

Roll. No.

Question Booklet Number

O.M.R. Serial No.

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**M.A./M.Sc. (SEM.-III) (NEP) (SUPPLE.)
EXAMINATION, 2024-25**

MATHEMATICS

(Functional Analysis)

Paper Code							
B	0	3	0	9	0	2	T

Question Booklet
Series

A

Time : 1 : 30 Hours

Max. Marks : 75

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.
4. Four alternative answers are mentioned for each question as - A, B, C & D in the booklet. The candidate has to choose the correct / answer and mark the same in the OMR Answer-Sheet as per the direction :

(Remaining instructions on last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, उसे तुरन्त बदल लें।
4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर- A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छानना है। उत्तर को OMR उत्तर-पत्रक में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

(शेष निर्देश अन्तिम पृष्ठ पर)

1. Normed on $\mathbb{N}$ is a map :
 - (A) $\mathbb{N}$ into $\mathbb{R}$
 - (B) $\mathbb{N}$ into $\mathbb{C}$
 - (C) $\mathbb{N}$ into $\mathbb{N}$
 - (D) None of the above
2. In a normed linear space, every convergent sequence is a :
 - (A) Cauchy sequence
 - (B) Divergent sequence
 - (C) Oscillatory sequence
 - (D) Constant sequence
3. Which of the following is true for Normed linear space $\mathbb{N}$:
 - (A) $\|x + y\| \geq \|x\| + \|y\|$
 - (B) $\|x + y\| = \|x\| + \|y\|$
 - (C) $\|x + y\| < \|x\| + \|y\|$
 - (D) $\|x + y\| \leq \|x\| + \|y\|$

where $x, y \in \mathbb{N}$
4. A complete normed linear space is known as :
 - (A) Metric space
 - (B) Compact space
 - (C) Banach space
 - (D) Euclidean space
5. Which of the following is not a property of norm in general :
 - (A) $\|kx\| = \|k\| \cdot \|x\|$
 - (B) $\|x\| \geq 0$
 - (C) $\|x + y\| \leq \|x\| + \|y\|$
 - (D) $\|x - y\| \leq \|x\| - \|y\|$
6. Consider the statements :
 - (i) Every finite dimensional normed linear space is a Banach space
 - (ii) Every Banach space is finite dimensional linear space
 - (A) Only (ii) is true
 - (B) Only (i) is true
 - (C) Both (i) and (ii) are true
 - (D) Neither (i) nor (ii) is true
7. Cauchy's inequality is a particular case of :
 - (A) Minkowski's inequality
 - (B) Schwarz inequality
 - (C) Halder's inequality
 - (D) All of the above
8. Relation $\sum_{i=1}^n |x_i y_i| \leq \|x\| \cdot \|y\|$, where x and y are n -tuples of scalars, is known as :
 - (A) Minkowski's inequality
 - (B) Holder's inequality
 - (C) Schwarz's inequality
 - (D) Cauchy's inequality
9. Let $f, g \in L_p$ where $1 \leq p < \infty$, then $\|f + g\|_p \leq \|f\|_p + \|g\|_p$ is known as :
 - (A) Minkowski's inequality
 - (B) Holder's inequality
 - (C) Cauchy's inequality
 - (D) Schwarz's inequality

10. Let $f \in L_p$ and $g \in L_p$ where $P > 1$, then $f \cdot g \in L_1$ and $\|fg\|_1 \leq \|f\|_p \|g\|_q$ is called Holder's inequality if :
- (A) $\frac{1}{p} - \frac{1}{q} = 1$
- (B) $\frac{1}{p} + \frac{1}{q} = 1$
- (C) $\frac{1}{p} + \frac{1}{q} > 1$
- (D) $\frac{1}{p} + \frac{1}{q} < 1$
11. The sign of equality holds in the inequality of Cauchy Schwarz's i.e. $|\langle x, y \rangle| = \|x\| \cdot \|y\|$ iff :
- (A) x and y are dependent vectors
- (B) x and y are vectors
- (C) x and y are linearly dependent vectors
- (D) x and y are linear independent vectors
12. A normed linear $\mathbb{N}$ is a Banach space iff every absolutely summable series:
- (A) Divergent
- (B) Convergent
- (C) Both (A) and (B)
- (D) None of the above
13. ℓ_p^n is :
- (A) has Convergent Cauchy sequence
- (B) Banach space
- (C) Both (A) and (B)
- (D) None of the above
14. Let Y be a proper closed subspace of a normed linear space $\mathbb{N}$ over the field k_0 . Let $0 < \alpha < 1$ then there exists some $X_\alpha \in N$ such that $\|X_\alpha\| = 1$ and $\inf_{y \in Y} \|X_\alpha - y\| \geq \alpha$, this statement is known as :
- (A) Reisz representation theorem
- (B) Hahn-Banach theorem
- (C) Baire category theorem
- (D) Reisz Lemma
15. Let N be a non-zero normed linear space and $M = \{x : x \in N \text{ and } \|x\| = 1\}$ then M is :
- (A) Complete
- (B) Independent
- (C) Divergent
- (D) None of the above
16. Which of the following is a Banach space?
- (A) Space of all polynomial functions on $[a, b]$ with the supremum norms
- (B) Space of all continuous functions on $[a, b]$ with the supremum norm
- (C) Space of all polynomial functions on $[a, b]$ with p-norm
- (D) All of the above

