Roll. No. ....

O.M.R. Serial No. | | | | | | | | |

Question Booklet Number

M.A./M.Sc. (SEM.-V)
(NEP) (SUPPLE.)EXAMINATION, 2024-25

MATHEMATICS

(Advanced Differential Equation)

Paper Code							
B	0	3	0	9	0	5	T

Question Booklet Series
A

Time : 1 : 30 Hours **Max. Marks : 75**

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.
4. Four alternative answers are mentioned for each question as - A, B, C & D in the booklet. The candidate has to choose the correct / answer and mark the same in the OMR Answer-Sheet as per the direction :

(Remaining instructions on last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, उसे तुरन्त बदल लें।
4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर- A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR उत्तर-पत्रक में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

(शेष निर्देश अन्तिम पृष्ठ पर)

1. The differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ has the auxiliary equation :

(A) $m^2 + 3m + 2 = 0$

(B) $m^2 - 3m + 2 = 0$

(C) $m^2 - 2m + 3 = 0$

(D) $m^2 + 2m - 3 = 0$

2. The equation $x^2y'' + xy' - y = 0$ is:

(A) Cauchy–Euler equation

(B) Exact equation

(C) Bernoulli equation

(D) Riccati equation

3. A second-order linear homogeneous equation always has:

(A) Exactly one solution

(B) Two linearly independent solutions

(C) Infinitely many unrelated solutions

(D) No general solution

4. The equation $y'' + p(x)y' + q(x)y = 0$ is linear with:

(A) Constant coefficients only

(B) Variable coefficients only

(C) Either constant or variable coefficients

(D) No coefficients

5. Two solutions y_1, y_2 of a linear ODE are linearly dependent if:

(A) $y_1 + y_2 = 0$

(B) One is a constant multiple of the other

(C) Both are nonzero everywhere

(D) Their Wronskian is nonzero

6. The Wronskian of $y_1 = e^x, y_2 = e^{-x}$ is:

(A) 0

(B) 1

(C) -2

(D) $-2e^0$

7. If the Wronskian of two functions is identically zero, then the functions are:

(A) Always linearly independent

(B) Always linearly dependent

(C) Sometimes dependent, sometimes independent

(D) Orthogonal

8. For $y_1 = x, y_2 = x^2$, the Wronskian is:

(A) 0

(B) x^2

(C) x^3

(D) x

9. The Wronskian is defined as:
(A) A determinant involving solutions and their derivatives
(B) Sum of solutions
(C) Product of solutions
(D) Ratio of solutions

10. For . $y''+y=\sin(x)$, the particular solution form is:
(A) $A\sin(x)+B\cos(x)$
(B) $Ax\sin(x)+Bx\cos(x)$
(C) Ae^{x}
(D) Ax^{2}

11. The method of undetermined coefficients is applicable when the RHS is:
(A) Arbitrary function
(B) Polynomial, exponential, sine, cosine (or combinations)
(C) Any discontinuous function
(D) A delta function

12. If the RHS term is already a solution of the homogeneous equation, the trial solution should be:
(A) Same form
(B) Multiplied by x
(C) Multiplied by x^2
(D) Excluded completely

13. Reduction of order is used to:
(A) Solve nonlinear equations
(B) Find a second solution when one solution is known
(C) Reduce order of coefficients
(D) Find integrating factor

14. If $y_1 = x$ is a known solution of a second-order ODE, the second solution is sought as:
(A) $y_2 = vy_1$
(B) $y_2 = y_1 + v$
(C) $y_2 = y_1^{2}$
(D) $y_2 = \ln(y_1)$

15. In reduction of order, substitution $y = v(x)y_1(x)$ reduces the equation to:
(A) First-order in y
(B) First-order in v
(C) Second-order in v
(D) Third-order in v

16. Reduction of order is generally applied to:
(A) First-order equations
(B) Second-order linear homogeneous equations
(C) Third-order equations only
(D) Partial differential equations

