Roll No. ....

O. M. R. Serial No. | | | | | | | | |

Question Booklet Number

M. A./M. Sc. (Second Semester) (NEP) EXAMINATION, 2022-23

MATHEMATICS

(Advanced Topology)

Paper Code							
B	0	3	0	8	0	2	T

Questions Booklet Series
A

Time : 1:30 Hours] [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

(Remaining instructions on the last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. If (X, T) be a topological space and $A \subset X$, then a class $C^* = \{G_i : i \in I\}$ of subsets of X is said to be the :

(A) Cover

(B) Open cover

(C) Refinement

(D) T-open cover

of A if and only if $A \subset \bigcup_i \{G_i : i \in I\}$.

2. "Every open cover of a closed and bounded interval A = [a, b] is reducible to a finite cover." This is the statement of :

(A) Bolzano theorem

(B) Tychonoff's theorem

(C) Heine-Borel theorem

(D) None of the above

3. A topological space in which every open cover has finite subcover is said to be a :

(A) Connected space

(B) Compact space

(C) Lindelof space

(D) Hausdorff space

4. A class of sets is said to be 'Fixed' if :

(A) it has non-empty intersection.

(B) its intersection is empty.

(C) its number of elements are fixed.

(D) None of the above

5. Any closed subspace of a compact space is :

(A) Hausdorff

(B) Not compact

(C) Compact

(D) Connected

6. A continuous image of a compact set is :

(A) closed

(B) compact

(C) not compact

(D) open

7. If A is a subset of a topological space (X, T) and A is compact with respect to T, then :

(A) A has finite intersection property.

(B) A is compact w. r. t. the relative topology T_A on A.

(C) A is not compact w. r. t. the relative topology T_A on A.

(D) None of the above

8. Every compact subset of a Hausdorff space is :

(A) Hausdorff

(B) Compact

(C) Closed

(D) All of the above

9. Every compact Hausdorff space is :

(A) normal

(B) regular

(C) Tychonoff

(D) All of the above

10. An infinite subset A of a discrete topological space (X, **D**) is :

(A) closed

(B) compact

(C) not compact

(D) None of the above

11. If (X, T) and (Y, T*) be two topological spaces, A is a compact subset of X and $f : X \rightarrow Y$ is a continuous function, then $f[A]$ is :

(A) compact

(B) not compact

(C) regular

(D) normal

12. The closed interval [0, 1] is :

(A) not compact

(B) compact

(C) separated

(D) None of the above

13. The Cantor's set Γ is :

(A) closed and bounded

(B) unbounded

(C) not compact

(D) None of the above

14. Let T = {ϕ, {1}, {1, 2,}, Y} be a topology defined on Y = {1, 2, 3}. Then (Y, T) is :

(A) Hausdorff space

(B) Normal space

(C) Compact space

(D) Not a compact space

15. If X be a compact space and Y be a Hausdorff space, then every bijective continuous mapping of X onto Y is :

(A) a homomorphism

(B) an automorphism

(C) an isomorphism

(D) None of the above

16. In a topological space, compactness is :

(A) a topological variant property

(B) a topological invariant property

(C) a hereditary property

(D) None of the above

17. The usual topological space (R, U) is :

(A) Hausdorff

(B) Normal

(C) Compact

(D) Not compact

18. In a topological space (X, T) if every open cover of X is reducible to a countable cover, then the space is called a :

(A) Countable compact space

(B) Sequentially compact space

(C) Lindelof space

(D) Locally compact space

19. 'Every bounded infinite set of real numbers has a limit point.' This is called :

(A) Heine-Borel Theorem

(B) Bolzano-Weierstrass Theorem

(C) Bolzano-Weierstrass property

(D) None of the above

20. A topological space with Bolzano-Weierstrass property is also known to be :

(A) Frechet compact

(B) Locally compact

(C) Sequentially compact

(D) None of the above

21. The set of real numbers in $[0, 1] \subset R$ is a :

(A) Countably compact set

(B) Sequentially compact set

(C) Locally compact set

(D) All of the above

22. A sequentially compact topological space (X, T) is :

(A) countably compact space

(B) locally compact space

(C) Lindelof space

(D) None of the above

23. Which one of the following statements is not true ?

(A) The compactness is a topological invariant.