17. Example of Banach space is/are :
 (A) Euclidean space
 (B) Unitary space
 (C) Both (A) and (B)
 (D) Neither (A) nor (B)
18. If M is a closed linear subspace of a normed linear space N and $T : N \rightarrow N / M$ is a natural maps such that $T(x) = x + M$. Then T is :
 (A) Continuous
 (B) Closed
 (C) Compact
 (D) Complete
19. $\sum S_n$ is said to be absolutely sumable if :
 (A) $\sum_{i=1}^{\infty} \|S_i\| = \infty$
 (B) $\sum_{i=1}^{\infty} \|S_i\| > \infty$
 (C) $\sum_{i=1}^{\infty} \|S_i\| < \infty$
 (D) None of the above
20. L^p space is a Banach space if :
 (A) $1 < p < \infty$
 (B) $1 \leq p \leq \infty$
 (C) $1 < p \leq \infty$
 (D) $1 \leq p < \infty$
21. The space ℓ_{∞}^n represents, for all n-tuples $x = (x_1, x_2, \dots, x_n)$.
 (A) $\|x\|_{\infty} = \max\{|x_1|, |x_2|, \dots, |x_n|\}$
 (B) $\|x\|_{\infty} = \min\{|x_1|, |x_2|, \dots, |x_n|\}$
 (C) $\|x\|_n = \max\{|x_1|, |x_2|, \dots, |x_n|\}$
 (D) $\|x\|_n = \min\{|x_1|, |x_2|, \dots, |x_n|\}$
22. Which of the following statement is false?
 (A) Every finite dimensional subspace Y of a normed space X is closed in X
 (B) Every finite dimensional subspace Y of a normed space X is complete in X
 (C) Every subspace Y of a normed space X is closed in X
 (D) Every complete subspace Y of a Banach space X is closed
23. The metric induced by the norm is :
 (A) $d(x, y) = \|x + y\|$
 (B) $d(x, y) = \|x - y\|$
 (C) $d(x, y) = \|x - y\|^2$
 (D) $d(x, y) = \sqrt{\|x - y\|}$
24. Linear transformation $T : N \rightarrow N'$ is said to be bounded if there exists a real no. $k \geq 0$ such that :
 (A) $\|T(x)\| = k \|x\| \forall x \in N$
 (B) $\|T(x)\| = \|x\|^k \forall x \in N$
 (C) $\|T(x)\| \leq k \|x\| \forall x \in N$
 (D) $\|T(x)\| \geq k \|x\| \forall x \in N$

25. Two norms on a Vector space are equivalent if they :
- (A) Induce the same topology
 - (B) Same continuous function
 - (C) Same Convergence
 - (D) All of the above
26. Which of the following is not a Banach space?
- (A) Linear space of all bounded sequence $x = (a_1, a_2, \dots)$ with $\|x\| = \sup |a_i|$
 - (B) Linear space of all continuous functions on $[0, 1]$ with $\|f\| = \int_0^1 |f(t)| dt$
 - (C) Linear space of all n -tuples $x = (a_1, a_2, \dots, a_n)$ with $\|x\| = \max |a_i|$
 - (D) All of the above
27. For any normed space X , the conjugate space X^* is :
- (A) Always a compact set
 - (B) Always a Banach space
 - (C) Always finite dimensional
 - (D) Always an infinite dimensional
28. An Isometric isomorphism of N in to N^{**} is :
- (A) Natural embedding
 - (B) Conjugate space transformation
 - (C) Sound conjugate space transformation
 - (D) None of the above
29. Hahn-Banach theorem is about :
- (A) Extension of functionals
 - (B) Compactness
 - (C) Completeness
 - (D) Norm invariance
30. Dual space is :
- (A) Metric space
 - (B) Inner product space
 - (C) Compact operator
 - (D) Set of all bounded linear functionals
31. A linear functional f with Domain $D(f)$ in a normed space X is bounded iff :
- (A) X is complete
 - (B) X is finite dimensional
 - (C) X is compact
 - (D) None of the above
32. Let N and N' be two normed linear space with same scalar and $T : N \rightarrow N'$ be the linear transformed and $x_n \rightarrow x$ in $N \Rightarrow T(x_n) \rightarrow T(x)$ in N' then T is :
- (A) Complete
 - (B) Compact
 - (C) Continuous
 - (D) All of the above

