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O.M.R. Serial No. | | | | | | | | |

Question Booklet Number

M.A./M.Sc. (SEM.-IV)(NEP) (SUPPLE.) EXAMINATION, 2024-25
MATHEMATICS
(Integral Equations & Boundary Value Problems)

Paper Code							
B	0	3	1	0	0	2	T

Question Booklet Series
A

Time : 1 : 30 Hours **Max. Marks : 75**

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.
4. Four alternative answers are mentioned for each question as - A, B, C & D in the booklet. The candidate has to choose the correct / answer and mark the same in the OMR Answer-Sheet as per the direction :

(Remaining instructions on last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, उसे तुरन्त बदल लें।
4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर- A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR उत्तर-पत्रक में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

(शेष निर्देश अन्तिम पृष्ठ पर)

1. Equation in which an unknown function appears under the integral sign is called as :

(A) Gauss integral equation

(B) Partial integral equation

(C) Ordinary integral equation

(D) An integral equation

2. Which of the following is not a Symmetric Kernel ?

(A) $K(x,t) = xt + x^2t^2 + x^3t^3$

(B) $K(x,t) = \sin(x^2 + t^2 + xt)$

(C) $K(x,t) = xt^2 + x^2t + x^3t^2$

(D) $K(x,t) = \begin{cases} x(t^2-1) & , \quad x < t \\ (x^2-1)t & , \quad x > t \end{cases}$

3. The kernel K(x,t) is separable or degenerate if :

(A) $K(x,t) = \sum_{r=1}^{n} a_r(x)\, b_r(t)$

(B) $K(x,t) = \sum_{r=1}^{\infty} a_r(x)\, b_r(t)$

(C) $K(x,t) = \sum_{r=1}^{n} a_r(x,t)\, b_r(x,t)$

(D) $K(x,t) = \sum_{r=1}^{\infty} a_r(x,t)\, b_r(x,t)$

4. Which of the following is not a linear integral equation ?

(A) $\phi(x) = x + \int_0^x \sin(x-t)^2 \;\; \phi(t)\, dt$

(B) $\phi(x) = x^2 + \int_0^x \sin\mathrm{h}(x+t) \;\; \phi(t)\, dt$

(C) $\phi(x) = x - \int_0^x (x-t)^2 \;\; \phi(t)\, dt$

(D) $\phi(x) = x + \int_0^x e^{(x-t)} \;\; \phi^2(t)\, dt$

5. If the derivative of the unknown function are involved in the integral of that function, then it is called a/an :

(A) Integral differential equation

(B) Integro-differential equation

(C) Non-linear integral equation

(D) None of these

6. In linear integral equation :

$$\phi(x) = f(x) + \lambda \int_a^b K(x,t)\, \phi(t)\, dt$$

if a and b are constants. The equation is known as :

(A) Volterra integral equation

(B) Abel's integral equation

(C) Fredholm integral equation

(D) Green's integral equation

7. Which of the following is the Singular integral equation ?

(A) $\phi(x) = f(x) + \lambda \int_0^x \sin(x-t)\, \phi(t)\, dt$

(B) $\phi(x) = \int_0^x \frac{1}{(x-t)^{\alpha}} \;\; \phi(t)\, dt, \; 0 < \alpha < 1$

(C) $\phi(x) = \int_0^x (x-t)^{\alpha} \;\; \phi(t)\, dt, \; 0 < \alpha < 1$

(D) $\phi(x) = f(x) + \lambda \int_0^x e^{-|x-t|} \;\; \phi(t)\, dt$

8. "Integral equation" for any equation involving the unknown function $\phi(x)$ under the integral sign was first coined by :

(A) Du Bois Reymond

(B) Laplace

(C) Heumann

(D) Ivar Fredholm

9. A integral equation in which the Kernel K(x,t) can be written in the form K(x,t) = (x-t) is known as :

(A) Abel's Type integral equation

(B) Fox Type integral equation

(C) Singular Type integral equaition

(D) Convolution Type integral equation

10. Which one is known as Hilbert Kernel?

(A) $K(x,t) = \text{cosec}\dfrac{(t-x)}{2}$, x and t are real variable

(B) $K(x,t) = \cot\dfrac{(t-x)}{2}$, x and t are real variable

(C) $K(x,t) = \cos\dfrac{(t-x)}{2}$, x and t are real variable

(D) $K(x,t) = \sin\dfrac{(t-x)}{2}$, x and t are real variable

11. Solution of initial value problem :

$y'' = 0, 0 \le x \le 1$

$y(0) = c_0$, $y'(0) = c_1$

may be reduced in general solution of some linear :