(B) The Lindelofness is a topological invarianat.

(C) Both (A) and (B) are true.

(D) None of the above is true.

24. Every locally compact Hausdorff space is :

(A) regular

(B) completely regular

(C) normal

(D) None of the above

25. In a topological space (X, T) if every point in X has a nbd whose closure is compact, then the space is :

(A) Lindeloff

(B) Locally compact

(C) Normal

(D) None of the above

26. An indiscrete topological space I = { ϕ , X} is always :

(A) locally connected

(B) disconnected

(C) connected

(D) None of the above

27. Let (X, T) is a topological space and A is a non-empty subset of X. If A is the union of two non-empty separated sets, then :

(A) A is connected

(B) A is disconnected

(C) A is maximal connected

(D) None of the above

28. A topological space (X, T) is disconnected if and only if $\exists$ a non-empty proper subset of X which is :

(A) T-open in X.

(B) T-closed in X.

(C) Both T-open and T-closed in X.

(D) None of the above

29. The closure of a connected set is :

(A) Connected

(B) Maximal connected

(C) Separated

(D) None of the above

30. A continuous image of a connected space is :

(A) disconnected

(B) locally connected

(C) connected

(D) None of the above

31. If (X, T) be a product topological space of two topological spaces (X_1, T_1) and (X_2, T_2), then X is connected, if :

(A) X_1 is connected.

(B) X_2 is connected.

(C) X_1 and X_2 both are connected.

(D) None of the above

32. If T = {ϕ, {1}, {2, 3}, X} is a topology on X = {1, 2, 3}, then (X, T) is a :

(A) connected space

(B) maximal connected space

(C) disconnected space

(D) None of the above

33. Every discrete topological space (X, **D**) when X consists of more than one point is :

(A) connected

(B) disconnected

(C) maximal connected

(D) None of the above

34. If (X, T) is disconnected and T_0 is finer than T, then (X, T_0) is :

(A) locally connected

(B) connected

(C) disconnected

(D) None of the above

35. The range of a continuous real-valued function defined on a connected space is :

(A) an interval

(B) a set

(C) a space

(D) None of the above

36. If S be a lower limit topology of the set of real numbers R, then the space (R, S) is :

(A) maximal connected space

(B) connected space

(C) disconnected space

(D) None of the above

37. The usually topological space is :

(A) a locally connected space

(B) a connected space

(C) a disconnected space

(D) None of the above

38. If (X, T) is connected space and T* is coarser than T, then (X, T*) :

(A) is connected.

(B) is disconnected.

(C) may be connected or disconnected

(D) None of the above

39. If X = {p, q, r, s}

and T = {ϕ, {p}, {p, q} {p, q, r},X},

then (X, T) is :

(A) disconnected

(B) locally connected

(C) connected

(D) None of the above

40. If (X, T) is a connected space then it has :

(A) only open component X itself.

(B) only open component ϕ itself.

(C) many open components.

(D) None of the above

41. If (X, T) be a topological space and $A \subset X$. If A is a maximal connected subset of X, then A is said to be :

(A) a non-empty frontier.

(B) a component of the space X.

(C) a complement.

(D) None of the above

42. If (X, T) be an arbitrary topological space, then :

(A) each component of X is open.

(B) each component of X is closed.

(C) each component of X is disconnected.

(D) None of the above

43. If T = { ϕ , {1}, {2, 3}, {1, 2, 3}, {2, 3, 4, 5}, X} is a topology on X = {1, 2, 3, 4, 5}, then the components of X are :

(A) {1} and {2, 3, 4, 5}.

(B) {2, 3} and {1, 2, 3}.

(C) {1, 2, 3} and X.