17. If $y_1 = e^x$ is a solution of $y'' - y' = 0$, the second solution using reduction of order is:

(A) $y_2 = e^{-x}$

(B) $y_2 = xe^x$

(C) $y_2 = e^x \ln(x)$

(D) $y_2 = x^2 e^x$

18. An initial value problem specifies:

(A) Only the differential equation

(B) The differential equation + boundary conditions at two points

(C) The differential equation + values of the solution at a single point

(D) Only coefficients of the equation

19. The IVP $y' = y, y(0) = 1$ has solution:

(A) $y = e^x$

(B) $y = e^{-x}$

(C) $y = x$

(D) $y = 1$

20. The IVP $y'' + y = 0, y(0) = 0, y'(0) = 1$ has solution:

(A) $\sin(x)$

(B) $\cos(x)$

(C) e^x

(D) $\sinh(x)$

21. Existence and uniqueness theorem guarantees solution if:

(A) Coefficients are continuous

(B) RHS is continuous and Lipschitz in y

(C) Initial condition is zero

(D) Equation is homogeneous

22. The IVP $y' = 2x, y(0) = 3$ gives:

(A) $y = x^2 + 3$

(B) $y = x^2 - 3$

(C) $y = 2x^2 + 3$

(D) $y = 2x + 3$

23. The equivalent integral equation ensures:

(A) Approximate solutions only

(B) Exact equivalence to IVP

(C) Only uniqueness, not existence

(D) Only existence, not uniqueness

24. Which method often uses equivalent integral equations for proofs?

(A) Laplace transforms

(B) Picard iteration

(C) Frobenius method

(D) Fourier transforms

25. If $y(x) = y_0 + \int_{x_0}^{x} f(t, y,(t))\,dt$ differentiating gives:

(A) $y' = 0$

(B) $y' = f(x, y)$

(C) $y' = y_0$

(D) $y' = y^2$

26. Picard's method is used for:

(A) Exact algebraic solution of ODEs

(B) Approximate successive solutions of IVPs

(C) Solving PDEs directly

(D) Reducing order of equations

27. The second Picard approximation for the same problem is:

(A) $1 + x + \frac{x^2}{2}$

(B) $1 + \frac{x^2}{2}$

(C) $1 + 2x$

(D) $1 + x^2$

28. Picard's iteration is based on the equivalent:

(A) Algebraic equation

(B) Integral equation

(C) Difference equation

(D) Polynomial approximation

29. Convergence of Picard iteration depends on:

(A) Continuity of $f(x, y)$ only

(B) Lipschitz condition in y

(C) Differentiability of f

(D) Existence of Green's function

30. The constant L in the Lipschitz condition is called:

(A) Integral constant

(B) Lipschitz constant

(C) Picard constant

(D) Bound constant

31. If $f(x, y) = y^2$, then f is:

(A) Globally Lipschitz in y

(B) Locally Lipschitz in y

(C) Not Lipschitz at all

(D) Lipschitz only for negative y

32. The Existence theorem requires:

(A) $f(x, y)$ continuous in a region around (x_0, y_0)

(B) f bounded only

(C) f differentiable everywhere

(D) f linear in y

33. The Uniqueness theorem requires:

(A) Continuity of f only

(B) Lipschitz condition in y

(C) Monotonicity of f

(D) Boundedness of f

34. Which theorem ensures both existence and uniqueness of IVP solutions?

(A) Cauchy's theorem

(B) Picard–Lindelöf theorem

(C) Taylor's theorem

(D) Sturm–Liouville theorem

35. Functions $\{\sin(nx)\}, n = 1, 2...$ are orthogonal on:

(A) $(0, \infty]$

(B) $[0, 2\pi]$

(C) $[0, 1]$

(D) $[-\infty, \infty]$

36. Orthogonality is defined with respect to:

(A) Differentiation

(B) Integration with a weight function

(C) Summation

(D) Limit process

37. The set $\{1, x, x^2\}$ is orthogonal on $[-1, 1]$:

(A) Always

(B) With respect to weight function $w(x) = 1$

(C) No, it is not orthogonal

(D) Only for odd powers

38. A boundary value problem (BVP) specifies conditions:

(A) At one point only

(B) At two or more points

(C) At infinity only

(D) Without any conditions

39. The problem $y'' + y = 0, y(\pi) = 0,$ $y(0 = 0)$ is a:

(A) Boundary value problem

(B) Initial value problem

(C) Sturm–Liouville problem

(D) Integral equation

40. Boundary value problems often arise in :

(A) Geometry

(B) Fourier series expansions

(C) Physics (heat, wave equations)

(D) All of the above

41. A Sturm–Liouville problem has the form:

(A) $(p(x)y')' + q(x)y + \lambda w(x)y = 0$

(B) $y' + p(x)y = q(x)$

(C) $y'' + y = 0$

(D) $y'' + p(x) = 0$

42. Eigenfunctions of a Sturm–Liouville problem form:

(A) A linearly dependent set

(B) An orthogonal set

(C) A redundant set

(D) A divergent set

43. The eigenvalues of a Sturm–Liouville problem are usually:

(A) Complex

(B) Real

(C) Arbitrary

(D) Zero only

44. Green's function is used to solve:

(A) Nonlinear IVPs

(B) Linear inhomogeneous BVPs

(C) Picard iterations

(D) Existence theorems

45. The Green's function represents:

(A) A kernel for an integral operator

(B) A Fourier transform

(C) A Laplace transform

(D) A polynomial basis

46. If $Ly = f(x)$, then the solution using Green's function is:

(A) $y(x) = \int G(x,t) f(t)\,dt$

(B) $y(x) = f(t)\,dt$

(C) $y(x) = G(x)$

(D) $y(x) = f(x)$

47. The Ascoli–Arzelà theorem provides conditions for:

(A) Orthogonality of functions

(B) Compactness in C [a, b]

