

Roll No.

Question Booklet Number

O. M. R. Serial No.

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M. A./M. Sc. (Fourth Semester)
(NEP) EXAMINATION, 2025-26
MATHEMATICS

(Differential Geometry of Manifolds) (Elective)

Paper Code							
B	0	3	1	0	0	6	T

Questions Booklet
Series

D

Time : 1:30 Hours]

[Maximum Marks : 75

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. If $A_{\alpha\beta} = A_{\beta\alpha}$, then :

- (A) $A_{\alpha\beta}$ is an anti-symmetric tensor
- (B) $A_{\alpha\beta}$ is an asymmetric tensor
- (C) $A_{\alpha\beta}$ is a symmetric tensor
- (D) None of the above

2. If $\vec{\omega}$ is any arbitrary contravariant vector and $A_{ij}\vec{\omega}$ is a covariant vector, then :

- (A) A_{ij} is a contravariant tensor of rank 2
- (B) A_{ij} is a covariant tensor of rank 2
- (C) A_{ij} is a covariant tensor of rank 1
- (D) None of the above

3. The length L of a tensor A^i is given by :

- (A) $L = \sqrt[3]{A^i A_j}$
- (B) $L = \sqrt[i]{A^i A_j}$
- (C) $L = \sqrt{A^i A_j}$
- (D) None of the above

4. The metric tensor for two dimensional plane in polar coordinates is given by :

- (A) $ds^2 = dr^2 + r^2 d\theta^2$
- (B) $ds^2 = dr^2 - r^2 d\theta^2$
- (C) $ds^2 = dr^2 + 2r^2 d\theta^2$
- (D) $ds^2 = dr^2 - 2r^2 d\theta^2$

5. If $x^1 = r$ and $x^2 = \theta$, then the metric tensor $g_{\mu\nu}$ in the matrix form is :

- (A) $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 1 & r^2 \end{bmatrix}$
- (B) $g_{\mu\nu} = \begin{bmatrix} 1 & 1 \\ 0 & r^2 \end{bmatrix}$
- (C) $g_{\mu\nu} = \begin{bmatrix} 0 & 1 \\ 0 & r^2 \end{bmatrix}$
- (D) $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$

6. If A^μ and B_μ are any two vectors, then $A^\mu \cdot B_\mu$ is the :
- (A) rank two tensor
 (B) invariant
 (C) rank one tensor
 (D) None of the above
7. Kronecker delta is a :
- (A) mixed tensor of rank 2
 (B) mixed tensor of rank 3
 (C) mixed tensor of rank 1
 (D) None of the above
8. If A_i and B_j be two vectors such that $K_{ij} A_i B_j$ is invariant, then K_{ij} is a :
- (A) First order tensor
 (B) Second order tensor
 (C) Third order tensor
 (D) Fourth order tensor
9. If $A_\sigma^{\alpha\beta}$ and B_ρ^γ are two tensors of rank 3 and 2 respectively, then the rank of $A_\sigma^{\alpha\beta} \cdot B_\rho^\gamma$ is equal to :
- (A) 1
 (B) 6
 (C) 5
 (D) 2
10. Outer product of two tensors is a tensor whose rank is :
- (A) the sum of the ranks of the given tensors
 (B) the product of the ranks of the given tensors
 (C) the difference of the ranks of the given tensors
 (D) None of the above