33. Let M be a linear subspace of a normed linear space N , and let f be a functional defined on M . Then f can be extended to functional F defined on the whole space N such that :
- (A) $\|F\| > \|f\|$
 (B) $\|F\| < \|f\|$
 (C) $\|F\| \neq \|f\|$
 (D) $\|F\| = \|f\|$
34. Let f be a bounded linear functional on a normed space X , then $\|f\|$ is :
- (A) $\sup_{\|x\|=1} |f(x)|$
 (B) $\sup_{x \neq 0} |f(x)|$
 (C) $\sup_{x \neq 0} \|f(x)\|$
 (D) None of the above
35. Let N be linear space, then a continuous linear map $f : N \rightarrow R$ is called :
- (A) Functional on N
 (B) Normed space
 (C) Vector space
 (D) None of the above
36. Conjugate space of normed linear space is :
- (A) Dual space
 (B) Reflexive space
 (C) Banach space
 (D) None of the above
37. A normed linear space N , if $N^{**} = N$, then N is :
- (A) Banach space
 (B) Reflexive space
 (C) Both (A) and (B)
 (D) None of the above
38. If T is continuous linear transformation on normed linear space N into normed linear space N' and if M is its null space and $T' : \frac{N}{M} \rightarrow N'$ then
- (A) $\|T'\| = \|T\|$
 (B) $\|T'\| \neq \|T\|$
 (C) $\|T'\| > \|T\|$
 (D) None of the above
39. Let N and N' be two normed linear space with the same scalars and $T : N \rightarrow N'$ be a linear transformation, which of the following is/are true for above statement :
- (A) T is continuous
 (B) T is continuous at origin
 (C) T is bounded
 (D) All of the above
40. Consider the following statements :
- (i) The dual space X^* of a normed space X is a Banach space
 (ii) The dual space X^* is a normed space X is a Hilbert space
- (A) Both (i) and (ii) are true
 (B) Neither (i) nor (ii) are true
 (C) Only (i) is true
 (D) Only (ii) is true

41. Let N be a real normed linear space and subspace $f(x) = 0 \quad \forall \quad f \in N^*$ then :
- (A) $x = 0$
 (B) $x = 1$
 (C) $x < 0$
 (D) $x \geq 0$
42. In Generalized Hahn-Banach theorem, P be a sublinear functional of L , then L can be :
- (A) Real linear space
 (B) Complex linear space
 (C) Both (A) and (B) are correct
 (D) None of the above
43. If M is closed linear subspace of a normed linear space N and $T : N \rightarrow \frac{N}{M}$ is a natural map such that $T(x) = x + M, \quad x \in N$ then
- (A) $\|T\| = 1$
 (B) $\|T\| \geq 1$
 (C) $\|T\| \leq 1$
 (D) None of the above
44. Which of the following statement is correct?
- (A) Conjugate space of separable space compulsory separable
 (B) Conjugate space of separable space need not be separable
 (C) Conjugate space is equal to separable space
 (D) Can't say
45. If T be a linear transformation of a normed linear space N into another normed linear space N' for any real no. $k \geq 0$, which is correct :
- (A) $\|T(x)\| \leq k \|x\| \quad \forall \quad x \in N$
 (B) $\|T(x)\| < k \|x\| \quad \forall \quad x \in N$
 (C) $\|T(x)\| = k \|x\| \quad \forall \quad x \in N$
 (D) $\|T(x)\| \geq k \|x\| \quad \forall \quad x \in N$
46. If T is bounded in linear transformation of N into N' , for any $k \geq 0$, which is true :
- (A) $\|T(x)\| \geq k \|x\| \quad \forall \quad x \in N$
 (B) $\|T(x)\| = k \|x\| \quad \forall \quad x \in N$
 (C) $\|T(x)\| \leq k \|x\| \quad \forall \quad x \in N$
 (D) None of the above
47. Let N and N' be normed linear space and let S be a linear transformation of N into N' . Then T is continuous :
- (A) At every point of N
 (B) At no point of N
 (C) Either (A) or (B)
 (D) Neither (A) nor (B)
48. To guarantee the existence of continuous linear extensions of continuous linear functionals we can use the theorem/lemma :
- (A) Baire category theorem
 (B) Riesz lemma
 (C) Riesz representation theorem
 (D) Hahn-Banach theorem