(A) Homogeneous Fredholm integral equation of Ist kind

(B) Non-homogeneous fredholm integral equation of IInd kind

(C) Homogeneous volterra integral equation of IInd kind

(D) Non-homogeneous volterra integral equation of Ist kind

12. If P and Q are absolute constants, then the value of $\dfrac{d}{dx}\int_P^Q F(x,t)dt$ is :

(A) $\int_P^Q \dfrac{\partial F}{\partial x}(x,t)dt$

(B) $\int_P^Q \dfrac{\partial F}{\partial x}(x,t)dt + F[x,P]\dfrac{dP}{dx} - F[x,Q]\dfrac{dQ}{dx}$

(C) $\int_P^Q \dfrac{\partial F(x,t)}{\partial x}dt + F[x,q]\dfrac{dQ}{dx} - F[x,P]\dfrac{dP}{dx}$

(D) None of these

13. A linear integral equation of the form

$\alpha(x)\phi x = f(x) + \lambda\int_a^b K(x,t)\phi(t)dt$

the unknown function of the above integral equation?

(A) $\alpha(x)$

(B) $\phi(x)$

(C) f(x) and K(x, t)

(D) Both $\alpha(x)$ and $\phi(x)$

14. The solution of the integral equation $\int_0^x (x-t)^2 \phi(t)dt = x^3$ is :

(A) $\phi(x)=1$

(B) $\phi(x)=2$

(C) $\phi(x)=3$

(D) $\phi(x)=\frac{1}{(\pi\sqrt{x})}$

15. Which of the following is solution of the integral equation?

$\int_0^x e^{x-t}\phi(t)dt = x$

(A) $\phi(x)=x$

(B) $\phi(x)=1+x$

(C) $\phi(x)=1-x$

(D) $\phi(x)=1-x^2$

16. Integral equation corresponding to differential equation $\frac{dy}{dx}-y=0$, y(0) = 1 is :

(A) y(x) = $1-\int_0^x y(t)dt$

(B) y(x) = $1+\int_0^x y(t)dt$

(C) y(x) = $2+\int_0^x y(t)dt$

(D) y(x) = $1+\int_0^x (x-t)y(t)dt$

17. The resolvent kernel of the integral equation $\phi(x)=x+\int_0^x \phi(t)dt$ is ?

(A) $e^{2(x-t)}$

(B) e^{x-t}

(C) $e^{3(x-t)}$

(D) $e^{\lambda(x-t)}$

18. For the linear integral equation $\phi(x)=x+\int_0^{1/2}\phi(t)dt$ the resolvent kernel R(x, t; 1) is :

(A) 1

(B) 1/2

(C) 3/2

(D) 2

19. Neumann series method is used on solving Volterra integral equation of second kind when :

(A) Iterated Kernel is closed form

(B) Resolvent Kernel is not closed form

(C) Resolvent Kernel is closed form

(D) Iterated Kernel is not closed form

20. The resolvent Kernel of the Kernel $K(x,\xi)=\sin(x-2\xi)$, $0\le x\le 2\pi$ and $0\le\xi\le 2\pi$ is :

(A) $\sin(x+2\xi)$

(B) $\sin(x-\xi)$

(C) $\sin(x-2\xi)$

(D) $\cos(x-2\xi)$

21. An integral equation corresponding to the differential equation given by : y'' + y = cosx with y(0) = 0, y'(0) =1 is:

(A) $(\cos x - x) - \int_0^x (x-t)\phi(t)dt = \phi(x)$

(B) $(\cos x - x) + \int_0^x (x-t)\phi(t)dt = \phi(x)$

(C) $(\cos x + x) - \int_0^x (x-t)\phi(t)dt = \phi(x)$

(D) None of these

22. The resolvent kernel of integral equation with kernel K(x,t) = 2x, (λ=1) is :

(A) $e^{x^2-t^2}$

(B) $2\, e^{x^2-t^2}$

(C) $2x\, e^{x^2-t^2}$

(D) $2(x-t)\, e^{x^2-t^2}$

23. The value of λ for which the determinant of homogeneous integral equation is zero are called :

(A) Eigenfunctions

(B) Trivial solutions

(C) Eigenvalues

(D) Unique solutions

24. Two $f_0(x)$ and $g_0(x)$, continuous functions on an closed interval of a and b are said to be orthogonal on closed interval of a and b if :

