Roll No. ....

O. M. R. Serial No. | | | | | | | |

Question Booklet Number

M. A./M. Sc. (Fourth Semester) (NEP) EXAMINATION, 2025-26

MATHEMATICS

(Calculus of Variations) (Elective)

Paper Code							
B	0	3	1	0	0	9	T

Questions Booklet Series

A

Time : 1:30 Hours] [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.

2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.

3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।

2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।

3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. The founder of the calculus of variations is :

   (A) Leonhard Euler

   (B) Nicolas Fuss

   (C) Joseph Louis Lagrange

   (D) Newton and de L' Hospital

2. The birth year of the Calculus of Variations is :

   (A) 1694

   (B) 1696

   (C) 1707

   (D) 1730

3. The solution of the brachistochrone problem was given by :

   (A) L. Euler

   (B) Leibnitz

   (C) Lagrange's

   (D) Johann Bernoulli

4. Every solution of Euler's equation which satisfies the boundary condition is called a/an :

   (A) Admissible solution

   (B) Stationary value

   (C) Extremal

   (D) Maximals

5. A necessary condition that the integral $v[y(x)] = \int_{x_0}^{x_1} F(x, y, y')dx,\ y(x_0) = y_0$ and $y(x_1) = y_1$ will be stationary is :

   (A) δv = Constant

   (B) $\delta v = 0$

   (C) $\delta v \neq 0$

   (D) None of the above

6. If $F(x, y, y')$ depends only on y and y' then Euler's equation for the functional $\int_{x_1}^{x_2} F(x, y, y')dx$ reduces to :

   (A) $F - y'F_{y'} = 0$

   (B) $F - y'F_{y'} = C$

   (C) $F + y'F_{y'} = C$

   (D) $F + y'F_{y'} = 0$

7. The path of the 'quickest descent' is :

   (A) Catenary

   (B) Equiangular spiral

   (C) Cycloid

   (D) Family of Circles

8. The shortest curve joining two points (x_1, y_1) and (x_2, y_2) is :

(A) Family of circles

(B) Equiangular spiral

(C) Straight line joining two points

(D) Family of straight lines

9. Necessary condition for existence of extermal of the functional

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y, y')dx$$

is :

(A) $\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$

(B) $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

(C) $\frac{\partial F}{\partial y} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y'}\right) = 0$

(D) None of the above

10. Euler's Lagrange's equation can also be represented in the form :

(A) $\frac{\partial}{\partial x}\left[F - y'\frac{\partial F}{\partial y'}\right] - \frac{\partial F}{\partial x} = 0$

(B) $\frac{\partial}{\partial x}\left[F - y'\frac{\partial F}{\partial y'}\right] - \frac{dF}{dx} = 0$

(C) $\frac{d}{dx}\left[F - y'\frac{dF}{\partial y'}\right] - \frac{\partial F}{dx} = 0$

(D) $\frac{d}{dx}\left[F - y'\frac{\partial F}{\partial y'}\right] - \frac{\partial F}{\partial x} = 0$

11. The extremal of the functional

$$v[y(x)] = \int_0^1 (x \sin y + \cos y)dx$$

with boundary condition $y(0) = 0$, $y(1) = \frac{\pi}{2}$ is :

(A) $y = \tan^{-1} x$

(B) $y = \cot^{-1} x$

(C) $y = \sin^{-1} x + \cos^{-1} x$

(D) No extremal exist

12. Functional :

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y, y')dx$$

if F is independent of y, then Euler's Lagrange's equation is :

(A) $\frac{\partial F}{\partial y'} = 0$

(B) $\frac{\partial F}{\partial y'} = \text{Constant}$

(C) $\frac{\partial F}{\partial y} = 0$

(D) $\frac{\partial F}{\partial y} = \text{Constant}$

13. The closed plane curve of given length that encloses maximum area is :

(A) Cycloid

(B) Catenary

(C) Circle

(D) Straight line

14. The extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} y^2 dx,\ y(x_0) = y_0, y(x_1) = y_1$$

is :

(A) Entire x-axis

(B) The portion of x-axis which satisfies the given boundary condition

(C) No extremal exists

(D) Entire y-axis

15. The extremals of the functional

$$v[y(x)] = \int_a^b (y + xy')dx,\ y(a) = y_0$$

and $y(b) = y_1$ is :