11. If $\bar{T}_\alpha = \frac{\partial x_l}{\partial \bar{x}_\alpha} T_l$ and $\bar{S}^{\alpha\beta} = \frac{\partial \bar{x}^\alpha}{\partial x^m} \frac{\partial \bar{x}^\beta}{\partial x^n} S^{mn}$, then :
- (A) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial \bar{x}^\beta}{\partial x^n} T_l S^{ln}$
- (B) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial \bar{x}_\beta}{\partial x^n} T_l S^{ln}$
- (C) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial x_\beta}{\partial x^n} T_l S^{ln}$
- (D) None of the above
12. In n -dimensional space, the distance between two neighbouring points with coordinates x^μ and $x^\mu + dx^\mu$ is given by :
- (A) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n g_{\mu\nu} x^\mu x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (B) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n x^\mu / x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (C) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n x^{-\mu} x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (D) None of the above
13. The distance between two adjacent points (x, y, z) and $(x + dx, y + dy, z + dz)$ in Cartesian Coordinate is given by:
- (A) $ds^2 = dx^2 - dy^2 + dz^2$
- (B) $ds^2 = dx^2 + dy^2 + dz^2$
- (C) $ds^2 = dx^2 + dy^2 - dz^2$
- (D) None of the above
14. If g_{ij} is a contravariant tensor of rank 2 and if $A_{ij} = \frac{1}{2} (g_{ij} + g_{ji})$, then :
- (A) A_{ij} is a negative tensor
- (B) A_{ij} is a asymmetric tensor
- (C) A_{ij} is a symmetric tensor
- (D) None of the above
15. If $dS^2 = g_{ij} dx^i dx^j$ is invariant, then :
- (A) g_{ij} is a symmetric covariant tensor of rank 2
- (B) g_{ij} is a non-symmetric covariant tensor of rank 2
- (C) g_{ij} is a symmetric covariant tensor of rank 3
- (D) None of the above

16. The acceleration is a rank of :
- (A) 3
 - (B) 2
 - (C) 4
 - (D) None of the above
17. The velocity is a rank of :
- (A) 1
 - (B) 2
 - (C) 3
 - (D) None of the above
18. The gradient of a field is a tensor named :
- (A) nonlinear
 - (B) covariant
 - (C) contravariant
 - (D) None of the above
19. The velocity is a tensor named :
- (A) contravariant
 - (B) covariant
 - (C) linear
 - (D) None of the above
20. If in an algebraic operation, the rank of mixed tensor is lowered by 2, then it is known as :
- (A) linear
 - (B) deformation
 - (C) contraction
 - (D) None of the above
21. The necessary and sufficient condition that a geodesics be a plane curve is that :
- (A) it is helix
 - (B) it is line of curvature
 - (C) it is parabola
 - (D) None of the above

22. The geodesics on any cylinder are :
- (A) lines
(B) parabolas
(C) ellipses
(D) helices
23. The curves of the family $v^2u^{-2} =$ constant are geodesics on a surface with a metric ($u > 0, v > 0$) given by :
- (A) $v^2du^2 - 2uvdudv - 2u^2dv^2$
(B) $v^2du^2 - 2uvdudv + 3u^2dv^2$
(C) $v^2du^2 - 2uvdudv + 2u^2dv^2$
(D) None of the above
24. The equation of the developable surface which passes through the curve $z = 0, y^2 = 4ax, x = 0, y^2 = 4bz$ is :
- (A) $y^2 = 4ax + 4bz$
(B) $y^2 = 4ax - 4bz$
(C) $y^2 + 4ax + 4bz = 0$
(D) $y^2 - 4ax + 4bz = 0$
25. On the surface $z = f(x, y)$, the asymptotic lines are :
- (A) $rdx^2 - 2sdx dy + tdy^2$
(B) $rdx^2 + 2sdx dy + tdy^2$
(C) $rdx^2 + 2sdx dy - tdy^2$
(D) None of the above
26. If Gaussian curvature K is zero, then :
- (A) the surface is generator
(B) the surface is principal curvature
(C) the surface is developable
(D) None of the above
27. If the curve is $x = 6t, y = 3t^2, z = 2t^3$ for its edge of regression of developable surface, then its equation is :
- (A) $(3xz - 4y^2)(12y - x^2) = (9z - xy)^2$
(B) $(3xz - 4y^2)(12y + x^2) = (9z - xy)^2$
(C) $(3xz - 4y^2)(12y - x^2) = (9z + xy)^2$
(D) None of the above

