Roll No. ....

O. M. R. Serial No. | | | | | | | |

Question Booklet Number

M. A./M. Sc. (Fourth Semester) (NEP) EXAMINATION, 2025-26

MATHEMATICS

(Operator Theory) (Elective)

Paper Code							
B	0	3	1	0	0	8	T

Questions Booklet Series
A

Time : 1:30 Hours] [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.

2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.

3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।

2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।

3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. A Hilbert space is a :

(A) Normed linear space

(B) Complete inner product space

(C) Metric space

(D) Linear space

2. Every Hilbert space is a Banach space because :

(A) It has a norm

(B) It is complete with respect to the norm induced by inner product

(C) It is finite dimensional

(D) It is compact

3. An operator T on a Hilbert space is self-adjoint if :

(A) $T = T^{-1}$

(B) $T^{*} = T$

(C) $T^{*}T = 1$

(D) $T^{2} = T$

4. A unitary operator U satisfies :

(A) $U^{*} = U$

(B) $UU^{*} = I$

(C) $U^{2} = 1$

(D) $U = 0$

5. A projection operator P satisfies :

(A) $P^{2} = P$

(B) $P^{*} = -P$

(C) P^{-1} exists

(D) $P^{2} = 1$

6. An operator T is compact if :

(A) It maps bounded sets into relatively compact sets

(B) It is invertible

(C) It is finite dimensional

(D) It is self-adjoint

7. In finite dimensional spaces, every linear operator is :

(A) Compact

(B) Self-adjoint

(C) Unitary

(D) Normal

8. The spectrum of a bounded linear operator T is :

(A) always finite

(B) always empty

(C) a non-empty set

(D) infinite only

9. The resolvent set of an operator T consists of λ, such that :

(A) $(T-\lambda I)$ is not invertible

(B) $(T-\lambda I)$ is invertible and bounded

(C) $\lambda = 0$

(D) $T = \lambda I$

10. The spectral radius $r(\mathrm{T})$ is given by :

(A) $\max \| \mathrm{T}x \|$

(B) $\sup \{ |\lambda| : \lambda \in \text{spectrum}\ (\mathrm{T}) \}$

(C) $\| \mathrm{T} \|$

(D) $\min\{ |\lambda| \}$

11. The spectral radius formula is :

(A) $r(\mathrm{T}) = \| \mathrm{T} \|$

(B) $r(\mathrm{T}) = \lim \| \mathrm{T}^n \| \wedge (1/n)$

(C) $r(\mathrm{T}) = \det(\mathrm{T})$

(D) $r(\mathrm{T}) = \mathrm{Trace}(\mathrm{T})$

12. A banach algebra is :

(A) A complete normed algebra

(B) A group

(C) A metric space only

(D) An inner product space

13. Gelfand Naimark theorem is related to :

(A) Banach algebra

(B) Differential equations

(C) Group theory

(D) Number theory

14. In a commutative Banach algebra with identity, maximal ideals correspond to :

(A) eigen values

(B) continuous homomorphism

(C) zero operator

(D) compact sets

15. An operator T is normal if :

(A) TT = 1

(B) $\mathrm{TT}^* = \mathrm{T}^*\mathrm{T}$

(C) $\mathrm{T} = \mathrm{T}^*$

(D) $\mathrm{T}^2 = \mathrm{T}$

16. If T is self-adjoint, then its spectrum lies in :

(A) Complex plane

(B) Unit circle

(C) Real line

(D) Imaginary axis

17. A unitary operator preserves :

(A) Norm only

(B) Inner product

(C) Determinant

(D) Eigen values

18. If P is an orthogonal projection, then P is :

(A) Self-adjoint and idempotent

(B) Unitary

(C) Compact always

(D) Invertible always

19. The non-zero spectrum of a compact operator (on infinite dimensional space) consists of :

(A) finite elements only