(A) $y = x$

(B) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(C) $y = x^3$

(D) No extremal exists

16. The curve joining two points that yields a surface of revolution of minimum area when revolved about x-axis is :

(A) Family of paraboloid

(B) Family of circles

(C) Family of catenaries

(D) Family of straight line

17. The stationary function of $\int_0^4 \left[xy' - (y')^2\right]dx$, which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$ is :

(A) $y = \frac{x}{2}(x-1)$

(B) $y = \frac{x}{4}(x-1)$

(C) $y = x(x - 1)$

(D) $y = x(x + 1)$

18. The extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{x} dx$ is :

(A) $y = c \cosh \frac{x-c}{c_1}$

(B) $x^2 = 4a(y - c)$

(C) $x^2 + (y - c_1)^2 = c_2^2$

(D) $y = c_1 x + c_2$

19. Which one of the following is linear functional ?

(A) $v[y(x)] = \int_{x_0}^{x_1} (y' + 2y^2)dx$

(B) $v[y(x)] = \int_{x_0}^{x_1} (y'^2 + 2y)dx$

(C) $v[y(x)] = \int_{x_0}^{x_1} (y' + yy')x^2 dx$

(D) $v[y(x)] = \int_{x_0}^{x_1} (y' + 2y)dx$

20. The extremals of the functional $v[y(x)] = \int_0^1 \left[(y')^2 + 12xy\right] dx,\ y(0) = 0$ and $y(1) = 1$ represents :

(A) Nodal cubic curve

(B) Family of straight line

(C) Family of circles

(D) Cycloid

21. Geodesics on a surface is a curve along which the distance between any two points of the surface is :

(A) Minimum

(B) Maximum

(C) Either maximum or minimum

(D) Fixed

22. Geodesics on a plane are :

(A) Concentric circles

(B) Straight lines

(C) Circular helix

(D) Cubic curves

23. Geodesics on a right circular cylinder of fixed radius is :

(A) Straight lines

(B) Family of paraboloid

(C) Nodal cubic curves

(D) Circular helix

24. If the area of the surface of revolution of a curve $y = y(x)$ is $2\pi \int_{x_1}^{x_2} y\sqrt{1+(y')^2}\,dx$ and is minimum, then the curve is :

(A) Family of catenaries

(B) Family of straight lines

(C) Family of concentric circles

(D) None of the above

25. Which curves can give an extremum of the functional $v[y(x)] = \int_0^1 \left[(y')^2 + 12xy\right] dx,\ y(0) = 0,$ $y(1) = 1$?

(A) $y(x) = x^3 - 1$

(B) $y(x) = x^3 - x$

(C) $y(x) = x^3$

(D) $y(x) = x^3 + 2x$

26. The geodesics on a sphere of a radius 'a' are its :

(A) Straight line

(B) Catenary

(C) Great circle

(D) Circular helix

27. An extremum of the functional $v[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')dx$ with $y(0) = 1$, $y(1) = 2$ is :

(A) $y = -\frac{x}{2}$

(B) $y = \frac{x}{2}$

(C) $y = x$

(D) No extremal exist

28. Extremal of the functional

$$v[y(x)] = \int_1^e \left(xe^y - ye^x\right)dx$$

with $y(1) = 1$, $y(e) = e$ is :

(A) $y(x) = \log x - x$

(B) $y(x) = x - \log x$

(C) $y(x) = x + \log x$

(D) No extremal exists

29. Euler's equation for the functional

$$\int_{x_1}^{x_2} \left[p(x)y'^2 + 2q(x)yy' + r(x)y^2 \right] dx$$

is :

(A) First order linear differential equation

(B) Second order linear differential equation

(C) Second order non-linear differential equation

(D) None of the above

30. Extremal for the variational problem

$$v[y(x)] = \int_0^{\log 2} \left(e^{-x}y'^2 - e^x y^2\right)dx$$

satisfy the differential equation :

(A) $y'' - y' + e^x y = 0$

(B) $y'' - y' + e^{2x} y = 0$

(C) $y'' - y' - e^{2x} y = 0$

(D) $y'' + y' + e^{2x} y = 0$

31. Extremal $y = y(x)$ for the variational problem $v[y(x)] = \int_0^1 [1 + (y'')^2]\, dx$ satisfies the ordinary differential equation is :