(A) $\int_a^b f_0(x).g_0(x)dx = 0$

(B) $\int_a^b f_0(x).g_0(x)dx = \frac{\pi}{2}$

(C) $\int_a^b f_0(x).g_0(x)dx = 1$

(D) $\int_a^b f_0(x).g_0(x)dx \neq \frac{\pi}{2}$

25. Any non-trivial solution of the homogeous integral equation for a definite value of λ is called :

(A) Eigenvalue

(B) Orthogonal solutions

(C) Unique solutions

(D) None of thse

26. The eigenvalues of a kernel if the type K(x, t) = $\overline{K(x,t)}$ are :

(A) Purely real

(B) Purely imaginary

(C) Either real or complex

(D) none of these

27. The eigenvalues of the homogeneous integral equation

$\phi(x) = \lambda \int_0^1 \sin \pi x \cos \pi t\, \phi(t)\, dt$ are :

(A) $\lambda = \pm \frac{1}{\pi}$

(B) $\lambda = \frac{2}{e^2 - 1}$

(C) $\lambda = \pm \pi$

(D) No eigenvalues exists

28. The solution of integral equation

$$\phi(x) = \cos x + \lambda \int_0^x \sin x \ \phi(t)\, dt \text{ is :}$$

(A) $\phi(x) =$ sin x, if $\lambda \neq 1/2$

(B) $\phi(x) =$ cos x, if $\lambda \neq 1/2$

(C) $\phi(x) =$ tan x, $\lambda \neq 1/2$

(D) $\phi(x) =$ cos x + sin x, if $\lambda \neq 1/2$

29. The integral equation

$$\phi(x) = F(x) + \lambda \int_0^1 (1-3xt) \ \phi(t)\, dt$$

exists unique solution if and only if :

(A) $\lambda \neq \pm 3$

(B) $\lambda \neq \pm 1$

(C) $\lambda \neq 0$

(D) $\lambda \neq \pm 2$

30. The eigenfunction of a symmetric Kernel corresponding to different eigenvalues are :

(A) real

(B) Linearly independent

(C) Orthogonal

(D) Linearly dependent

31. The eigenfunctions of a eigenvalues λ then eigenfunction correspond to same eigenvalue (λ) can be expressed as :

(A) Arbitary multiples of eigenfunction

(B) Sum of eigenfunctions

(C) Difference of eigenfunctions

(D) None of these

32. A function f(x) is said to be square integrable if :

(A) $\int_a^b |f(x)|\, dx < \infty$

(B) $\int_a^b |f(x)|^2\, dx = \infty$

(C) $\int_a^b |f(x)|^2\, dx < \infty$

(D) None of these

33. The integral operator :

$$T = \int_a^b K(x,t)\, dt$$

is said to be self adjoint, if it satisfies the condition :

(A) (Tf, g) = (T, fg)

(B) (Tf, g) = (f, Tg)

(C) (Tf, g) = (fg, T)

(D) (Tf, g) = (f, T+g)

34. If f(x) and g(x) be any two functions in a Hilbert space (H), then Schwarz inequality is :

(A) $\| \langle f,g \rangle \| \leq \|f\| \cdot \|g\|$

(B) $\| \langle f,g \rangle \| \leq |f| \cdot |g|$

(C) $| \langle f,g \rangle | \leq \|f\| \cdot \|g\|$

(D) $| \langle f,g \rangle |^2 \leq \|f\| \cdot \|g\|$

35. All iterated Kernels of a symmetric Kernal are :

(A) Skew symmetric

(B) Hermition

(C) Symmetric

(D) Symmetric or Skew symmetric

36. The multiplicity of any non-zero eigenvalue is finite for every symmetric Kernel for which $\int_a^b \int_a^b |K(x,t)|^2 \, dx \, dt$ is :

(A) Finite

(B) Infinite

(C) Can't be determinded absolutely

(D) None of these

37. Every symmetric kernel with a norm not equal to zero has at least :

(A) one eigenvalue

(B) two eigenvalues

(C) no eigenvalue exist

(D) none of these

38. Hilbert-Schmidt the solution of a symmetric non-homogeneous Fredholm integral equation of second kind having no solution is :

