

Roll No.

Question Booklet Number

O. M. R. Serial No.

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M. A./M. Sc. (Fourth Semester)
(NEP) EXAMINATION, 2025-26
MATHEMATICS

(Differential Geometry of Manifolds) (Elective)

| Paper Code | | | | | | | |
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Questions Booklet
Series

A

Time : 1:30 Hours]

[Maximum Marks : 75

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. A real valued function f defined over a real interval I is said to be of class n , n being a positive integer, if it possesses :
- (A) continuous derivatives of n th order at each point of I
- (B) partial derivatives of $n + 1$ th order at each point of I
- (C) semi continuous derivatives of n th order at each point of I
- (D) continuous derivatives of $n + 1$ th order at each point of I
2. The equation of the tangent line to the curve $x = t$, $y = t^2$, $z = t^3$ at the point $t = 1$, is :
- (A) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$
- (B) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$
- (C) $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z-1}{3}$
- (D) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z+1}{3}$
3. The necessary and sufficient condition for the curve to be a plane curve is :
- (A) $[\dot{r} \ddot{r} \ddot{r}] = 0$
- (B) $[\dot{r} \ddot{r} \ddot{r}] = 0$
- (C) $[\dot{r} \ddot{r} \ddot{r}] = 0$
- (D) $[\dot{r} \ddot{r} \ddot{r}] = 0$
4. The true relation is :
- (A) $\tau = \frac{[\dot{r} \ddot{r} \ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
- (B) $\tau = \frac{[\dot{r} \ddot{r} \ddot{r}]}{[\dot{r} \times \ddot{r}]^{1/2}}$
- (C) $\tau = -\frac{[\dot{r} \ddot{r} \ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
- (D) $\tau = \frac{[\dot{r} \ddot{r} \ddot{r}]}{[\dot{r} \times \ddot{r}]^2}$
5. If the position vector r of a current point on the curve is a function of t , then the true relation is :
- (A) $\dot{r} = \dot{s} t$
- (B) $\dot{r} = \dot{s} \times t$
- (C) $\dot{r} = -\dot{s} t$
- (D) None of the above
6. If m_1, m_2, m_3 are the moments about the origin of unit vectors t, n, b localized in the tangent, principal normal, and binormal, then the following relation exists :
- (A) $m_1' = \tau m_2$
- (B) $m_1' = -\kappa m_2$
- (C) $m_1' = \kappa m_2$
- (D) None of the above

7. If $x = a \cos \theta$, $y = a \sin \theta$ and $z = f(\theta)$, and determines a plane curve, then :
- (A) $\frac{df}{d\theta} - \frac{d^2f}{d\theta^2} = 0$
- (B) $\frac{df}{d\theta} + \frac{d^2f}{d\theta^2} = 0$
- (C) $\frac{df}{d\theta} + 2\frac{d^2f}{d\theta^2} = 0$
- (D) None of the above
8. If $x = 3u$, $y = 3u^2$, $z = 2u^3$, then there exist a relation :
- (A) $\rho = \frac{3}{2}(1 + 2u^2)^2$
- (B) $\rho = \frac{3}{2}(1 - 2u^2)^2$
- (C) $\rho = \frac{5}{2}(1 + 2u^2)^2$
- (D) None of the above
9. For a cubic curve, the torsion τ is :
- (A) $\frac{3}{(9u^4 - 9u^2 + 1)}$
- (B) $\frac{3}{(9u^4 + 9u^2 + 1)}$
- (C) $\frac{3}{(9u^4 + 9u^2 - 1)}$
- (D) $\frac{3}{(9u^4 - 9u^2 - 1)}$
10. The distance between corresponding points of two involutes is :
- (A) indefinite
- (B) variable
- (C) constant
- (D) None of the above
11. The tangent to two different evolutes corresponding to the value a_1 and a_2 of the parameter c drawn from the same point of the given curve are inclined to each other at the constant angle :
- (A) $a_1 + a_2$
- (B) $2c(a_1 + a_2)$
- (C) $2c(a_1 - a_2)$
- (D) $(a_1 - a_2)$
12. If a plane curve has only a single evolutes in its own plane, then it is :
- (A) the locus of the centre of curvature
- (B) the locus of the half of centre of curvature
- (C) the locus of the twice of centre of curvature
- (D) None of the above

13. If the surface is given by $z = x^2 + y^2$, then the equation of tangent plane at point $(1, -1, 2)$ is :

- (A) $2x - 2y + z = 2$
- (B) $2x - 2y - z = 2$
- (C) $2x + 2y - z = 2$
- (D) None of the above

14. The equation of the edge of regression is :

- (A) $F(a) = \text{constant}, \frac{\partial}{\partial a} F(a) = \text{constant}$
- (B) $F(a) = \text{constant}, \frac{\partial}{\partial a} F(a) = 0$
- (C) $F(a) = 0, \frac{\partial}{\partial a} F(a) = 0$
- (D) $F(a) = 0, \frac{\partial}{\partial a} F(a) = \text{constant}$

15. If L, MN vanish everywhere on a surface, then :

- (A) the surface is part of a plane
- (B) the surface is a plane
- (C) there is no surface
- (D) None of the above

16. If on the paraboloid $x^2 - y^2 = z$, the sections are by the plane $z = \text{constant}$, then orthogonal trajectories of the section are :