28. If the given curve is a line of curvature, there exists :
- (A) $\frac{dN}{ds} - \kappa \frac{d\tau}{ds} = 0$
- (B) $\frac{dN}{ds} + 2\kappa \frac{d\tau}{ds} = 0$
- (C) $\frac{dN}{ds} + \kappa \frac{d\tau}{ds} = 0$
- (D) None of the above
29. The edge of regression of the rectifying developable of a space curve has its equation is :
- (A) $R - r = \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'}$
- (B) $R + r = \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'}$
- (C) $R - r + \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'} = 0$
- (D) $R + r - \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'} = 0$
30. The generators of the osculating developable of a twisted curve are tangents to the curve and its edge of regression is :
- (A) normal
- (B) tangent
- (C) curve itself
- (D) None of the above
31. If Gaussian curvature at a point P on the surface is positive, then the surface is :
- (A) elliptic
- (B) parabolic
- (C) linear
- (D) None of the above
32. If ψ is the angle between the direction (du, dv) of the normal section and the normal curvature κ_n at a point on a surface is given in terms of principal curvature κ_a and κ_b is given by :
- (A) $\kappa_n = \kappa_a \cos^2 \psi - \kappa_b \sin^2 \psi$
- (B) $\kappa_n = \kappa_a \sin^2 \psi - \kappa_b \cos^2 \psi$
- (C) $\kappa_n = \kappa_a \cos^2 \psi + \kappa_b \sin^2 \psi$
- (D) None of the above
33. The necessary and sufficient condition that a curve on a surface be a line of curvature is that :
- (A) $\kappa d\tau + dN = 0$ at each of its points
- (B) $\kappa d\tau - dN = 0$ at each of its points
- (C) $\kappa d\tau \pm dN + 1 = 0$ at each of its points
- (D) None of the above

34. The principal curvature for the surface $z = f(x, y)$ is :

(A) $\kappa_n^2(EG - F^2) - \kappa_n(EN + LG - 2FM) + (LN + M^2) = 0$

(B) $\kappa_n^2(EG - F^2) - \kappa_n(EN - LG - 2FM) + (LN - M^2) = 0$

(C) $\kappa_n^2(EG + F^2) - \kappa_n(EN + LG - 2FM) + (LN - M^2) = 0$

(D) $\kappa_n^2(EG - F^2) - \kappa_n(EN + LG - 2FM) + (LN - M^2) = 0$

35. If on the paraboloid $x^2 - y^2 = z$, the sections are by the plane $z = \text{constant}$, then orthogonal trajectories of the section are :

(A) $\frac{x}{y} = \text{constant}$

(B) $xy = \text{constant}$

(C) $xy^2 = \text{constant}$

(D) $x^2y = \text{constant}$

36. If L, MN vanish everywhere on a surface, then :

(A) the surface is part of a plane

(B) the surface is a plane

(C) there is no surface

(D) None of the above

37. The equation of the edge of regression is :

(A) $F(a) = \text{constant}, \frac{\partial}{\partial a} F(a) = \text{constant}$

(B) $F(a) = \text{constant}, \frac{\partial}{\partial a} F(a) = 0$

(C) $F(a) = 0, \frac{\partial}{\partial a} F(a) = 0$

(D) $F(a) = 0, \frac{\partial}{\partial a} F(a) = \text{constant}$

38. If the surface is given by $z = x^2 + y^2$, then the equation of tangent plane at point $(1, -1, 2)$ is :

(A) $2x - 2y + z = 2$

(B) $2x - 2y - z = 2$

(C) $2x + 2y - z = 2$

(D) None of the above

39. If a plane curve has only a single evolute in its own plane, then it is :
- (A) the locus of the centre of curvature
 (B) the locus of the half of centre of curvature
 (C) the locus of the twice of centre of curvature
 (D) None of the above
40. The tangent to two different evolutes corresponding to the value a_1 and a_2 of the parameter c drawn from the same point of the given curve are inclined to each other at the constant angle :
- (A) $a_1 + a_2$
 (B) $2c(a_1 + a_2)$
 (C) $2c(a_1 - a_2)$
 (D) $(a_1 - a_2)$
41. The distance between corresponding points of two involutes is :
- (A) indefinite
 (B) variable
 (C) constant
 (D) None of the above
42. For a cubic curve, the torsion τ is :
- (A) $\frac{3}{(9u^4 - 9u^2 + 1)}$
 (B) $\frac{3}{(9u^4 + 9u^2 + 1)}$
 (C) $\frac{3}{(9u^4 + 9u^2 - 1)}$
 (D) $\frac{3}{(9u^4 - 9u^2 - 1)}$
43. If $x = 3u, y = 3u^2, z = 2u^3$, then there exist a relation :
- (A) $\rho = \frac{3}{2}(1 + 2u^2)^2$
 (B) $\rho = \frac{3}{2}(1 - 2u^2)^2$
 (C) $\rho = \frac{5}{2}(1 + 2u^2)^2$
 (D) None of the above
44. If $x = a \cos \theta, y = a \sin \theta$ and $z = f(\theta)$, and determines a plane curve, then :
- (A) $\frac{df}{d\theta} - \frac{d^2f}{d\theta^2} = 0$
 (B) $\frac{df}{d\theta} + \frac{d^2f}{d\theta^2} = 0$
 (C) $\frac{df}{d\theta} + 2\frac{d^2f}{d\theta^2} = 0$
 (D) None of the above