(A) $\lambda \neq \lambda K; \int_a^b f(K) \, \phi_K(x) \, dx \neq 0$

(B) $\lambda = \lambda K; \int_a^b f(x) \, \phi_K(x) \, dx \neq 0$

(C) $\lambda = \lambda K; \int_a^b f(x) \, \phi_K(x) \, dx = 0$

(D) $\lambda \neq \lambda K; \int_a^b f(x) \, \phi_K(x) \, dx = 0$

39. For the homogeneous fredholm integral equation

$$f(x) = \lambda \int_0^1 e^{x+t} f(t) \, dt$$

a non-zero solution exists, when λ has the value :

(A) $\lambda = \frac{2}{e^2 - 1}$

(B) $\lambda = \frac{1}{e^2 - 1}$

(C) $\lambda = \frac{2e}{e^2 - 1}$

(D) $\lambda = \frac{2}{e^2 + 1}$

40. A function $\phi(x)$ is said to be normalised if :

(A) $\|\phi(x)\| = 0$

(B) $\|\phi(x)\| = 1$

(C) $\|\phi(x)\| = \infty$

(D) $\|\phi(x)\| = 0$ or 1

41. The eigen values of a symmetric Kernel form a finite or infinite sequence $\langle\lambda n\rangle$ has :

(A) finite limit point

(B) no finite limit point

(C) either finite or infinite limit point exists

(D) zero is the only limit point

42. For Fredholm integral equation of second kind

$$\phi(x) = f(x) + \lambda \int_a^b K(x,t)\phi(t)\,dt$$

will have a unique continuous :

(A) Kernel is bounded (K) and $|\lambda| = \frac{1}{K(a-b)}$

(B) Kernel is bounded (K) and $|\lambda| < \frac{1}{K(a-b)}$

(C) Kernel is bounded (K) and $|\lambda| > \frac{1}{K(a-b)}$

(D) None of these

43. The nth iterated Kernal Kn(x, t) satisfies the relation

$$Kn(x,t) = \int_a^b K_m(x,y) \cdot K_n - m(y,t)\,dy$$

then :

(A) m is any positive integer less than n

(B) m is any positive integer greater than n

(C) m is any positive integer less than any p

(D) p is any positive integr less than m

44. The iterated Kernel of the Kernel K(x,t) = sin(x-t) , $0 \le x \le 2\pi$, $0 \le t \le 2\pi$:

(A) sin (x - 2t)

(B) cos (x - 2t)

(C) 2 sin (x - 2t)

(D) 0

45. Solution of the integral equation $\phi(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\,\phi(t)\,dt$ is :

(A) $\phi(x) = \frac{5x}{6}$

(B) $\phi(x) = x$

(C) $\phi(x) = \frac{2x}{3}$

(D) $\phi(x) = 0$

46. Solution of the integral equation

$$\phi(x) = (1+x) - \int_0^x x\,\phi(t)\,dt \text{ , } \phi_0(x) = 1$$

is :

(A) $\phi(x) = 0$

(B) $\phi(x) = 1$

(C) $\phi(x) = 2$

(D) $\phi(x) = 3$

47. Solution of the integral equation

$$\phi(x) = 2x + 2 - \int_0^x x\,\phi(t)\,dt \text{ , } \phi_0(x) = 2$$

is :