(A) Homogeneous linear differential equation of fourth order

(B) Homogeneous non-linear differential equation of fourth order

(C) Homogeneous linear differential equation of more than fourth order

(D) None of the above

32. The extremals of the functional $v[y(x)] = \int_0^{\pi/4} [(y'')^2 - y^2 + x^2]\, dx$, $y(0) = 0, y'(0) = 1,\ y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ is :

(A) $y(x) = \sin x$

(B) $y(x) = \cos x$

(C) $y(x) = \tan x$

(D) $y(x) = x$

33. How many extremals are possible for the extremals $v[y(x)] = \int_0^1 (y'' + y)\, dx$, $y(0) = 1$, $y(1) = -1$, $y'(0) = 1$, $y'(1) = 1$?

(A) Only one

(B) Exactly two

(C) Infinite many

(D) None of the above

34. Extremal of the functional $I[y(x)] = \int_1^2 \frac{x^3}{y'^2}\, dx$, $y(1) = 0$ and $y(2) = 3$ is :

(A) $y = x^2 + 1$

(B) $y = x^2 - 1$

(C) $y = x^3 - 1$

(D) $y = x - 1$

35. Extremal of the functional $I[y(x)] = \int_0^1 \left[(y')^2 + 4y^2\right] dx,\ y(0) = e^2$ and $y(1) = 1$ lies on the curve :

(A) $y = e^{2x}$

(B) $y = e^{x}$

(C) $y = e^{2-2x}$

(D) $y = e^{2+2x}$

36. The functional $I[y(x)] = \int_0^{\pi/2} \left[(y')^2 - y^2\right] dx;\ y(0) = 0,$ $y\left(\frac{\pi}{2}\right) = 1$ has :

(A) Unique extremal

(B) Two extremal

(C) No extremal

(D) Infinite extremal

37. The solutions of Euler-Poisson equation are called as :

(A) Stationary solution

(B) Extremals

(C) Trial solution

(D) Admissible solution

38. For the functional $I[y(x)] = \int_a^b \left(y^2 + 2xyy'\right) dx;\ y(a) = y_a$ and $y(b) = y_b$, the number of extremals equals to :

(A) 0

(B) 1

(C) 2

(D) Infinite

39. The Euler's equation for a functional of the form $\int_a^b F(x, y) dx$ is :

(A) $F_{y'} = c_1$

(B) $F_y - \frac{d}{dx}\{F_{y'}\} = c_1$

(C) $F_y = c_1$

(D) $F_y = 0$

40. The Euler equation for the functionals $I[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy$ satisfy :

(A) Wave equation

(B) Poisson's equation

(C) Laplace equation

(D) Heat equation

41. The necessary conditions for the existence of extremals for functional $I[u(x,y)] = \iint_D F(x, y, u, u_x, u_y)\,dxdy$ is :

(A) $\frac{\partial F}{\partial u} + \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

(B) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

(C) $\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u_x}\right) - \frac{d}{dy}\left(\frac{\partial F}{\partial u_y}\right) = 0$

(D) None of the above

42. The necessary condition for the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y', y'')\,dx$ to be an extremum is :

(A) $F_y - \frac{d}{dx}\{F_{y'}\} = 0$

(B) $F_y - y' F_{y'} = C$

(C) $F_y - \frac{d}{dx}\{F_{y'}\} - \frac{d^2}{dx^2}\{F_{y''}\} = 0$

(D) $F_y - \frac{d}{dx}\{F_{y'}\} + \frac{d^2}{dx^2}\{F_{y''}\} = 0$

43. The extremals of the functional $I[y(x), z(x)] = \int_0^{\pi/2}\left(y'^2 + z'^2 + 2yz\right)dx$ are the solution of simultaneous differential equation :

(A) $y'' + z = 0,\ z'' + y = 0$

(B) $y'' - z = 0,\ z'' + y = 0$

(C) $y'' - z = 0,\ z'' - y = 0$

(D) $y'' + z = 0,\ z'' - y = 0$

44. Euler's equations for the functional $\int_1^2 (y'^2 + z^2 + z'^2)\,dx$ are given by :

(A) $y'' + y = 0,\ z'' + z = 0$

(B) $y'' = 0,\ z'' - z = 0$

(C) $y'' - y = 0,\ z'' = 0$

(D) None of the above

45. The extremal of the functional $I[y(x)] = \int_0^{\pi}\left[(y'^2 - y^2)\right]dx$, $y(0) = 1$, $y(\pi) = \alpha$ has :