49. Let N and N' be normed linear space and $B(N, N')$ denote the set of all bounded linear transformations from N into N' , then which is/are true for $B(N, N')$:
- (A) $B(N, N')$ is itself a normed linear space
 (B) Banach space
 (C) Both (A) and (B)
 (D) Neither (A) nor (B)
50. The correct statement is :
- (A) Every Hilbert space is Banach space
 (B) Every Banach space is Hilbert space
 (C) Every Vector space is Banach space
 (D) Every Vector space is Hilbert space
51. Let B and B' be Banach space and T is open mapping if :
- (A) T is a continuous linear transformation of B into B'
 (B) T is a, continuous linear transformation of B onto B'
 (C) Both (A) and (B)
 (D) Neither (A) nor (B)
52. Let X be an inner product space. Then the orthogonal complement of $\{0\}$ is :
- (A) X
 (B) $\{0\}$
 (C) $X/\{0\}$
 (D) $X^\perp$
53. Let B and B' be Banach space and let T be one-one continuous linear transformation of B onto B' then T is :
- (A) Isomorphism
 (B) Homeomorphism
 (C) T^{-1} is continuous
 (D) Both (B) and (C)
54. Let B be Banach space then a map $P: B \rightarrow B$ is said to projection on B then which one is true :
- (A) $P^2 = P$ idempotent
 (B) $P: B \rightarrow B$ is not continuous
 (C) P is not linear map
 (D) All of the above
55. Let X be an inner product space and $x \in X$, α be any scalar then $\|\alpha x\|^2$ is :
- (A) $\alpha \|x\|^2$
 (B) $|\alpha| \|x\|^2$
 (C) $\alpha^2 \|x\|^2$
 (D) $|\alpha|^2 \|x\|^2$
56. Which of the following is the Schwartz inequality?
- (A) $|\langle x, y \rangle| \geq \|x\| \cdot \|y\|$
 (B) $|\langle x, y \rangle| \leq \|x\| \cdot \|y\|$
 (C) $\|x + y\| \leq \|x\| + \|y\|$
 (D) $\|x + y\| \geq \|x\| + \|y\|$

57. In an inner product space X , which of the following is not true in general?
- (A) $x_n \rightarrow x$ and $y_n \rightarrow y \Rightarrow (x_n, y_n) \rightarrow (x, y)$
- (B) $|(x, y)| \leq \|x\| \cdot \|y\|$
- (C) $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$
- (D) $\|x+y\|^2 = \|x\|^2 + \|y\|^2$
58. Which of the following statement is false?
- (A) All normed spaces are inner product spaces
- (B) All Banach spaces are metric space
- (C) All Hilbert spaces are Topological spaces
- (D) All inner product spaces are normed spaces
59. Which is true for inner product space:
- (A) $\langle \overline{x}, y \rangle = \langle y, x \rangle$
- (B) $\langle \alpha x + \beta y - z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$
- (C) $\langle x, x \rangle \geq 0$
- (D) All of the above
60. Which one is correct?
- (A) ℓ_1^m is Banach space and Hilbert space
- (B) ℓ_1^m is not Banach space but a Hilbert space
- (C) ℓ_1^m is neither Banach space nor Hilbert space
- (D) ℓ_1^m is Banach space but not Hilbert space
61. The properly $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$ is known as :
- (A) Polarisation identity
- (B) Parallelogram law
- (C) Triangle law
- (D) All of the above
62. Which one of the following known as polarization property?
- (A) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$
- (B) $\|x+y\|^2 < \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 = 4\langle x, y \rangle$
- (C) $\|x+y\|^2 + \|x-y\|^2 = 4\langle x, y \rangle$
- (D) $\|x+y\|^2 - \|x-y\|^2 = 2\|y\|^2$
63. A complex Banach space is Hilbert space of :
- (A) Complex no associated
- (B) Real no. associated
- (C) Both (A) and (B)
- (D) None of the above
64. According to Baire category theorem following statement is true :
- (A) A complete metric space is a first category
- (B) A complete metric space is a second category
- (C) Both (A) and (B)
- (D) None of the above