(A) $\phi(x) = 0$

(B) $\phi(x) = 1$

(C) $\phi(x) = 2$

(D) $\phi(x) = e^{x}$

48. The Volterra's integral of first kind can be converted into a :

(A) Homogeneous Volterra's integral equation of IInd kind

(B) Homogeneous Fredholm integral equation of IInd kind

(C) Non-homogeneous Volterra's integral equaition of IInd kind

(D) None of these

49. The solution of non-zero Volterra's integral equation of 2nd kind :

$$\phi(x) = f(x) + \lambda \int_0^x K(x,t)\, \phi(t)\, dt \text{ is :}$$

(A) $\phi(x) = f(x) + \lambda \int_0^x R(x,t;\lambda)\, \phi(t)\, dt$

(B) $\phi(x) = f(x) + \lambda \int_0^x R(x,t,\lambda)\, f(t)\, dt$

(C) $\phi(x) = f(x) + \lambda \int_0^x R(x,t;\lambda)\, f(t)\phi(t)\, dt$

(D) None of these

50. Solution of Volterra's integral equation of first kind $\int_0^x e^{x-t}\phi(t)\, dt = \sin x$:

(A) $\phi(x) =$ sinx

(B) $\phi(x) =$ cosx

(C) $\phi(x) =$ sinx – cos x

(D) $\phi(x) =$ cosx– sinx

51. The convrgence of Fredholm's determinant D(λ) is :

(A) Absolutely and uniformly converging power series in λ

(B) Conditionally and uniformly converging power series in λ

(C) Absolutely and diverging power series in λ

(D) None of these

52. Which one is the solution of the homogenous integral equation when $D(\lambda_0) = 0$, $D(\lambda_0) \neq 0$?

(A) $\dfrac{D(x, y_0, \lambda_0)}{D(\lambda_0)}$

(B) $\dfrac{D(\lambda_0)}{D(x, y_0, \lambda_0)}$

(C) $D(\lambda_0) - D(x, y_0, \lambda_0)$

(D) $D(x, y_0, \lambda_0)$

53. The Kernel $K(t,x)$ and $K(x,t)$ have the same :

(A) Characteristic constant

(B) Characteristic vectors

(C) Characteristics function

(D) Characteristic solution

54. If λ_0 is eigenvalue of $K(x,t)$ of index q and λ_0 is a eivenvalue of $K(t,x)$ of index $\overline{q}$ then :

(A) $q > \overline{q}$

(B) $q = \overline{q}$

(C) $q < \overline{q}$

(D) Either $q < \overline{q}$ or $q > \overline{q}$

55. If Kernel $k(x,t)=1$, the Fredholm determinant $D(\lambda)$ is given by :

(A) λ

(B) $1-\lambda, \quad \lambda \neq 1$

(C) $\frac{\lambda}{1-\lambda}, \quad \lambda \neq 1$

(D) $\frac{\lambda}{1+\lambda}, \quad \lambda \neq -1$

56. If Kernel $K(x, t) = \sin x$, a = 0, $b=\pi$, then the value of $D(x,t,\lambda)$ is :

(A) $1-2\lambda, \quad \lambda \neq 1/2$

(B) $\sin x$

(C) $1-\lambda, \quad \lambda \neq 1$

(D) $2\sin x$

57. The Fredholm homogenus integral equation $\phi(x)=\lambda\int_a^b K(x,t)\phi(t)dt$ has one and only one continuous solution given by $\phi(x)=0$, if :

(A) $D(\lambda)\neq 0$

(B) $D(\lambda)=1$

(C) $D(\lambda)=0$

(D) $D(\lambda)=\infty$

58. Resolvent Kernel of the integral equation $\phi(x)=\sec^2 x+\lambda\int_0^1 \phi(t)dt$ is given by :

(A) $\frac{\lambda}{1+\lambda}, \quad \lambda \neq -1$

(B) $\frac{\lambda}{1-\lambda}, \quad \lambda \neq 1$

(C) $\frac{\lambda}{2+\lambda}, \quad \lambda \neq -2$

(D) $\frac{\lambda}{2-\lambda}, \quad \lambda \neq 2$

59. The integral equation given by $f(x)=\int_0^x \frac{\phi(t)dt}{(x-t)^\alpha}, \quad 0<\alpha<1$ is known as :

(A) Volterra integral equation

(B) Abel's integral equation

(C) Dirac integral equation

(D) Fox integral equation

60. An integral equation in which Kernel is unbounded but the integral of the square of its modulus exists,then the Kernel is called as :

(A) Abel's Singular Kernel

(B) Hilbert Type Singular Kernel

(C) Strongly Singular Kernel

(D) Weakly Singular Kernel

61. If there exist constants M and μ, $0<\mu\geq 1$ such that for every pair of points x_1, x_2 lying in the range $a\leq x\leq b$. The condition $|f(x_1)-f(x_2)|<M|x_1-x_2|^\mu$ is called :