(A) A unique extremals if $\alpha = 1$

(B) Infinite many extremal if $\alpha = 1$

(C) A unique extremal if $\alpha = -1$

(D) Infinite many extremals if $\alpha = -1$

46. The extremal of the functional $\int_0^2 \frac{y'^2}{x} dx$ with $y(0) = a$, $y(2) = b$ is parabola passing through origin. Then a and b are :

(A) $a = 0, b = 1$

(B) $a = 1, b = 2$

(C) $a = -1, b = 2$

(D) $a = 0, b = 2$

47. If $F(x, y, y')$ is a function of y' alone, then the extremal of the functional $\int_{x_1}^{x_2} F(x, y, y') dx$ is :

(A) Circle

(B) Straight line

(C) Nodal cubic curve

(D) Catenary

48. For the functional

$$I[y(x)] = \int_a^b (x - y)^2 dx$$

the extremal is a :

(A) Straight line

(B) Circle

(C) Catenary

(D) Parabola

49. Euler's equation for the functional

$$I[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy$$

is :

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(B) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

(C) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \cdot \frac{\partial z}{\partial x} = 0$

(D) None of the above

50. Extremal of the functional

$$I[y(x)] = \int_{x_0}^{x_1} (2xy + y'''^2) dx$$

lies on the :

(A) Four parameter family of curves

(B) Five parameter family of curves

(C) Six parameter family of curves

(D) Seven parameter family of curves

51. The simplification of the Euler-Lagrange equation is known as :

(A) Belframi identity

(B) Hamilton's identity

(C) Liouville's identity

(D) Legendre's condition

52. F(x, y, y') is not functional because :

(A) Its range is infinite

(B) Its range is **R**

(C) Its range is not **R**

(D) None of the above

53. Solid figure of revolution which, for a given surface area, has maximum volume is :

(A) A circle

(B) A sphere

(C) An ellipse

(D) A parabola

54. In the equation H = $f + \lambda g$, where $\int_1^2 \mathrm{H}dx$ to be an extremum, λ is called as :

(A) Isoperimetric constant

(B) Kernels

(C) Lagrange Multiplier

(D) Green's function

55. The curve made by a cable of fixed length suspended from two points for minimum gravitational potential energy is :

(A) Circular

(B) Parabolic

(C) Hyperbolic

(D) Catenary

56. The problem where a curve of given perimeter and it encloses the maximum area is known as :

(A) Geodesics

(B) Brachistochrone

(C) Isoperimetric

(D) Weierstrass

57. The transversality condition which together with the relation $y(x_1) = \phi(x_1)$ is :

(A) $\left[F+(\phi'+y')\frac{\partial F}{\partial y'}\right]_{x=x_1}=0$

(B) $\left[F+(\phi'-y')\frac{\partial F}{\partial y'}\right]_{x=x_1}=0$

(C) $\left[F+(\phi'-y')\frac{\partial F}{\partial y}\right]_{x=x_1}=0$

(D) None of the above

58. The function

$E(x, y, y', p) = F(x, y, y') - F(x, y, p) - (y' - p) F_p(x, y, p)$ is known as :

(A) Weiersbrass function

(B) Lagrange's function

(C) Jacobi condition

(D) Legendre condition

59. The Jacobi equation is given by :

(A) $\left(F_{yy}-\frac{d}{dx}F_{yy'}\right)u-\frac{d}{dx}(F_{y'y'}u')=0$

(B) $\left(F_{yy}-\frac{d}{dx}F_{yy'}\right)u+\frac{d}{dx}(F_{y'y'}u')=0$

(C) $\left(F_{yy}-\frac{d}{dx}F_{y'y'}\right)u-\frac{d}{dx}(F_{yy'}u')=0$

(D) None of the above

60. The Euler's-Ostrogradsky equation for

$$I[U(x,y)]=\iint_D\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]dxdy$$

where the value of u are prescribed on the boundary C of the domain D :