(D) All of the above

44. If {X, T} be an arbitrary topological space, then :

(A) each component of X is disconnected.

(B) each component of X is locally connected.

(C) the components of X form a partition of X.

(D) None of the above

45. The continuous image of an arcwise connected set is :

(A) connected

(B) arcwise connected

(C) disconnected

(D) None of the above

46. The components of a totally disconnected space (X, T) :

(A) are singleton sets

(B) are null sets

(C) are open sets

(D) None of the above

47. Every discrete topological space (X, **D**) is :

(A) connected

(B) arcwise connected

(C) totally disconnected

(D) None of the above

48. Every component of a locally connected space is :

(A) open

(B) closed

(C) disconnected

(D) None of the above

49. Any discrete space (X, **D**) containing a single point is :

(A) totally disconnected

(B) locally connected

(C) disconnected

(D) None of the above

50. The product X × Y of locally connected sets X and Y is :

(A) locally connected

(B) separated

(C) totally disconnected

(D) None of the above

51. Which one of the following is the weakest topology defined on a non-empty set X ?

(A) Product topology

(B) Discrete topology

(C) Indiscrete topology

(D) None of the above

52. The product of two second axiom (second countable) space is a :

(A) Hausdorff space

(B) Second axiom space

(C) Normal space

(D) None of the above

53. The product of two Hausdorff spaces is a :

(A) Hausdorff space

(B) Regular space

(C) Normal space

(D) None of the above

54. If X = X $\{X_n : n \in I\}$, then the function $\Pi_n : X \to X_n$ defined by $\Pi_n(x) = x_n \ \forall x \in X$ is said to be the :

(A) evaluation map

(B) projection map

(C) diagonal map

(D) None of the above

55. A map f : X → Y defining a homeomorphism of X onto Y is said to be :

(A) an embedding

(B) a diagonal map

(C) evaluation map

(D) a projection of a topological space X into another space Y.

56. The evaluation map of X into X $\{Y_n : n \in I\}$ is :

(A) constant

(B) unique

(C) not unique

(D) None of the above

57. If $\{(X_\alpha, T_\alpha) : d \in I\}$ be an arbitrary collection of topological spaces and T is a topology on X = X $\{X_\alpha : \alpha \in I\}$, then T is :

(A) the product topology for X

(B) the quotient topology for X

(C) the metric topology for X

(D) None of the above

58. Each projection mapping Π_α is :

(A) constant

(B) closed

(C) open

(D) None of the above

59. Let X be any set and τ = X, ϕ. Then τ is called :

(A) co-finite topology

(B) co-complement topology

(C) Indiscrete topology

(D) None of the above

60. The product of any non-empty class of Hausdorff spaces is :

(A) open

(B) closed

(C) a Hausdorff

(D) None of the above

61. The product space X = X $\{X_\alpha : \alpha \in I\}$ is regular if and only if each co-ordinate space X_α is :

(A) continuous

(B) normal

(C) open

(D) regular

62. Every Tychonoff space X = X $\{X_\alpha : \alpha \in I\}$ can be embedded as :

(A) a subspace of X

(B) a subspace of a cube

(C) a subspace of co-ordinate space

(D) None of the above

63. Projection maps are :

(A) always closed

(B) not always closed

(C) always open

(D) None of the above

64. If X is a T_1-space which is regular and also satisfies the second axiom of countability, then X is :

(A) metrizable and separable

(B) connected space

(C) quotient space

(D) None of the above

65. Which one of the following topologies is the smallest topology of which the projections are continuous ?

(A) Quotient topology

(B) Discrete topology

(C) Product topology

(D) None of the above

66. If (X, T) be a topological space, Y is a set and f is a mapping of X onto Y, then the largest topology say T* for Y such that $f : T \rightarrow T^*$ is continuous, is said to be the :

(A) Tychonoff topology

(B) Product topology

(C) Quotient topology

(D) Discrete topology for Y relative to f and T.