(A) Lipschitz's conditions

(B) Holder's conditions

(C) Weakly singular conditions

(D) Plemeli's formula

62. The solution of singular integral equation $f(x) = \int_0^x \frac{\phi(t)dt}{(x-t)^{1/2}}$ is :

(A) $\frac{\pi}{2} t^{1/2}$

(B) $\frac{3\pi}{2} t^{1/2}$

(C) $\frac{2}{\pi} t^{1/2}$

(D) $\frac{1}{\pi} t^{1/2}$

63. The characteristic values λ_0 and λ_n of the Kernel k and k_n of singular integral equation are :

(A) Same eigenvalue

(B) Same determinant

(C) Same rank

(D) Same trace

64. The solution of integral equation $\sin x = \int_0^x J_0(x-t)\phi(t)dt$ is :

(A) $2J_0(x)$

(B) $J_0(x)$

(C) $\pm 8J_0(x)$

(D) $\pm uJ_0(x)$

65. Solution of the integral equation of first kind $\int_0^x \phi(t)\phi(x-t)dt = 4\sin 4x$ is :

(A) $\pm J_0(4x)$

(B) $\pm 2J_0(4x)$

(C) $\pm 4J_0(4x)$

(D) $\pm 8J_0(4x)$

66. Solution of the integral equation $\phi(x) = a\sin x - 2\int_0^1 \cos(x-t)\phi(t)dt$ is :

(A) axe^{-x}

(B) axe^{x}

(C) ax^2e^{-x}

(D) axe^{-2x}

67. The integral equation given by $f(x) = \int_{-\infty}^{\infty} k(x-t)\phi(t)dt$ is known as:

(A) Weiner-Hopf integral equation

(B) Fox integral equation

(C) Mellin transform equation

(D) Heaviside intnegral equation

68. Solution of integral equation $\int_0^\infty F(t)\cos pt = e^{-p}$ is :

(A) $\frac{2}{\pi(1-t^2)}$

(B) $\frac{2}{\pi(1+t)}$

(C) $\frac{\pi}{2(1+t^2)}$

(D) $\frac{2}{\pi(1+t^2)}$

69. The solution of integral $\int_0^2 e^{4t}\delta(2t-3)dt$ is :

(A) $\frac{e^6}{3}$

(B) $\frac{e^6}{6}$

(C) $\frac{e^6}{2}$

(D) $\frac{e^6}{4}$

70. Evaluate the integral $\int_{-\infty}^{\infty}\frac{\cos x\delta(x)}{2e^x+1}dx$ is :

(A) 5

(B) 1/5

(C) 3

(D) 1/3

71. Dirac-delta function is also known as:

(A) Unit step function

(B) Unit impulse function

(C) Unit modulus function

(D) Unit even function

72. Dirac-delta function $\delta(x)$ is a/an :

(A) even function

(B) odd function

(C) Both even and odd function

(D) prime function

73. The derivative property of Dirac-delta function is :

(A) $\int_{-\infty}^{\infty} f(x)\delta'(x)dx = f(0)$

(B) $\int_{-\infty}^{\infty} f(x)\delta'(x)dx = f'(0)$

(C) $\int_{-\infty}^{\infty} f(x)\delta'(x)dx = -f'(0)$

(D) None of the above

74. If a real Kernel $k(x, t)$ has a Complex eigenvalue $\lambda_0 = \alpha + i\beta$, then the entire function $D(\lambda)$ follows :

(A) If $D(\alpha+i\beta)=0$, then $D(\alpha-i\beta)\neq 0$

(B) If $D(\alpha+i\beta)\neq 0$, then $D(\alpha-i\beta)=0$

(C) If $D(\alpha+i\beta)\neq 0$, then $D(\alpha-i\beta)\neq 0$

(D) If $D(\alpha+i\beta)=0$, then $D(\alpha-i\beta)=0$

75. Order of a pole of the Resolvent Kernel $R(x,t;\lambda)$ is :

(A) at most equal to the order of zero of the $D(\lambda)$

(B) equal to identity

(C) equal to the order of the zero of the $D(\lambda)$

(D) None of the above

76. The Green's function of the boundary value problem $u''+xu=1$, $u(0)=u(1)=0$ is :

(A) $G(x,t)=\begin{cases} x(1-t); & 0\le x<t \\ t(1-x); & t<x\le t \end{cases}$

(B) $G(x,t)=\begin{cases} x(1+t); & 0\le x<t \\ t(1+x); & t<x\le t \end{cases}$

(C) $G(x,t)=\begin{cases} \left(1-\dfrac{2t}{x}\right)\pi; & 0\le x<t \\ \left(1-\dfrac{2x}{t}\right)\pi; & t<x\le t \end{cases}$

(D) None of the above

77. The use of Green's function enables the conservation of a self-adjoint eigenvalue problem to :

(A) a Volterra integral equation of first kind

(B) a Fredholm integral equation of first kind

(C) a Volterra integral equation of second kind

(D) a Fredholm integral equation of second kind

78. The Green's function of the boundary value problem $u''+u=x$, $u(0)=0$, $u'(1)=0$ is :

(A) $G(x,t)=\begin{cases} x; & 0\le x<t \\ t; & t<x\le 1 \end{cases}$

(B) $G(x,t)=\begin{cases} -\alpha-\log t; & 0\le x<t \\ -\alpha-\log x; & t<x\le 1 \end{cases}$