(C) Lipschitz continuity

(D) Picard iteration convergence

48. The theorem states that a family of functions is relatively compact if:

(A) It is bounded and Equi continuous

(B) It is orthogonal

(C) It is differentiable

(D) It satisfies Lipschitz condition

49. A first-order system of linear differential equations can be written as:

(A) $Y' = AY + f(x)$

(B) $Y'' = AY$

(C) $AY' = Y$

(D) $Y' = f(Y)$

50. For the homogeneous system $Y' = AY$, the solution is:

(A) $Y = e^{At}Y(0)$

(B) $Y = AY$

(C) $Y = Y(0)$

(D) $Y = \int AY dx$

51. A fundamental set of solutions of a system corresponds to:

(A) Any two solutions

(B) Linearly independent solutions spanning the solution space

(C) Arbitrary constant functions

(D) Orthogonal polynomials only

52. The dimension of the solution space of nth order linear homogeneous system is:

(A) n

(B) n^2

(C) $2n$

(D) Infinite

53. A fundamental matrix solution is:

(A) Any square matrix of solutions

(B) A matrix whose columns form a fundamental set of solutions

(C) Determinant of solutions

(D) Inverse of A

54. The Wronskian of solutions of a linear system is:

(A) The sum of solutions

(B) The determinant of the fundamental matrix

(C) Always zero

(D) Ratio of solutions

55. If the Wronskian of solutions is nonzero at some point, then:

(A) Solutions are dependent

(B) Solutions are independent

(C) Solutions vanish

(D) None of the above

56. The trace of a matrix is:

(A) Sum of diagonal elements

(B) Product of diagonal elements

(C) Determinant

(D) Rank

57. The particular solution can be found using:

(A) Variation of parameters

(B) Method of undetermined coefficients

(C) Both (A) and (B)

(D) None of the above

58. If A has distinct real eigenvalues, the fundamental solutions are:

(A) Polynomials

(B) Exponentials involving eigenvalues

(C) Sinusoids

(D) Arbitrary constants

59. For repeated eigenvalues, fundamental solutions include:

(A) Only exponentials

(B) Exponentials and polynomial multiples

(C) Only polynomials

(D) None of the above

60. Complex eigenvalues yield solutions involving:

(A) Pure exponentials

(B) Sine and cosine functions

(C) Polynomials

(D) Only real constants

61. A self-adjoint second-order equation has form:

(A) $\left(p(x)y'\right)' + q(x)y = 0$

(B) $y'' + y = 0$

(C) $y' + py = 0$

(D) None of the above

62. Sturm theory deals with:

(A) Orthogonality of eigenfunctions

(B) Convergence of Picard iteration

(C) Existence and uniqueness

(D) Nonlinear chaos

63. The Abel formula gives relation between:

(A) Solutions and their Wronskian

(B) Eigenvalues and eigenfunctions

(C) Green's function

(D) Phase-plane paths

64. Sturm separation theorem states:

(A) Zeros of two solutions interlace

(B) All solutions are orthogonal

(C) All solutions vanish

(D) Solutions coincide

65. Sturm comparison theorem gives:

(A) Relation between zeros of solutions of two equations

(B) Relation between Wronskians

(C) Orthogonality criterion

(D) Eigenvalue formula

66. Fundamental matrix at $t = 0$ is usually chosen as:

(A) Identity matrix

(B) Zero matrix

(C) Determinant of A

(D) Eigenvector matrix

67. Linearly dependent solutions give a fundamental matrix with determinant:

(A) Nonzero

(B) Zero

(C) One

(D) Infinite

68. The Wronskian is zero everywhere if:

(A) Solutions are independent

(B) Solutions are dependent

(C) Matrix is diagonal

(D) Trace is zero

69. Non-homogeneous systems can also be solved using:

(A) Laplace transforms

(B) Fourier transforms

(C) Both (A) and (B)

(D) None of the above

70. A non-linear system has the form:

(A) $Y' = AY$

(B) $Y' = f(Y)$ (non-linear f)

(C) $y'' + py = 0$

(D) None of the above

71. The phase plane represents:

(A) Graph of solution in (t,y)

(B) Graph of solution in (y,y')

(C) Graph of solution in (x,y)

(D) Graph of eigenvalues

72. A path in phase plane corresponds to:

(A) Integral curve of system

(B) Straight line only

(C) Eigenvector

(D) Determinant curve

73. An autonomous system has:

(A) No explicit dependence on t

(B) No dependence on y

(C) Constant coefficients only

(D) Random forcing

74. Critical points are defined as points where:

(A) $f(x,y) = 0$

(B) $dy/dt = 0,\ dx/dt = 0$

(C) Both (A) and (B)

