{t_1}(2E)\,dt = 0$

(C) $\delta\int_{t_0}^{t_1}(2T)\,dt = 0$

(D) $\delta\int_{t_0}^{t_1}(2V)\,dt = 0$

85. In a simple dynamical conservative system sum of kinetic energy and potential energy is :

(A) Not constant

(B) Constant

(C) Zero

(D) Varying

86. Hamiltonian (H) is defined as :

(A) the total energy of the system

(B) the difference in energy of the system

(C) the product of energy of the system

(D) None of the above

87. Hamiltonian function is :

(A) $H = L\sum_{k} p_k \dot{q}_k$

(B) $H = -L + \sum_{k} p_k \dot{q}_k$

(C) $H = -L + \sum_{k} p_k q_k$

(D) $H = L + \sum_{k} p_k \dot{q}_k$

88. In the Poisson Bracket condition which is true?

(A) $[Q, P] = -1$

(B) $[Q, P] = 2$

(C) $[Q, P] = 0$

(D) $[Q, P] = 1$

89. Which transformation is canonical?

(A) $Q = \frac{1}{p}, P = qp^2$

(B) $Q = \frac{1}{p^2}, P = q^2 p$

(C) $Q = \frac{1}{p^3}, P = p^2 q^2$

(D) None of the above

90. Which transformation is canonical?

(A) $P=\frac{1}{2}\left(p^2+q^2\right),\ Q=\tan^{-1}\left(\frac{p}{q}\right)$

(B) $P=-\frac{1}{2}\left(p^2+q^2\right),\ Q=\tan^{-1}\left(\frac{q}{p}\right)$

(C) $P=\frac{1}{2}\left(p^2+q^2\right),\ Q=\tan^{-1}\left(\frac{q}{p}\right)$

(D) $P=-\frac{1}{2}\left(p^2+q^2\right),\ Q=\tan^{-1}\left(\frac{p}{q}\right)$

91. The value of α and β to be equations $Q=q^{\alpha}\cos\beta p,\ P=q^{\alpha}\sin\beta p$ represent a canonical transformation:

(A) $\alpha=2, \beta=y_2$

(B) $\alpha=\frac{1}{2}, \beta=2$

(C) $\alpha=y_2, \beta=y_2$

(D) $\alpha=2, \beta=1$

92. The transformation $Q=\log\left(\frac{1}{q}\sin p\right),\ p=q\,cot\,p$ is canonical then generating function is:

(A) $F=\left(p+\cot p\right)q$

(B) $F=\left(p-\cot p\right)q$

(C) $F=\left(p^2+\cot p\right)q$

(D) None of the above

93. The transformation Q = aq + bp, P = cq + dp is canonical if :

(A) (ad – bc) $\neq$ 1

(B) (ad – bc) = 1

(C) –ad –bc = 0

(D) $\left(\frac{a}{d}-\frac{b}{c}\right)=1$

94. The curve along which the integral $I\left[y(x)\right]=\int_0^{\frac{\pi}{2}}\left(x^2-y^2+y'^2\right)dx$ with y(0) = 1, $y\left(\frac{\pi}{2}\right)=0$ is :

(A) $y(x) = \sin x$

(B) $y(x) = \sin x + \cos x$

(C) $y(x) = \cos x$

(D) $y(x) = \sin x - \cos x$

95. The extremals of the functional $I\left[y(x)\right]=\int_a^b\left(y+xy'\right)dx$, $y(a) = y_0$, $y(b) = y_1$ is :

(A) family of straight line

(B) family of circles

(C) family of catenaries

(D) no extremal exists

96. The extremal of the functional $I[y(x)] = \int_{x_0}^{x_1} y^2 dx$, $y(x_0) = y_0$, $y(x_1) = y_1$ is :

(A) y - axis

(B) Straight line

(C) No extremal exists

(D) None of the above

97. The extremum of the functional $I[y(x)] = \int_{o}^{a} (y')^3 dx$, $y\ (0) = 0$, $y(0) = b$:

(A) $y = \frac{b}{a}x$

(B) $y = \frac{a}{b}x$

(C) $y = \frac{b}{a}x^2$

(D) None of the above

98. The extremum of the functional $I[y(x)] = \int_{1}^{2} \frac{x^3}{(y')^2} dx$, $y(1) = 1$, $y(2) = 4$ is :

(A) weak minimum exists along $y = x^3$

(B) weak minimum exists along $y = x^2$

(C) Strong minimum exists along $y = x^2$

(D) Strong maximum exists along $y = x^2$

99. The extremal of the functional $I[y(x)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \right] dxdydz$ is :

(A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(B) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(D) None of the above

100. In calcul of variation gives method to determine maxima or minima of some mathematical terms known as :

(A) functions

(B) admissible functions

(C) functionals

(D) general relativity

Rough Work

Example :

Question :

Q.1 Ⓐ ● Ⓒ Ⓓ

Q.2 Ⓐ Ⓑ ● Ⓓ

Q.3 Ⓐ ● Ⓒ Ⓓ

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.

6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.

7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.

8. After the completion of the examination, candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.

9. There will be no negative marking.

10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.

11. To bring and use of log-book, calculator, pager & cellular phone in examination hall is prohibited.

12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ

प्रश्न 2 Ⓐ Ⓑ ● Ⓓ

प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।

6. सभी उत्तर केवल ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।

7. ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।

8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।

9. निगेटिव मार्किंग नहीं है।

10. कोई भी रफ कार्य, प्रश्न-पुस्तिका में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।

11. परीक्षा-कक्ष में लॉग-बुक, कैल्कुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।

12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्णः प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्नपुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्नपुस्तिका प्राप्त कर लें।