67. If X is a non-empty set and D a decomposition of X, then the mapping Π from X onto D, such that $\Pi(x)$ is the unique member of D to which x belongs, is said to be the :

(A) canonical map

(B) evaluation map

(C) diagonal map

(D) None of the above

68. If X be a topological space such that the quotient space X/R is Hausdorff, then :

(A) R is a closed subset of the space X.

(B) R is a closed subset of the product space X × X.

(C) R is an open subset of the product space X × X.

(D) R is an open subset of the space X.

69. A subset A of Y is open in the quotient topology relative to $f : X \rightarrow Y$, if and only if :

(A) f^{-1} [A] is open in Y.

(B) f^{-1} [A] is open in X.

(C) f [A] is open in Y.

(D) None of the above

70. If f be a continuous mapping of a topological space (X, T) onto another space (Y, T*) such that f is either open or closed, then T* must be the :

(A) Tychonoff topology for Y

(B) Tychonoff topology for X

(C) Quotient topology for Y

(D) Quotient topology for X

71. If X be a topological space, X/R be a quotient space such that R is closed in X × X and Π is an open quotient map, then X/R is :

(A) normal space

(B) regular space

(C) Hausdorff space

(D) None of the above

72. The usual topological space (R, U) is :

(A) Normal space

(B) Hausdorff space

(C) Regular space

(D) None of the above

73. The pair (A, ≥) consisting of a non-empty set A and a binary relation ≥ defined on A is called a :

(A) directed system

(B) cofinal subset

(C) residual subset

(D) None of the above

74. A sequence of natural numbers < N > is an example of a :

(A) residual subset

(B) net

(C) directed set

(D) None of the above

75. If (X, T) be an indiscrete topological space, then every net (f, X A, ≥) in X converges to :

(A) a unique point of X

(B) only two points of X

(C) every point of X

(D) None of the above

76. Let a net $\{f_a : a \in A\}$ be in a set X, if ∀ subset V of X, the net is eventually in V or eventually in V′, then the net is called :

(A) universal net

(B) subnet

(C) cofinal subnet

(D) None of the above

77. If (A, ≥) and (B, ≥*) be two directed sets, then a mapping Ψ : A → B defined as $a_1 \geq a_2 \Rightarrow \Psi(a_1) \geq^* \Psi(a_2),\ a_1, a_2 \in A$, is known as :

(A) projection mapping

(B) isotone mapping

(C) homomorphism

(D) None of the above

78. If (X, T) be a topological space and f be a net in X and x_0 be a point in X. If f is frequently in every T-nbd of x_0, then the point x_0 is said to be :

(A) cluster point of the net f.

(B) interior point of the net f.

(C) neighborhood point of the net f.

(D) All of the above

79. If each net in X converges to at most one point, then the topological space (X, T) is :

(A) Tychonoff space

(B) Lindelof space

(C) Hausdorff space

(D) None of the above

80. A subspace Y of a topological space (X, T) is closed if and only if no net in Y converges to :

(A) a point in X

(B) a point in Y

(C) a point in X ~ Y

(D) None of the above

81. If $\{f_a : a \in A\}$ be an ultranet in X and g a mapping of X into Y, then $\{g(f_a) : a \in A\}$ is :

(A) an ultranet in X

(B) an ultranet in Y

(C) an ultranet in X ~ Y

(D) None of the above

82. If (f, X, A, ≥) be a net and Ψ be an isotone map of a directed set (B, ≥) into the directed set (A, ≥) such that Ψ [B] is cofinal in A, then f o Ψ is :

(A) a subset of f

(B) a subset of Ψ

(C) an ultranet of f

(D) None of the above

83. A topological space (X, T) is compact if and only if each net in X has :

(A) an interior point

(B) an exterior point

(C) a cluster point

(D) None of the above

84. A point p in a topological space (X, T) is a cluster point of the net (f, X, A, ≥) if and only if :

(A) some subnet (g, X, B, $\geq^*$) of f converges to p.

(B) some subnet (g, X, B, $\geq^*$) of f converges except p.