45. If m_1, m_2, m_3 are the moments about the origin of unit vectors t, n, b localized in the tangent, principal normal, and binormal, then the following relation exists :
- (A) $m_1' = \tau m_2$
 (B) $m_1' = -\kappa m_2$
 (C) $m_1' = \kappa m_2$
 (D) None of the above
46. If the position vector r of a current point on the curve is a function of t , then the true relation is :
- (A) $\dot{r} = \dot{s}t$
 (B) $\dot{r} = \dot{s} \times t$
 (C) $\dot{r} = -\dot{s}t$
 (D) None of the above
47. The true relation is :
- (A) $\tau = \frac{[\dot{r}\ddot{r}\ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
 (B) $\tau = \frac{[\dot{r}\ddot{r}\ddot{r}]}{[\dot{r} \times \ddot{r}]^{1/2}}$
 (C) $\tau = -\frac{[\dot{r}\ddot{r}\ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
 (D) $\tau = \frac{[\dot{r}\ddot{r}\ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
48. The necessary and sufficient condition for the curve to be a plane curve is :
- (A) $[\dot{r}\ddot{r}\ddot{r}] = 0$
 (B) $[\dot{r}\ddot{r}\ddot{r}] = 0$
 (C) $[\dot{r}\ddot{r}\ddot{r}] = 0$
 (D) $[\dot{r}\ddot{r}\ddot{r}] = 0$
49. The equation of the tangent line to the curve $x = t, y = t^2, z = t^3$ at the point $t = 1$, is :
- (A) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$
 (B) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$
 (C) $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z-1}{3}$
 (D) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z+1}{3}$
50. A real valued function f defined over a real interval I is said to be of class n , n being a positive integer, if it possesses :
- (A) continuous derivatives of n th order at each point of I
 (B) partial derivatives of $n + 1$ th order at each point of I
 (C) semi continuous derivatives of n th order at each point of I
 (D) continuous derivatives of $n + 1$ th order at each point of I

51. If α be a geodesic on a 2-surface S ,
Then a vector field X tangent to S along
 α is parallel along α if and only if :
- (A) $\|X\|$ and the angle between $\|X\|$
and $\dot{\alpha}$ are constants
- (B) $\|X\|$ is constant
- (C) $\|X\|$ and $\dot{\alpha}$ are constants
- (D) $\|X'\|$ and $\dot{\alpha}'$ are constants
52. If X is Euclidean parallel, then :
- (A) X is constant
- (B) X is Levi-Civita parallel
- (C) X is oscillatory
- (D) None of the above
53. If X and Y be two vector fields along a
parameterized curve α on an n -surface,
then $(X.Y)'$ is equal to :
- (A) $\dot{X}.Y + X.\dot{Y}$
- (B) $X.Y + X.Y'$
- (C) $\dot{X}.Y' + X'.\dot{Y}$
- (D) Both (A) and (B)
54. What is the relationship between the
acceleration $\ddot{\alpha}$ and the covariant
acceleration $\dot{\alpha}'$ of a parameterized
curve α on an n -surface :
- (A) one is a scalar multiple of the
other
- (B) $\dot{\alpha}'$ is the projection of $\ddot{\alpha}$ to the
normal space
- (C) $\ddot{\alpha}$ is the projection of $\dot{\alpha}'$ to the
tangent space
- (D) $\dot{\alpha}'$ is the projection of $\ddot{\alpha}$ to the
tangent space
55. What is the relationship between the
ordinary derivative \dot{X} and X' of a vector
field X on an n -surface ?
- (A) \dot{X} is the projection of X' to the
tangent space
- (B) \dot{X} is the projection of X' to the
normal space
- (C) X' is the projection of \dot{X} to the
tangent space
- (D) One is a scalar multiple of the
other