(C) $G(x,t)=\begin{cases} e^{x}+e^{-x}; & 0\le x<t \\ e^{t}+e^{-t}; & t<x\le 1 \end{cases}$

(D) None of the above

79. The modified Green's function exists only for those homogeneous linear differential equation possesses :

(A) zero solution

(B) non-zero solution

(C) either zero or non-zero solution

(D) Both zero and non-zero solution

80. The modified Green's function $G_m(x, t)$ is symmetric function if :

(A) $G_m(x, t)$ is not orthogonal to the eigenfunction $\phi_0(x)$

(B) $G_m(x, t)$ is orthogonal to the eigenfunction $\phi_0(x)$

(C) $G_m(x, t)$ is not normalized to the eigenfunction $\phi_0(x)$

(D) None of the above

81. The solution of the integral equation

$\phi_0'(x) = x + \int_0^x \cos t + \phi(x-t)dt$,

$\phi(0) = 4$ is :

(A) $\phi(x) = \frac{1}{2}x^2$

(B) $\phi(x) = \frac{5}{2}x^2 + \frac{x^4}{24} + 4$

(C) $\phi(x) = \frac{5}{2}x^2 + \frac{x^4}{12} + 4$

(D) $\phi(x) = 5x^2 + \frac{x^4}{24} + 4$

82. Solution of the integral equation

$f(x) = \int_0^x \frac{\phi(t)dt}{\sqrt{x-t}}$ is :

(A) $\phi(x) = \frac{2}{\pi}\frac{d}{dx}\int_0^x \frac{f(t)dt}{\sqrt{x-t}}$

(B) $\phi(x) = \frac{1}{2\pi}\frac{d}{dx}\int_0^x \frac{f(t)dt}{\sqrt{x-t}}$

(C) $\phi(x) = \frac{1}{\pi}\frac{d}{dx}\int_0^x \frac{f(t)dt}{\sqrt{x-t}}$

(D) None of the above

83. The Mellin transform of function $\phi(x)$ is :

(A) $\Phi(x) = \int_0^\infty x^{u-1}\phi(x)dx$

(B) $\Phi(x) = \int_{-\infty}^\infty x^{u-1}\phi(x)dx$

(C) $\Phi(u) = \int_{-\infty}^\infty x^{u-1}\phi(x)dx$

(D) $\Phi(u) = \int_0^\infty x^{u-1}\phi(x)dx$

84. The solution of integral equation

$\phi(x) = c\sin x - 2\int_0^1 \cos(x-\xi)\phi(\xi)d\xi$

(A) $\phi(x) = cxe^{-x}$

(B) $\phi(x) = cxe^{x}$

(C) $\phi(x) = cxe^{-2x}$

(D) $\phi(x) = cx^2e^{-x}$

85. If $\phi(x) = x + x\int_0^1 t\,\phi(t)\,dt$ then $\phi(x)$ at x = 2 is :

(A) 1

(B) 3/4

(C) 3/2

(D) 3

86. The integral equation

$\phi(x) = 1 + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\phi(t)\,dt$

has :

(A) Infinite solution

(B) No solution

(C) Unique solution

(D) None of these

87. Integral equation

$\phi(x) = \lambda\int_a^b K(x,t)\,\phi(t)\,dt$ where

$K(x,t) = \sum_{r=1}^{n} a_r(x)b_r(t)$ has :

(A) At least n- eigenvalues

(B) Exactly n-eigenvalues

(C) Atmost n-eigenvalues

(D) None of these

88. The value of α for which the integral equation $\phi(x) = \alpha\int_0^1 e^{x-t}\,\phi(t)\,dt$ has trivial solution :

(A) -2

(B) -1

(C) 1

(D) 2

89. Which one of the following is true ?

(A) $\int_a^b D(x,x;\lambda) = -\frac{1}{\lambda}D'(\lambda)$

(B) $\int_a^b D(x,x;\lambda) = -\lambda D'(\lambda)$

(C) $\int_a^b D(x,x;\lambda) = \frac{1}{\lambda}D'(\lambda)$

(D) $\int_a^b D(x,x;\lambda) = \lambda D'(\lambda)$

90. The solution of integral equation $\phi(x) = x + \int_0^1 xt^2\phi(t)\,dt$ is :

(A) $\phi(x) = \frac{2x}{3}$

(B) $\phi(x) = \frac{3x}{2}$

(C) $\phi(x) = \frac{3x}{4}$

(D) $\phi(x) = \frac{4x}{3}$

91. Given Kernel K(x,t) = xt; a = 0, b = 10 then Fredholm minor D(x,t;λ) is :

(A) $1-\frac{10^3}{3}\lambda$

(B) xt

(C) $1+\frac{10^3}{3}\lambda$

(D) x + cos t

92. Solution of integral equation:

$\phi x = \lambda\int_a^b K(x,t)\phi(t)\,dt + F(x)$ is :