(C) some ultranet (g, X, B, $\geq^*$) of f converges except p.

(D) None of the above

85. If N be directed by $\geq$, then N – {1, 2} is a :

(A) closed subset of N

(B) residual subset of N

(C) cofinal subset of N

(D) None of the above

86. If the set of all natural numbers N be directed by $\geq$, then the subset S = {1, 3, 5, ………} is :

(A) cofinal subset of N

(B) residual subset of N

(C) closed subset of N

(D) None of the above

87. Let f be a net in N i. e. $f : N \to N$, defined by $f(x) = 2n - 1$, then the set of cluster points of f being :

(A) {1, 2, 1, 3, 1, 4, …………}

(B) {2, 3, 4, 3, 5, 3, …………}

(C) {1, 3, 5, ……….}

(D) {2, 4, 6, ………}

88. If (X, D) be a discrete topological space, then a net (f, X A, $\geq$) converges to a point $p \in X$ if and only if :

(A) f is eventually in p.

(B) f is frequently in p.

(C) f is eventually in $\{p\}$.

(D) f is frequently in $\{p\}$.

89. In a topological space (X, T) if every convergent net in X has a unique cluster point and this is the unique limit point of the net then the space, is :

(A) Connected space

(B) Compact space

(C) Lindelof space

(D) Hausdorff space

90. If X be an infinite set, then the family F = {A : X ~ A is finite} defines a filter on X, known as :

(A) cofinite filter

(B) free filter

(C) fixed filter

(D) None of the above

91. If (X, T) be a topological space and N (x) be a collection of all T-nbds of a point $x \in X$, then N (x) defines a filter on X, is called :

(A) discrete filter

(B) indiscrete filter

(C) cofinite filter

(D) neighbourhood filter

92. A filter F on a non-empty set X is known as free filter if and only if :

(A) $\cap\{A : A \in F\} = \phi$

(B) $\cap\{A : A \in F\} \neq \phi$

(C) $\cap\{A : A \in X\} = \phi$

(D) None of the above

93. If X be a non-empty set, then {X} is always a filter on X and known as :

(A) fixed filter

(B) indiscrete filter

(C) discrete filter

(D) None of the above

94. A filter F on X is said to be an ultrafilter on X if and only if there exists :

(A) no filter on X strictly finer than F.

(B) no filter on X strictly coarser than F.

(C) no filter on X equal to F.

(D) None of the above

95. If (X, T) be a topological space and F be a filter on X, then F is said to converge to a point $p \in$ X, if and only if F is eventually in each nbd of *p*. Here *p* is known as the :

(A) cluster point of F

(B) limit point of F

(C) interior point of F

(D) None of the above

96. Two filter bases B_1 and B_2 are equivalent if *p* is :

(A) a limit point of both B_1 and B_2.

(B) an interior point of both B_1 and B_2.

(C) an adherent point of both B_1 and B_2.

(D) None of the above

97. Every filter on a set X is contained in :

(A) a discrete filter

(B) a cofinite filter

(C) a neighbourhood filter

(D) None of the above

98. Every filter F on a set X is the intersection of :

(A) all the ultrafilters coarser than F.

(B) all the ultrafilters finer than F.

(C) all the cofinite filters coarser than F.

(D) None of the above

99. A topological space (X, T) is Hausdorff if and only if every convergent filter on X has :

(A) a limit point

(B) a cluster point

(C) a unique limit

(D) None of the above

100. If (X, T) be a topological space and every ultrafilter on X converges, then the space (X, T) is :

(A) a Hausdorff space

(B) a compact space

(C) a connected space

(D) None of the above

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 (A) ● (C) (D)

Q. 2 (A) (B) ● (D)

Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.

6. All answers are to be given on OMR Answer sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.

7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.

8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.

9. There will be no negative marking.

10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.

11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.

12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is ny discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।

6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।

7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।

8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।

9. निगेटिव मार्किंग नहीं है।

10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।

11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।

12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।