(A) $\nabla^2 U = f(x, y)$

(B) $\nabla U = 0$

(C) $\nabla^2 U = 0$

(D) None of the above

61. External of the isometric problem $\int_a^b (y')^2 dx$ subject to $\int_a^b y dx$ = C is :

(A) $y = \lambda x^2 + ax + b$

(B) $y = \lambda x^3 + ax + b$

(C) $y = \lambda x + b$

(D) $y = \lambda x^3 + ax^2 + bx + C$

62. The extremal of the functional $I[z(x, y)] =$

$$\iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x, y)\right] dxdy$$

where $f(x, y)$ is known function :

(A) $\nabla^2 z = 0$

(B) $\nabla^2 z = 2f(x, y)$

(C) $\nabla^2 z = f(x, y)$

(D) $\nabla^2 z = f(x, y)z$

63. The length of a curve expressed in the parametric from $x = x(t)$, $y = y(t)$ as 't' increases from t_1 to t_2 is given by :

(A) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

(B) $\pi\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

(C) $2\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

(D) None of the above

64. For what curve, the time taken along which the least, when velocity at any point of it is $v = x$:

(A) A family of circles

(B) A family of straight lines

(C) Catenary

(D) Cycloid

65. The shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$ is :

(A) $\frac{4}{\sqrt{5}}$

(B) $2\sqrt{2} - 1$

(C) $\frac{19\sqrt{2}}{8}$

(D) $\frac{19}{8\sqrt{2}}$

66. The shortest distance between parabola $y^2 = 4x$ and the straight line $x + y = -5$ is :

(A) $2\sqrt{2}$

(B) $2\sqrt{2} - 1$

(C) $3\sqrt{2}$

(D) $2\sqrt{3}$

67. The shortest distance between point A (1, 0) and ellipse $4x^2 + 9y^2 = 36$ is :

(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{4}{\sqrt{5}}$

68. The minimum distance between circle $x^2 + y^2 = 1$ and straight line $x + y = 4$ is :

(A) $2\sqrt{2}$

(B) $2\sqrt{2}-1$

(C) $2\sqrt{2}+2$

(D) $3\sqrt{2}-1$

69. The function of the form

$$I[y(x)]=\int_{x_1}^{x_2} g(x,y)\sqrt{1+(y')^2}dx$$

where $g(x, y)$ does not vanish at the movable boundary point x_2 then transversality conditions reduce to :

(A) Legendre's condition

(B) Orthogonality condition

(C) Jacobi condition

(D) Weierstrass condition

70. The extremum of the functional

$$I[y(x)]=\int_0^2 (ey'+3)dx, y(0) = 0,$$

$y(2) = 1$:

(A) $y^2 = \frac{x}{2}$

(B) $y = \frac{x}{2}$

(C) $y = 2x$

(D) $y = x$

71. Use the equation for the functional

$$I[y(x)] = \int_{x_1}^{x_2} F(x, y, y', y'')dx$$

to be an extremum is :

(A) Euler's Lagrange's Equation

(B) Euler's Ostrogradasky Equation

(C) Euler's Poisson Equation

(D) Euler's Equation

72. The extremal of the functional

$$I[y(x)] = \int_{0}^{\log 2} (e^{-x} y'^2 - e^x y^2)dx$$

possesses solution if we substitute :

(A) $x = \log v, y = u$

(B) $x \log u, y = v$

(C) $x = \log u, y = v^2$

(D) $x = \log u, y = u^2$

73. Function $y(x)$ for which $\int_0^1 (x^2 + y'^2)dx$ is stationary given that $\int_1^2 y^2 dx = 2$; $y(0) = 0$, is :

(A) $y = \sin m\pi x$

(B) $y = \pm 2 \sin m\pi x$

(C) $y = \pm 3 \sin m\pi x$

(D) $y = 4 \sin m\pi x$

74. Extremal of the functional

$$I[y(x)] = \int_1^2 \frac{x^3}{(y')^2} dx \quad y(1) = 0 \text{ and}$$

$y(2) = 3$ is :

(A) $y = x^2 - 1$

(B) $y = x^3 - 1$

(C) $y = x - 1$

(D) $y = x^3 - x$

75. For a functional involving n^{th} order derivatives how many boundary conditions are typically required to solve the Euler's-Ostrogradsky equation ?