(B) eigen values with finite multiplicity

(C) only zero

(D) continuous spectrum only

20. Zero is always in the spectrum of a compact operator on infinite dimensional space because :

(A) It is self-adjoint

(B) It is normal

(C) It is not invertible

(D) Spectrum is empty

21. For a bounded operator T, the spectrum is :

(A) closed

(B) open

(C) always infinite

(D) empty

22. The resolvent operator is defined as :

(A) $(T+\lambda I)$

(B) $(T-\lambda I)^{-1}$

(C) $T\lambda$

(D) T^{-1}

23. The resolvent function is :

(A) Discontinuous

(B) Analytic on resolvent set

(C) Constant

(D) Periodic

24. The spectrum of an operator is divided into :

(A) Two parts

(B) Three parts

(C) Four parts

(D) Five parts

25. Let H be a Hilbert space and $T: H \to H$ be a bounded linear operator. Then T is continuous if and only if :

(A) T is compact

(B) T is linear

(C) T is bounded

(D) T is self-adjoint

26. The operator T^* of a bounded operator T on a Hilbert space satisfies :

(A) $< Tx, y > = < x, T^* y >$

(B) $< Tx, y > = < T^* x, y >$

(C) $< Tx, y > = < y, T^* x >$

(D) None of the above

27. What is an operator ?

(A) A function between vector spaces

(B) A matrix representation of a linear transformation

(C) A bounded linear transformation between normed spaces

(D) A differential equation

28. Which of the following is an example of a linear operator ?

(A) Differentiation operator

(B) Integration operator

(C) Both (A) and (B)

(D) None of the above

29. What is the spectrum of an operator ?

(A) The set of all eigen values

(B) The set of all eigen vectors

(C) The set of all complex numbers λ such that $(T - \lambda I)$ is not invertible

(D) The set of all bounded linear functionals

30. Which of the following is a property of a self-adjoint operator ?

(A) $T = T^*$

(B) $T = -T^*$

(C) $T = T^*(-1)$

(D) $T = 0$

31. Which of the following types of operators on a Hilbert space is always bounded ?

(A) Self-adjoint operator

(B) Unitary operator

(C) Projection operator

(D) All of the above

32. What is the spectrum of a self-adjoint operator on a Hilbert space ?

(A) A subset of the real numbers

(B) A subset of the complex numbers

(C) A subset of the integers

(D) A subset of the rational numbers

33. A Banach algebra is a :

(A) Normed linear space only

(B) Complete normed algebra over R or C

(C) Inner product space

(D) Metric space with multiplications

34. In a Banach algebra A, the norm satisfies :

(A) $|xy| = |x| + |y|$

(B) $|xy| \leq |x||y|$

(C) $|xy| \geq |x||y|$

(D) None of the above

35. Every Banach algebra is :

(A) Complete with respect to its norm

(B) Finite dimensional

(C) Commutative

(D) Unital

36. The spectral radius of an element a in a Banach algebra is defined as :

(A) $\sup|\lambda|:\lambda\in\sigma(a)$

(B) $\inf|\lambda|:\lambda\in\sigma(a)$

(C) $|a|$

(D) $|a^{-1}|$

37. The spectrum of an element in a complex Banach algebra is :

(A) Always empty

(B) Always finite

(C) Non-empty and compact

(D) Always countable

38. The Gelfand-Mazur theorem states that every complex Banach division algebra is isometrically isomorphic to :

(A) R

(B) C

(C) R^2

(D) None of the above

39. If A is a Banach algebra with identity e, then $|e|$ is :

(A) always 0

(B) always infinite

(C) always 1 (in standard normalization)