56. Covariant derivative of a vector field X on an n -surface S is given by :

- (A) $\dot{X}(t) + \{\dot{\alpha}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (B) $\dot{X}(t) - \{\dot{X}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (C) $\dot{X}(t) + \{\dot{X}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (D) $\dot{X}(t) - \{\dot{\alpha}(t) \cdot N(\alpha(t))\}N(\alpha(t))$

57. An n -surface S in R^{n+1} is said to be geodesically complete, if :

- (A) domain of every maximal geodesic is R
- (B) image of every maximal geodesic is S
- (C) image of every maximal geodesic is complete subset of S
- (D) domain of every maximal geodesic is complete subset of R

58. $\alpha : I \rightarrow S$ is a geodesic on an n -surface S in R^{n+1} , then :

- (A) α is orthogonal to S
- (B) speed of α is constant
- (C) $\dot{\alpha} \perp \ddot{\alpha}$
- (D) All of the above

59. $\alpha : I \rightarrow S$ is a geodesic if and only if it satisfies the differential equation :

- (A) $\ddot{\alpha} + (\dot{\alpha} \cdot \dot{N}\alpha)(N\alpha) = 0$
- (B) $\ddot{\alpha} + (\dot{\alpha} \cdot N\alpha)(\dot{N}\alpha) = 0$
- (C) $\dot{\alpha} + (\ddot{\alpha} \cdot \dot{N}\alpha)(N\alpha) = 0$
- (D) $\dot{\alpha} + (\ddot{\alpha} \cdot N\alpha)(N\alpha) = 0$

60. On the sphere, a maximal geodesic in the 2-sphere is always a part of :

- (A) a circle
- (B) an ellipse
- (C) great circles
- (D) straight lines

61. A geodesic in the cylinder $x_1^2 + x_2^2 = 1$ is always of form :
- (A) $(\cos(at + b), \sin(at + b), ct + d)$
 (B) $(\cos t, \sin t, \cos t + \sin t)$
 (C) $(\cos at, \sin bt, 0)$
 (D) $(\cos(at + b), \sin(at + b), at + b)$
62. If $\alpha : I \rightarrow S$ is a geodesic on an n -surface S in \mathbb{R}^{n+1} , then :
- (A) acceleration of α is tangent to S
 (B) acceleration of α is orthogonal to S
 (C) velocity of α is orthogonal to S
 (D) velocity of α is tangent to S
63. The relationship between the spherical images of the surface S with orientation N and S with orientation $-N$ is :
- (A) reflection through origin
 (B) reflection about x -axis
 (C) reflection about y -axis
 (D) None of the above
64. What is the spherical image of surface ?
- (A) Image under Weingarten map
 (B) Image under Gauss map
 (C) Image under Lagrange's map
 (D) Image of sphere
65. The special linear group $SL(2)$ is :
- (A) 1-surface in \mathbb{R}^2
 (B) 2-surface in \mathbb{R}^3
 (C) 3-surface in \mathbb{R}^4
 (D) 4-surface in \mathbb{R}^5
66. The unit n -sphere is disconnected for the condition :
- (A) $n > 0$
 (B) $n = 1$
 (C) $n > 2$
 (D) $n = 0$

67. The positive rotation in the tangent space of a 2-surface in \mathbb{R}^3 is given by :

(A) $R_\theta(v) = (\cos \theta)v + (\sin \theta) N(p) \times v$

(B) $R_\theta(v) = (\sin \theta)v - (\cos \theta) N(p) \times v$

(C) $R_\theta(v) = (\cos \theta)v - (\sin \theta) N(p) \times v$

(D) $R_\theta(v) = (\sin \theta)v + (\cos \theta) N(p) \times v$

68. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be left handed for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

69. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be right handed for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

70. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be inconsistent for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

71. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be consistent for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

72. A disconnected 2-surface in \mathbb{R}^3 having two components has exactly how many choices for orientation ?

- (A) 2
- (B) 5
- (C) 3
- (D) 4

73. The maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $x_1^2 + 2x_2^2 = 1$ are :

- (A) 2 and 1 respectively
- (B) 1 and 1/2 respectively
- (C) 3 and 2 respectively
- (D) 1 and 0 respectively

74. The cylinder $x_1^2 + x_2^2 = 1$ in \mathbb{R}^3 is obtained by rotating :

- (A) The circle $x_1^2 + x_2^2 = 1$ about x_1 -axis
- (B) The line $x_1 = 1$ in the plane x_1x_3 about x_1 -axis
- (C) The line $x_1 = 1$ in the plane x_1x_2 about x_2 -axis
- (D) The line $x_2 = 1$ in the plane x_2x_3 about x_3 -axis

75. The set of all unit vectors at all points of \mathbb{R}^2 is a :
- (A) 1-surface in \mathbb{R}^2
 - (B) 2-surface in \mathbb{R}^3
 - (C) 3-surface in \mathbb{R}^4
 - (D) 4-surface in \mathbb{R}^5
76. What is the surface of revolution obtained by rotation the plane curve $-x^2 + y^2 = 1, y > 0$ about x -axis ?
- (A) Paraboloid
 - (B) 1-sheeted hyperboloid
 - (C) 2-sheeted hyperboloid
 - (D) Torus
77. What is the surface of revolution obtained by rotation the plane curve $x_2 = 1$ about x_2 -axis ?
- (A) Plane
 - (B) Sphere
 - (C) Cylinder
 - (D) Torus
78. What is the surface of revolution obtained by rotation the plane curve $x_2 = 1$ about x_1 -axis ?
- (A) Plane
 - (B) Sphere
 - (C) Cylinder
 - (D) Torus
79. What is the surface of revolution obtained by rotation the plane curve $x_1^2 + (x - 2)^2 = 1$ about x_2 -axis ?
- (A) Sphere
 - (B) Plane
 - (C) Cylinder
 - (D) Torus

80. What is the surface of revolution obtained by rotating the plane curve $x_1^2 + (x-2)^2 = 1$ about x_1 -axis ?

- (A) Plane
- (B) Sphere
- (C) Cylinder
- (D) Torus

81. An n -surface in \mathbb{R}^{n+1} is called a hypersurface when :

- (A) $n = 1$
- (B) $n > 1$
- (C) $n > 2$
- (D) $n > 3$

82. Let f be a smooth real valued function on \mathbb{R}^{n+1} . A point $p \in \mathbb{R}^{n+1}$ such that $\nabla f(p) \neq 0$ is called :

- (A) Critical point
- (B) Regular point
- (C) Terminal point
- (D) Point of inflection

83. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ be smooth and $\alpha : I \rightarrow U$ be a parameterized curve, then, which is a necessary and sufficient condition for the image of α is contained in a level set of ?

- (A) $\alpha(t)$ is constant
- (B) $f(\alpha(t)) = 0$
- (C) $\dot{\alpha}(t) = k \Delta f(\alpha(t))$
- (D) $\dot{\alpha}(t) \perp k \Delta f(\alpha(t))$