(A) $n + 1$

(B) n

(C) $2n$

(D) $n - 1$

76. The functional :

$$I[y(x_1)] = \int_0^1 (1+x)(y')^2 dx, \; y(0) = 0, \; y(1) = 1$$

possesses :

(A) Strong maxima

(B) Strong minima

(C) Weak minima but not a strong maxima

(D) Weak maxima but not a strong minima

77. In a conservative field a system moves from t_1 to t_2 in such a way that

$$\int_{t_1}^{t_2} (T - V) dt$$

is an extremum, when :

(A) Maximum

(B) Minimum

(C) Neither Maximum nor Minima

(D) None of the above

78. The Hamilton's cannonical equation of motion is :

(A) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_j = \frac{\partial H}{\partial q_i}$

(B) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_j = -\frac{\partial H}{\partial q_i}$

(C) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_j = -\frac{\partial H}{\partial q_i}$

(D) None of the above

79. In a conservative field the Hamilton's principle is :

(A) $\delta\int_{t_1}^{t_2} \text{L}dt = 0$

(B) $\delta\int_{t_1}^{t_2} \text{L}dt \neq 0$

(C) Both (A) and (B)

(D) None of the above

80. Let q_1, q_2 .... q_n denote generalised coordinate of material system and L = T – V is the Lagrange's function. Then the Lagrange's equations of motion are :

(A) $\frac{\partial \text{L}}{\partial qr} - \frac{d}{dt}\left(\frac{\partial \text{L}}{\partial \dot{q}r}\right) = 0, (r = 1, 2, ...n)$

(B) $\frac{\partial \text{L}}{\partial \dot{q}r} - \frac{d}{dt}\left(\frac{\partial \text{L}}{\partial qr}\right) = 0, (r = 1, 2, ...n)$

(C) $\frac{\partial \text{L}}{\partial \dot{q}r} + \frac{d}{dt}\left(\frac{\partial \text{L}}{\partial qr}\right) = 0, (r = 1, 2, ...n)$

(D) None of the above

81. The mechanical system in which 't' the time, does not enter explicitly in $r_v = r_v$ (q_1, q_2, ... q_n, t) is called :

(A) Holonomic system

(B) Rheonomic system

(C) Scleronomic system

(D) None of the above

82. If H is the Hamiltonian and 'f' is any function depending upon on position, momenta and time, if [H, F] is the Poisson Bracket, then :

(A) $\frac{d\text{F}}{dt} = \frac{\partial \text{F}}{\partial t} - [\text{H}, \text{F}]$

(B) $\frac{d\text{F}}{dt} = -\frac{\partial \text{F}}{\partial t} + [\text{H}, \text{F}]$

(C) $\frac{d\text{F}}{dt} = -\frac{\partial \text{F}}{\partial t} + [\text{H}, \text{F}]$

(D) $\frac{d\text{F}}{dt} = -\frac{\partial \text{F}}{\partial t} - [\text{H}, \text{F}]$

83. For a conservative field, the Lagrange's function (L) of the system is defined as :

(A) L = K.E. – P.E.

(B) L = K.E. + P.E.