(D) Less than 1

40. The resolvent set of an element $a\in A$ is :

(A) set of eigen values

(B) set of λ such that $a-\lambda\in$ is invertible

(C) set where inverse does not exist

(D) empty set

41. In a Banach algebra, the spectral radius formula is :

(A) $r(a)=|a|$

(B) $r(a)=\lim_{n\to\infty}|a^n|$

(C) $r(a)=\lim_{n\to\infty}|a^n|^{\frac{1}{n}}$

(D) None of the above

42. A commutative Banach algebra with identity allows the use of :

(A) Fourier transform only

(B) Gelfand transform

(C) Laplace transform

(D) Z-transform

43. The maximal ideal space of a commutative Banach algebra consists of :

(A) All ideals

(B) All maximal left ideals

(C) Non-zero multiplicative linear functionals

(D) All bounded operators

44. If a is invertible in a Banach algebra, then :

(A) $0 \in \sigma(a)$

(B) $0 \notin \sigma(a)$

(C) $\sigma(a) = \{0\}$

(D) $\sigma(a)$ is empty

45. The unitization of a Banach algebra is needed when the algebra :

(A) is not complete

(B) has no identity element

(C) is commutative

(D) is finite dimensional

46. Which of the following is a Banach Algebra ?

(A) C[0, 1] with sup. norm and point wise multiplication

(B) Any incomplete normed algebra

(C) Any vector space

(D) Any Hilbert space with no multiplication

47. In a Banach algebra, the spectrum $\sigma(a)$ is contained is :

(A) $\lambda : |\lambda| > |a|$

(B) $\lambda : |\lambda| \leq |a|$

(C) R only

(D) Empty set

48. Which of the following is an example of a Banach algebra ?

(A) The space of bounded linear operator on a Hilbert space

(B) The space of continuous functions on a compact Hausdorff space

(C) The space of integrable functions on a measure space

(D) All of the above

49. Which of the following is a property of the Gual'fand transform ?

(A) It is an isomorphism

(B) It is a homomorphism

(C) It is a linear functional

(D) None of the above

50. Let T be a bounded linear operator on a finite-dimensional vector space. The determinant of T is equal to :

(A) sum of eigen values

(B) Product of eigen values (counting multiplicity)

(C) Maximum eigen value

(D) None of the above

51. If T is an invertible bounded linear operator on a Banach space, then 0 belongs to :

(A) Point spectrum

(B) Continuous spectrum

(C) Resolvent set

(D) Residual spectrum

52. The spectrum $\sigma(T)$ of a bounded operator T on a Banach space is :

(A) Always empty

(B) Always finite

(C) Always non-empty and compact

(D) Always countable

53. For a matrix $A \in Mn(c)$ the determinant satisfies :

(A) $\det(\lambda I - A) = 0$ for all λ

(B) zeros of $\det(\lambda I - A)$ are eigenvalues of A

(C) det(A) = trace(A)