(D) None of the above

75. An isolated critical point means:

(A) Only one solution passes through it

(B) No other critical point nearby

(C) Always unstable

(D) None of the above

76. Linearization near critical point uses:

(A) Jacobian matrix

(B) Wronskian

(C) Sturm comparison

(D) Green's function

77. The type of critical point depends on:
(A) Trace and determinant of Jacobian
(B) Rank of Jacobian
(C) Only determinant
(D) Eigenfunction set

78. If eigenvalues are real and distinct of same sign, critical point is:
(A) Node
(B) Saddle
(C) Spiral
(D) Center

79. If eigenvalues are real, of opposite sign:
(A) Saddle
(B) Node
(C) Spiral
(D) Center

80. If eigenvalues are purely imaginary:
(A) Spiral
(B) Center
(C) Saddle
(D) Node

81. If eigenvalues are complex with real part nonzero:
(A) Spiral (focus)
(B) Node
(C) Saddle
(D) Center

82. Stability of a critical point requires:
(A) Negative real parts of eigenvalues
(B) Positive real parts
(C) Purely imaginary eigenvalues
(D) None of the above

83. Asymptotically stable point means:
(A) Paths diverge
(B) Paths approach and remain
(C) Paths oscillate only
(D) Paths are periodic

84. Unstable point corresponds to eigenvalues with:
(A) Positive real part
(B) Negative real part
(C) Zero
(D) Pure imaginary

85. Saddle point stability:

(A) Neutral

(B) Always stable

(C) Always unstable

(D) Oscillatory

86. Spiral point trajectories:

(A) Spiral toward or away from critical point

(B) Straight lines

(C) Closed circles

(D) None of the above

87. Center point trajectories:

(A) Closed orbits

(B) Spiral inward

(C) Spiral outward

(D) Diverge to infinity

88. Node point is stable if eigenvalues are:

(A) Positive

(B) Negative

(C) Imaginary

(D) Zero

89. Degenerate node occurs when:

(A) Eigenvalues equal, only one eigenvector

(B) Eigenvalues distinct

(C) Eigenvalues imaginary

(D) Determinant = 0

90. If $\text{Re}(\lambda) < 0$, path:

(A) Approaches critical point

(B) Diverges

(C) Oscillates

(D) Periodic

91. If $\text{Re}(\lambda) > 0$, path:

(A) Approaches

(B) Diverges from critical point

(C) Remains closed

(D) None of the above

92. If eigenvalues purely imaginary, path:

(A) Remains closed (canter)

(B) Diverges

(C) Approaches

(D) Unstable spiral

93. Stable spiral has eigenvalues:

(A) Complex with negative real part

(B) Complex with positive real part

(C) Pure imaginary

(D) Real equal values

94. Unstable spiral has eigenvalues:

(A) Complex with positive real part

(B) Complex with negative real part

(C) Pure imaginary

(D) Real

95. Linearization theorem states:

(A) Non-linear system near critical point behaves like linearized system

(B) System is always unstable

(C) Only linear systems are stable

(D) None of the above

96. Lyapunov's method is used to study:

(A) Sturm comparison

(B) Orthogonality

(C) Wronskian behaviour

(D) Stability of nonlinear systems

97. If all solutions remain bounded but not convergent, the equilibrium is:

(A) Unstable

(B) Asymptotically stable

(C) Stable (Lyapunov stable)

(D) None of the above

98. If solutions converge to equilibrium, it is:

(A) Unstable

(B) Asymptotically stable

(C) Neutrally stable

(D) None of the above

99. If solutions diverge for arbitrarily small perturbations:

(A) Unstable equilibrium

(B) Stable

(C) Neutral

(D) Periodic

100. For diagonalizable A, solution is:

(A) $y = Pe^{dt}p^{-1}y(0)$

(B) $y = a^{-1}y(0)$

(C) $y = Dt$

(D) None of the above

Rough Work

Example :

Question :

Q.1 Ⓐ ● Ⓒ Ⓓ

Q.2 Ⓐ Ⓑ ● Ⓓ

Q.3 Ⓐ ● Ⓒ Ⓓ

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.

6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.

7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.

8. After the completion of the examination, candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.

9. There will be no negative marking.

10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.

11. To bring and use of log-book, calculator, pager & cellular phone in examination hall is prohibited.

12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ

प्रश्न 2 Ⓐ Ⓑ ● Ⓓ

प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।

6. सभी उत्तर केवल ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।

7. ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।

8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।

9. निगेटिव मार्किंग नहीं है।

10. कोई भी रफ कार्य, प्रश्न-पुस्तिका में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।

11. परीक्षा-कक्ष में लॉग-बुक, कैल्कुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।

12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्णः प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्नपुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्नपुस्तिका प्राप्त कर लें।