(C) L = (K.E. + P.E.)^2

(D) None of the above

84. For a conservative holonomic dynamical system principle of least action is :

(A) $\delta\int_{t_0}^{t_1}(2\mathrm{L})dt=0$

(B) $\delta\int_{t_0}^{t_1}(2\mathrm{E})dt=0$

(C) $\delta\int_{t_0}^{t_1}(2\mathrm{T})dt=0$

(D) $\delta\int_{t_0}^{t_1}(2\mathrm{V})dt=0$

85. In a simple dynamical conservative system sum of kinetic energy and potential energy is :

(A) Not constant

(B) Constant

(C) Zero

(D) Varying

86. The equation of motion of one-dimensional harmonic oscillator is :

(A) $m\dot{x}-kx=0$

(B) $m\ddot{x}+kx=0$

(C) $m\dot{x}+kx=0$

(D) $k\dot{x}+mx=0$

87. Hamiltonian function is :

(A) $\mathrm{H}=\mathrm{L}\sum_k p_k\dot{q}_k$

(B) $\mathrm{H}=-\mathrm{L}+\sum_k p_k\dot{q}_k$

(C) $\mathrm{H}=-\mathrm{L}+\sum_k p_k q_k$

(D) $\mathrm{H}=\mathrm{L}+\sum_k p_k\dot{q}_k$

88. In the Poisson-Bracket Condition, which is true ?

(A) [Q, P] = – 1

(B) [Q, P] = 2

(C) [Q, P] = 0

(D) [Q, P] = 1

89. Which transformation is canonical ?

(A) $Q=\frac{1}{p},\ P=qp^2$

(B) $Q=\frac{1}{p^2},\ P=q^2p$

(C) $Q=\frac{1}{p^3},\ P=p^2q^2$

(D) None of the above

90. Which transformation is cannonical ?

(A) $P=\frac{1}{2}(p^2+q^2),\ Q=\tan^{-1}\left(\frac{p}{q}\right)$

(B) $P=-\frac{1}{2}(p^2+q^2),\ Q=\tan^{-1}\left(\frac{q}{p}\right)$

(C) $P=\frac{1}{2}(p^2+q^2),\ Q=\tan^{-1}\left(\frac{q}{p}\right)$

(D) $P=-\frac{1}{2}(p^2+q^2),\ Q=\tan^{-1}\left(\frac{p}{q}\right)$

91. The value of α and β to be equations $Q = q^{\alpha} \cos \beta p$, $P = q^{\alpha} \sin \beta p$ represent a cannonical transformation :

(A) $\alpha = 2, \beta = \frac{1}{2}$

(B) $\alpha=\frac{1}{2}, \beta=2$

(C) $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$

(D) $\alpha = 2, \beta = 1$

92. The transformation

$$Q = \log\left(\frac{1}{q}\sin p\right),\ p = q \cot p$$

is canonical. Then generating function is :

(A) $F = (p + \cot p)q$

(B) $F = (p - \cot p)q$

(C) $F = (p^2 + \cot p)q$

(D) None of the above

93. The transformation Q = $aq + bp$, P = $cq + dp$ is canonical, if :

(A) $(ad - bc) \neq 1$

(B) $(ad - bc) = 1$

(C) $ad - bc = 0$

(D) $\left(\frac{a}{d} - \frac{b}{c}\right) = 1$

94. The curve along which the integral $I[y(x)] = \int_0^{\pi/2} (x^2 - y^2 + y'^2)dx$ with $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 0$ is :

(A) $y(x) = \sin x$

(B) $y(x) = \sin x + \cos x$

(C) $y(x) = \cos x$

(D) $y(x) = \sin - \cos x$

95. The extremals of the functional $I[y(x)] = \int_a^b (y + xy')dx$, $y(a) = y_0$, $y(b) = y_1$ is :

(A) Family of straight lines

(B) Family of circles

(C) Family of catenaries

(D) No extremal exists

96. For what value of the parameter α the transformation P = $q \sin \alpha + p \cos \alpha$, Q = $q \cos \alpha - p \sin \alpha$ is canonical ?

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) 0

(D) For all values of α

97. The extremum of the functional

$$I[y(x)] = \int_0^a (y')^3 dx, y(0) = 0, y(a) = b$$

(A) $y = \frac{b}{a} x$

(B) $y = \frac{a}{b} x$

(C) $y = \frac{b}{a} x^2$

(D) None of the above

98. The extremum of the functional

$$I[y(x)] = \int_1^2 \frac{x^3}{(y')^2} dx, \ y(1) = 1, \ y(2) = 4$$

is :

(A) Weak minimum exists along $y = x^3$

(B) Weak minimum exists along $y = x^2$

(C) Strong minimum exist along $y = x^2$

(D) Strong maximum exists along $y = x^2$

99. The extremal of the functional

$$I[y(x)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \right] dxdydz$$

is :

(A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(B) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(D) None of the above

100. In calculus of variation gives method to determine maxima or minima of some mathematical terms known as :

(A) Functions

(B) Admissible functions

(C) Functionals

(D) General relativity

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 Ⓐ ● Ⓒ Ⓓ

Q. 2 Ⓐ Ⓑ ● Ⓓ

Q. 3 Ⓐ ● Ⓒ Ⓓ

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ

प्रश्न 2 Ⓐ Ⓑ ● Ⓓ

प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमत: जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।