65. Let B be Banach space and N be normed linear space. Let $\{T_i\}$ be non-empty set of bounded linear transformations from B to N with the property that $\{T_i(x)\}$ is bounded set of numbers for each x in B , then $\{\|T_i\|\}$ is bounded set of numbers, above statements is related to :
- Open mapping theorem
 - Closed graph theorem
 - Riesz lemma
 - Uniform bounded principle
66. Let B and B' be Banach space and $T : B \rightarrow B'$ be linear transformation from B into B' . Then T is continuous if and if its graph T_G is closed, above statement is related to :
- Open mapping theorem
 - Hilbert space
 - Banach space
 - Closed group theorem
67. ℓ_2^h is an example of :
- Hilbert space
 - Banach space
 - Both (A) and (B)
 - None of the above
68. In a Hilbert space the norm is generated by :
- Metric
 - Inner product
 - Modulus
 - None of the above
69. The norm is a :
- Continuous map
 - Discontinuous map
 - Differentiable map
 - None of the above
70. A linear subspace of a Hilbert space is :
- Not convex set
 - Convex set
 - Complete set
 - No option is true
71. A linear transformation is bounded if $K > 0$ set.
- $\|T(X)\| \leq K \|X\|$
 - $T(X) = KX$
 - $T(x) \leq K$
 - None of the above
72. Pythagorean theorem holds in a Hilbert space if :
- The vectors are parallel
 - The vectors are orthogonal
 - The vectors are arbitrary
 - None of the above
73. If $x \in H$ where H is Hilbert space such that $\langle x, y \rangle = 0, \forall y \in H$, then :
- x is null vector
 - $\|x\| = 1$
 - $\|x\| \neq 0$
 - All of the above

74. The graph of a linear transformation $T : V \rightarrow V$ over F is :
- (A) $\{(x, T(x)) : x \in V\}$
 (B) A curve on $T(x)$ lines
 (C) It is a design
 (D) None of the above
75. Let S be a set in V over field F then orthogonal complement of S over the inner product is :
- (A) $\{x \in V : (x, y) = 0 \text{ where } y \in S\}$
 (B) $\{x \in V : (x, x) = 0\}$
 (C) $\{x \in V : (x, y) = 0 \quad \forall x, y \in V\}$
 (D) None of the above
76. Let V be a normed linear space then orthogonal complement of null set $S = \{0\}$ is :
- (A) S
 (B) V
 (C) A proper subset of V
 (D) ϕ
77. Let $V(F)$ is a n.l.s. then orthogonal complement of V is :
- (A) $\{0\}$
 (B) V
 (C) A proper subspace of V
 (D) ϕ
78. Let S be a linear subspace of the n.l.s. $V(F)$ then $S \cap S^\perp$ is :
- (A) $\{\phi\}$
 (B) $\{0\}$
 (C) V
 (D) S
79. Let S be a set in the n.l.s. $V(F)$ then the orthogonal complement of S in $S^\perp$ then:
- (A) $S^\perp$ is a only a subset of V
 (B) $S^\perp$ is a linear subspace
 (C) $S^\perp = \{0\}$
 (D) $S^\perp = \phi$
80. Let $S = \{\hat{i}, \hat{j}\}$ in 3 dim space then $S^\perp$ is : (where $\hat{i}, \hat{j}, \hat{k}$ are unit vector along axis system)
- (A) $\{\hat{i}\}$
 (B) $\{\hat{j}\}$
 (C) $\{\hat{k}\}$
 (D) None of the above
81. Let $S_1 \subset S_2$ in the n.l.s. $V(F)$ then :
- (A) $S_2^\perp \subset S_1^\perp$
 (B) $S_1^\perp \subset S_2^\perp$
 (C) $S_1^\perp = V$
 (D) $S_2^\perp = V$
82. The orthogonal complement of a set S is :
- (A) Set but not subspace
 (B) Closed linear subspace
 (C) $\{0\}$
 (D) ϕ