(A) $\phi x = \lambda + \sum_{n=1}^{m} C_n f_n(x) + F(x)$

(B) $\phi x = \lambda\sum_{n=1}^{m} C_n f_n(x) + F'(x)$

(C) $\phi x = \lambda\sum_{n=1}^{m} C_n f_n(x) + F(x)$

(D) $\phi x = \lambda + \sum_{n=1}^{m} f_n(x) + C_n$

93. If a real Kernel K(x, t) has a complex eigenvalue $\lambda_0 = a + ib$, then its another eigenvalue is equal to :

(A) $\lambda_1 = 2a + ib$

(B) $\lambda_1 = a - 2ib$

(C) $\lambda_1 = 2a - ib$

(D) $\lambda_1 = a - ib$

94. The Fourier transform of $\delta(t-a)$, a ≥ 0 is equal to :

(A) $\dfrac{e^{-iap}}{p}$

(B) e^{iap}

(C) e^{-iap}

(D) $\dfrac{e^{iap}}{p}$

95. Fourier transform of $\sin p_0 t$ is equal to :

(A) $\dfrac{\pi i}{2}\left[\delta(p+p_0)+\delta(p-p_0)\right]$

(B) $\dfrac{\pi i}{2}\left[\delta(p+p_0)-\delta(p-p_0)\right]$

(C) $\pi i\left[\delta(p+p_0)+\delta(p-p_0)\right]$

(D) $\pi i\left[\delta(p+p_0)-\delta(p-p_0)\right]$

96. The integral equation :

$f(x)=\int_0^\infty \sin(x,t)u(t)dt$ is :

(A) a singular integral equation of first kind

(B) a singular integral equation of inverse kind

(C) a singular integral equation of zero kind

(D) a singular integral equation of second kind

97. The Green's function exists only for those homogeneous linear differential equation possession :

(A) Trivial solution

(B) Non-trivial solution

(C) Both trivial and non-trivial solution

(D) None of these

98. The eigenvalue of the integral equation

$\phi(x)=\lambda\int_0^{2\pi}\sin(x+t)\,\phi(t)dt$ are :

(A) $\dfrac{1}{2\pi},\ -\dfrac{1}{2\pi}$

(B) $\pi,\ -\pi$

(C) $2\pi,\ -2\pi$

(D) $\dfrac{1}{\pi},\ -\dfrac{1}{\pi}$

99. The integral equation :

$\phi(t)=\sin t+\lambda\int_0^t (s^2t+e^{x^2+t^2})\phi(s)ds$,

$0\leq t\leq 1,\ \lambda\in R,\ \lambda\neq 0$:

(A) All non-zero value of λ

(B) No value of λ

(C) Only countable many positive value of λ

(D) Only countable many negative value of λ

100. Solution of the integral equation :

$3\sin 2x=\phi(x)+\int_0^x (x-t)\,\phi(t)\,dt$ is

(A) 2sinx – sin 2x

(B) sin x – sin 2x

(C) sin x – $\sin^2$x

(D) None of these

Rough Work

Rough Work

Example :

Question :

Q.1 Ⓐ ● Ⓒ Ⓓ

Q.2 Ⓐ Ⓑ ● Ⓓ

Q.3 Ⓐ ● Ⓒ Ⓓ

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.

6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.

7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.

8. After the completion of the examination, candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.

9. There will be no negative marking.

10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.

11. To bring and use of log-book, calculator, pager & cellular phone in examination hall is prohibited.

12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ

प्रश्न 2 Ⓐ Ⓑ ● Ⓓ

प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।

6. सभी उत्तर केवल ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।

7. ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।

8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।

9. निगेटिव मार्किंग नहीं है।

10. कोई भी रफ कार्य, प्रश्न-पुस्तिका में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।

11. परीक्षा-कक्ष में लॉग-बुक, कैल्कुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।

12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्णः प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्नपुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्नपुस्तिका प्राप्त कर लें।