(D) det(A) = 0 for all singular matrices

54. If T is a compact operator on an infinite dimensional Banach space, then its non-zero spectrum consists of :

(A) Only continuous spectrum

(B) Only eigen values with finite multiplicity

(C) Only residual spectrum

(D) Entire complex plane

55. If $\det(T) \neq 0$ (finite dimensional case), then T is :

(A) Compact

(B) Normal

(C) Invertible

(D) Self-adjoint

56. Which of the following is always true for the spectrum of a bounded operator T ?

(A) $\sigma(T) \subset R$

(B) $\sigma(T)$ is closed

(C) $\sigma(T)$ is open

(D) $\sigma(T) = 0$

57. For a finite-dimensional operator T, the spectral radius satisfies :

(A) $r(T) = (T)$ always

(B) $r(T) = \max|\lambda_i|$ where λ_i are eigen values

(C) $r(T) = \det(T)$

(D) $r(T) = \text{trace}(T)$

58. If T is a bounded operator and $\lambda \in \sigma(T)$, then $(T - \lambda I)^{-1}$ is :

(A) unbounded

(B) not defined

(C) bounded and everywhere defined

(D) compact

59. In infinite-dimensional Banach spaces, determinant of a general bounded operator is :

(A) Always defined

(B) Defined only for normal operators

(C) Not defined in general

(D) Equal to spectral radius

60. The resolvent set of an operator T is :

(A) $c \backslash \sigma(T)$

(B) set of eigen values

(C) Kernel of T

(D) Range of T

61. Let $T : V \to V$ be a linear operator on a finite-dimensional vector space V. The determinant of T is defined as :

(A) Trace of the matrix of T

(B) Determinant of any matrix representation of T

(C) Rank of T

(D) Norm of T

62. If T is a linear operator on an *n*-dimensional vector space and $\det(T) \neq 0$, then T is :

(A) Nilpotent

(B) Invertible

(C) Idempotent

(D) Singular

63. If T is a linear operator with eigen values $\lambda_1, \lambda_2, ..., \lambda_n$, then :

(A) $\det(T) = \Sigma\lambda_i$

(B) $\det(T) = \max \lambda_i$

(C) $\det(T) = \pi\lambda_i$ (counting multiplicity)

(D) $\det(T) = 0$ always

64. If T is a linear operator on an infinite dimensional Banach space, then the determinant of T :

(A) is always defined

(B) is defined only for compact operators

(C) is not defined in general

(D) equal the spectral radius

65. The spectral theorem for finite dimensional inner product spaces applied primarily to :

(A) Nilpotent operators

(B) Self-adjoint operators

(C) Arbitrary linear operators

(D) Rank-one operators

66. According to the spectral theorem, every self adjoint operator on a finite-dimensional inner product space is :

(A) Nilpotent

(B) Diagonalizable with real eigen values

(C) Singular

(D) Non-invertible

67. It T is a normal operator on a finite-dimensional complex Hilbert space, then by the spectral theorem :

(A) T is diagonalizable by a unitary matrix

(B) T is always nilpotent

(C) T has only zero eigen values

(D) T is not diagonalizable

68. The spectral theorem for bounded self-adjoint operators on a Hilbert space states that T can be represented as :

(A) A Jordon matrix

(B) A sum of projections weighted by eigen values (spectral decomposition)