84. The dimension of the normal space to an n -surface in \mathbb{R}^{n+1} at a point is :

- (A) $n - 1$
- (B) 1
- (C) n
- (D) $n + 1$

85. The dimension of the tangent space to an n -surface in \mathbb{R}^{n+1} at a point is :
- (A) 1
 (B) $n - 1$
 (C) n
 (D) $n + 1$
86. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ and $\alpha : I \rightarrow U$, then, $\frac{d}{dt} (f \circ \alpha) (t)$ is :
- (A) $f'(\alpha(t)) \alpha'(t)$
 (B) $\Delta f(\alpha(t)) \dot{\alpha}(t)$
 (C) $f'(\alpha(t)) \dot{\alpha}(t)$
 (D) $\Delta f(\alpha(t)) \cdot \alpha'(t)$
87. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ be smooth, then the tangent space to $f^{-1}(p)$, $p \in U$ is :
- (A) $\{\nabla f(p)\}$
 (B) $\text{span} \{\nabla f(p)\}$
 (C) $[\nabla f(p)]^\perp$
 (D) \mathbb{R}_p^{n+1}
88. If $(x_1, x_2) = x_1^2 + x_2^2$, then the level set of f at height -1 is :
- (A) a pair of straight lines
 (B) a point
 (C) a circle of radius 1
 (D) empty
89. If $(x_1, x_2) = x_1^2 - x_2^2$, then the level set of f at height 1 is :
- (A) a point
 (B) empty
 (C) a rectangular hyperbola
 (D) a pair of straight lines
90. Integral curve of the vector field $X(x_1, x_2) = (0, 1)$ through $(0, 0)$ is :
- (A) the line $x_1 = 0$
 (B) the line $x_1 = 1$
 (C) the line $x_2 = 0$
 (D) the line $x_2 = 1$

91. Integral curve of the vector field

$X(x_1, x_2) = (1, 0)$ through $(0, 0)$ is :

- (A) the line $x_1 = 0$
- (B) the line $x_2 = 0$
- (C) the line $x_1 = 1$
- (D) the line $x_2 = 1$

92. Integral curve of the vector field

$X(x_1, x_2) = (-x_2, x_1)$ through (a, b) is :

- (A) $\alpha(t) = (a \cos t - b \sin t, a \sin t + b \cos t)$
- (B) $\alpha(t) = (a \cos t + b \sin t, a \sin t + b \cos t)$
- (C) $\alpha(t) = (a \cos t - b \sin t, a \sin t - b \cos t)$
- (D) $\alpha(t) = (a \cos t + b \sin t, a \sin t - b \cos t)$

93. Integral curve of the vector

field $X(x_1, x_2) = (-x_2, x_1)$ through

$(1, 0)$ is :

- (A) $\alpha(t) = (\sin t, \cos t)$
- (B) $\alpha(t) = (\cos t, \sin 2t)$
- (C) $\alpha(t) = (\cos t, \sin t)$
- (D) $\alpha(t) = (\cos 2t, \sin t)$

94. If α and β are two integral curves of a

given vector field through a point p ,

then :

- (A) $\alpha = \beta$
- (B) α is the restriction of β
- (C) β is the restriction of α
- (D) Both (B) and (C)

95. The graph of a real valued function in \mathbb{R}^n is a level set for some function in \mathbb{R}^{n+1} , which is :
- (A) always true
 (B) always false
 (C) true only if $n = 2$
 (D) false only if $n = 2$
96. The graph of $f(x_1, x_2) = x_1^2 + x_2^2$ is :
- (A) circle
 (B) paraboloid
 (C) cylinder
 (D) sphere
97. The graph of a smooth real valued function in \mathbb{R}^n is :
- (A) an $n - 1$ – surface in \mathbb{R}^{n+1}
 (B) an $1 - 1$ – surface in \mathbb{R}^n
 (C) an $(n - 1) - 1$ – surface in \mathbb{R}^{n+1}
 (D) Not a surface
98. If $f(x_1, x_2) = x_1^2 - x_2^2$. Then the level set of f at height 0 is :
- (A) a point
 (B) empty
 (C) a pair of straight lines
 (D) a rectangular hyperbola
99. If $f(x_1, x_2) = x_1^2 + x_2^2$ then the level set of f at height 0 is :
- (A) empty
 (B) a point
 (C) a circle of radius 1
 (D) None of the above
100. If $v(\alpha, \gamma)$ and $w(\beta, \delta)$ are two vectors in \mathbb{R}^{n+1} at α and β respectively, then $v + w = :$
- (A) $(\alpha + \beta, \gamma + \delta)$
 (B) $(\alpha, \gamma + \delta)$
 (C) $(\beta, \gamma + \delta)$
 (D) Not defined

(Only for Rough Work)

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 (A) ● (C) (D)

Q. 2 (A) (B) ● (D)

Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।