83. Let M is a closed linear subspace of Hilbert space H then :
- (A) There exists a non zero-element z_0 in H.S.T. $z_0 \perp M$
- (B) No orthogonal element exists
- (C) Every orthogonal complement is empty
- (D) None of the above
84. Let M and N are closed linear transformation in a Hilbert space H and $M \perp N$ then there exists linear subspace $M + N$ which is :
- (A) Open
- (B) Closed
- (C) Complete
- (D) None of the above
85. Let M is a set in the Hilbert space H then $M^\perp \cap M^{\perp\perp}$ is equal to :
- (A) ϕ
- (B) $\{0\}$
- (C) M
- (D) $\{\phi\}$
86. $S^{\perp\perp\perp}$ in H is equal to :
- (A) $S^{\perp\perp}$
- (B) S
- (C) $S^\perp$
- (D) $\{0\}$
87. A set S is orthogonal set in H if:
- (A) $\langle x, y \rangle = 0 \quad \forall x, y \in S \text{ and } x \neq y$
- (B) $\langle x, x \rangle = 0 \quad \forall x \in S$
- (C) $\langle x, y \rangle = 1 \text{ if } x = y$
- (D) None of the above
88. A set S is orthonormal if :
- (A) $\langle e_i, e_i \rangle = 1$ if $i = j$ and 0 if (f) where $S = e_1, e_2, \dots, e_n$
- (B) $\langle e_i, e_j \rangle = 0$ for $(1, 2, \dots, n)$
- (C) $\langle e_i, e_j \rangle = 0$ if $(\neq)$
- (D) $\langle e_i, e_j \rangle = 1$ if $(\neq)$
89. Complete orthonormal set is the set in which :
- (A) Number of elements is equal $\dim$ of V
- (B) Any independent set is complete orthonormal set
- (C) Every non-empty set is orthonormal
- (D) All of the above
90. According to Bessel's inequality if $\{e_v\}$ is an orthonormal basis then :
- (A) $\sum |\langle x, e_i \rangle|^2 \leq \|x\|^2$
- (B) $\sum |\langle x, e_i \rangle|^2 = \|x\|^2$
- (C) $\sum |\langle x, e_i \rangle|^2 \geq \|x\|^2$
- (D) None of the above
91. Let $\{e_i\}$ is complete orthonormal set then if $x \perp \{e_i\}$ then :
- (A) $x = 1$
- (B) $x = 0$
- (C) $x = -1$
- (D) $x = \infty$

92. According to Gram Schmidt Orthogonalization process :
- (A) From any set we get linearly indep. set
 - (B) Any linearly indep. set is equivalent to an orthonormal set
 - (C) Gram Schmidt process is not valid in Hilbert space
 - (D) None of the above
93. A basis is complete if it has :
- (A) A basis that can generate the whole space
 - (B) It has all Cauchy sequence convergent
 - (C) Every basis generates the whole space
 - (D) None of the above
94. Let $f_y(x) = \langle x, y \rangle$ is a map in Hilbert space H then :
- (A) $\|f_y\| = \|x\|$
 - (B) $\|f_y\| = \|y\|$
 - (C) $\|f_y\| = 1$
 - (D) $\|f_y\| = \infty$
95. Let f is a functional in the Hilbert space H then :
- (A) $f = f_y$ for some unique $y \in H$
 - (B) $f = f_y$ for more than one y in H
 - (C) $f = f_x \quad \forall x \in H$
 - (D) None of the above
96. Let $f_x, f_y \in H^*$ then :
- (A) $\langle f_x, f_y \rangle = \langle x, y \rangle$
 - (B) $\langle f_y, f_x \rangle = \langle x, y \rangle$
 - (C) $\langle f_x, y \rangle = 0$
 - (D) $\langle f_y, x \rangle = 0$
97. Let H is a Hilbert space then :
- (A) H is not reflexive
 - (B) H is reflexive
 - (C) $H = H^*$
 - (D) $H = H^{**}$
98. Every weak convergence is :
- (A) Not strong convergence
 - (B) Same as strong convergence
 - (C) Strong convergence
 - (D) None of the above
99. Weak convergence is :
- (A) Convergence through sequence
 - (B) Convergence through functional
 - (C) Convergence through $\varepsilon - \delta$ criteria
 - (D) None of the above
100. A functional is :
- (A) Function of functions
 - (B) Space
 - (C) Set
 - (D) A linear map

Rough Work

Example :

Question :

Q.1 (A) ● (C) (D)

Q.2 (A) (B) ● (D)

Q.3 (A) ● (C) (D)

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination, candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager & cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ०एम०आर० उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैल्कुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण: प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्नपुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरिज की दूसरी प्रश्नपुस्तिका प्राप्त कर लें।