(C) A nilpotent operator

(D) A shift operator

69. Which of the following operators always satisfies the spectral theorem ?

(A) Unitary operator

(B) Self-adjoint operator

(C) Normal operator

(D) All of the above

70. It T is self-adjoint, then eigenvectors corresponding to distinct eigen values are :

(A) Linearly dependent

(B) Orthogonal

(C) Equal

(D) Zero vectors

71. The spectral theorem in infinite-dimensional Hilbert spaces uses :

(A) Determinants

(B) Spectral measures and projection-valued measure

(C) Only eigenvalues

(D) Jordan canonical form

72. Let A be a Banach-algebra with identity e. Then the norm of the identity element satisfies :

(A) $|e| = 0$

(B) $|e| \geq 1$

(C) $|e| \leq 1$

(D) $|e| = 1$

73. Let A be a Banach algebra with identity e if $|a| < 1$, then $e - a$ is :

(A) Nilpotent

(B) Invertible

(C) Idempotent

(D) Zero Divisor

74. Let A be a unital Banach algebra. The spectral radius $r(a)$ of $a \in \mathrm{A}$ is defined as :

(A) $|a^{-1}|$

(B) $\sup. |\lambda| : \lambda \in \sigma(a)$

(C) $\inf |\lambda| : \lambda \in \sigma(a)$

(D) a^2

75. If A is a Banach algebra with identity and a is invertible, then the set of invertible elements in A is :

(A) Closed

(B) Open

(C) Compact

(D) Dense

76. If A is a Banach algebra with identity e, the resolvent set of an element a is :

(A) $\lambda \in c : a - \lambda e$ is invertible

(B) $\lambda \in \mathrm{R} : a = \lambda e$

(C) $\sigma(a)$

(D) The set of eigen values only

77. Which of the following is a division algebra over R ?

(A) R

(B) C

(C) The quaternions H

(D) All of the above

78. A bounded linear operator on a Hilbert-space is always :

(A) Closed and Continuous

(B) Continuous only

(C) Compact

(D) Self-adjoint

79. If $T = T^*$, then T is called :

(A) Normal operator

(B) Unitary operator

(C) Self-adjoint operator

(D) Projection operator

80. A unitary operator U on a Hilbert space satisfies :

(A) $U^2 = U$

(B) $UU^* = I$

(C) $U = U^*$

(D) $|U_x| = 0$

81. Every unitary operator is :

(A) Self adjoint

(B) Normal

(C) Compact

(D) Nilpotent

82. A projection operator P on a Hilbert-space satisfies :

(A) $P^2 = I$

(B) $P^2 = P$

(C) $P^* = -P$

(D) $|P| = 0$

83. A compact operator on a Hilbert-space maps :

(A) Closed set into open sets

(B) Bounded sets into relatively compact sets

(C) Compact sets into bounded sets

(D) Dense sets into closed sets

84. If T is normal, then :

(A) $TT^* = T^*T$

(B) $T = T^*$

(C) $T^2 = T$

(D) None of the above

85. Every self-adjoint operator is :

(A) Normal

(B) Unitary

(C) Compact

(D) Nilpotent

86. The identity operator on a Hilbert space is :

(A) Compact

(B) Self adjoint and unitary

(C) Nilpotent

(D) Unbounded

87. The adjoint of an operator exists for :

(A) any linear operator

(B) only compact operators

(C) every bounded linear operator on a Hilbert-space

(D) only self-adjoint operators

88. An operator T is positive if :

(A) $< T_x, x > \geq 0$ for all $x \in H$

(B) $|T| > 0$

(C) $T = I$

(D) $T^2 = T$

89. The spectral theorem applies to :

(A) Any bounded operator

(B) Normal operators

(C) Nilpotent operators

(D) None of the above

90. A bounded operator on a Hilbert space is automatically :

(A) Closed and continuous

(B) Compact

(C) Self-adjoint

(D) Finite rank

91. An orthogonal projection P on a Hilbert space satisfies :

(A) $P^2 = I$

(B) $P^* = -P$

(C) $P^2 = P$ and $P^* = P$

(D) $P^* P = 0$

92. If P is an orthogonal projection on a Hilbert space H, then the range and null space satisfies :

(A) Ran(P) = Ker(P)

(B) Ran(P) $\perp$ Ker(P)

(C) Ker(P) = 0

(D) None of the above

93. Let M be a closed subspace of a Hilbert-space H. There exists a unique orthogonal projection such that :

(A) Ran(P) = M

(B) Ker(P) = M

(C) $P(H) = M^{\perp}$

(D) P = 0

94. If P is an orthogonal projection, then the spectrum $\sigma(P)$ is :

(A) 1

(B) 0, 1

(C) [– 1, 1]

(D) 0

95. For an orthogonal projection P on a Hilbert-space, then norm $|P|$ is :

(A) 0

(B) $1 \text{ (if } P \neq 0)$

(C) 2

(D) depends on the dimension of H

96. Which of the following operators on a Hilbert-space is always self-adjoint ?

(A) Any Projection

(B) Orthogonal projection

(C) Unitary operator

(D) None of the above

97. If P is an orthogonal projection on H, then for any $x \in H$:

(A) $|P_x| = |x|$

(B) $P_x = 0$

(C) $< P_x, x - P_x > = 0$

(D) None of the above

98. The residual spectrum of an operator T consists of λ such that :

(A) $T - \lambda I$ is not injective

(B) $T - \lambda I$ has dense range

(C) $T - \lambda I$ is injective but range is not dense

(D) $T - \lambda I$ is invertible

99. If T is a bounded self-adjoint operator then :

(A) $|T| = r(T)$

(B) $|T| = r(T)^2$

(C) $|T| = 0$

(D) $r(T) = 0$

100. For a compact operator on a Hilbert space, zero is always :

(A) In the resolvent set

(B) In the spectrum

(C) An eigen value necessarily

(D) Outside the spectrum

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 (A) ● (C) (D)
Q. 2 (A) (B) ● (D)
Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)
प्रश्न 2 (A) (B) ● (D)
प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।