- (A) $\frac{x}{y} = \text{constant}$
- (B) $xy = \text{constant}$
- (C) $xy^2 = \text{constant}$
- (D) $x^2y = \text{constant}$

17. The principal curvature for the surface $z = f(x, y)$ is :

- (A) $\kappa_n^2(EG - F^2) - \kappa_n(EN + LG - 2FM) + (LN + M^2) = 0$
- (B) $\kappa_n^2(EG - F^2) - \kappa_n(EN - LG - 2FM) + (LN - M^2) = 0$
- (C) $\kappa_n^2(EG + F^2) - \kappa_n(EN + LG - 2FM) + (LN - M^2) = 0$
- (D) $\kappa_n^2(EG - F^2) - \kappa_n(EN + LG - 2FM) + (LN - M^2) = 0$

18. The necessary and sufficient condition that a curve on a surface be a line of curvature is that :
- (A) $\kappa d\tau + dN = 0$ at each of its points
 (B) $\kappa d\tau - dN = 0$ at each of its points
 (C) $\kappa d\tau \pm dN + 1 = 0$ at each of its points
 (D) None of the above
19. If ψ is the angle between the direction (du, dv) of the normal section and the normal curvature κ_n at a point on a surface is given in terms of principal curvature κ_a and κ_b is given by :
- (A) $\kappa_n = \kappa_a \cos^2 \psi - \kappa_b \sin^2 \psi$
 (B) $\kappa_n = \kappa_a \sin^2 \psi - \kappa_b \cos^2 \psi$
 (C) $\kappa_n = \kappa_a \cos^2 \psi + \kappa_b \sin^2 \psi$
 (D) None of the above
20. If Gaussian curvature at a point P on the surface is positive, then the surface is :
- (A) elliptic
 (B) parabolic
 (C) linear
 (D) None of the above
21. The generators of the osculating developable of a twisted curve are tangents to the curve and its edge of regression is :
- (A) normal
 (B) tangent
 (C) curve itself
 (D) None of the above
22. The edge of regression of the rectifying developable of a space curve has its equation is :
- (A) $R - r = \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'}$
 (B) $R + r = \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'}$
 (C) $R - r + \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'} = 0$
 (D) $R + r - \kappa \frac{(\tau t + \kappa_b)}{\kappa' \tau - \kappa \tau'} = 0$
23. If the given curve is a line of curvature, there exists :
- (A) $\frac{dN}{ds} - \kappa \frac{d\tau}{ds} = 0$
 (B) $\frac{dN}{ds} + 2\kappa \frac{d\tau}{ds} = 0$
 (C) $\frac{dN}{ds} + \kappa \frac{d\tau}{ds} = 0$
 (D) None of the above

24. If the curve is $x = 6t$, $y = 3t^2$, $z = 2t^3$ for its edge of regression of developable surface, then its equation is :
- (A) $(3xz - 4y^2)(12y - x^2) = (9z - xy)^2$
 (B) $(3xz - 4y^2)(12y + x^2) = (9z - xy)^2$
 (C) $(3xz - 4y^2)(12y - x^2) = (9z + xy)^2$
 (D) None of the above
25. If Gaussian curvature K is zero, then :
- (A) the surface is generator
 (B) the surface is principal curvature
 (C) the surface is developable
 (D) None of the above
26. On the surface $z = f(x, y)$, the asymptotic lines are :
- (A) $rdx^2 - 2sdx dy + tdy^2$
 (B) $rdx^2 + 2sdx dy + tdy^2$
 (C) $rdx^2 + 2sdx dy - tdy^2$
 (D) None of the above
27. The equation of the developable surface which passes through the curve $z = 0$, $y^2 = 4ax$, $x = 0$, $y^2 = 4bz$ is :
- (A) $y^2 = 4ax + 4bz$
 (B) $y^2 = 4ax - 4bz$
 (C) $y^2 + 4ax + 4bz = 0$
 (D) $y^2 - 4ax + 4bz = 0$
28. The curves of the family $v^2 u^{-2} = \text{constant}$ are geodesics on a surface with a metric ($u > 0$, $v > 0$) given by :
- (A) $v^2 du^2 - 2uv du dv - 2u^2 dv^2$
 (B) $v^2 du^2 - 2uv du dv + 3u^2 dv^2$
 (C) $v^2 du^2 - 2uv du dv + 2u^2 dv^2$
 (D) None of the above
29. The geodesics on any cylinder are :
- (A) lines
 (B) parabolas
 (C) ellipses
 (D) helices

30. The necessary and sufficient condition that a geodesics be a plane curve is that :
- (A) it is helix
 - (B) it is line of curvature
 - (C) it is parabola
 - (D) None of the above
31. If in an algebraic operation, the rank of mixed tensor is lowered by 2, then it is known as :
- (A) linear
 - (B) deformation
 - (C) contraction
 - (D) None of the above
32. The velocity is a tensor named :
- (A) contravariant
 - (B) covariant
 - (C) linear
 - (D) None of the above
33. The gradient of a field is a tensor named :
- (A) nonlinear
 - (B) covariant
 - (C) contravariant
 - (D) None of the above
34. The velocity is a rank of :
- (A) 1
 - (B) 2
 - (C) 3
 - (D) None of the above
35. The acceleration is a rank of :
- (A) 3
 - (B) 2
 - (C) 4
 - (D) None of the above

36. If $ds^2 = g_{ij}dx^i dx^j$ is invariant, then :
- (A) g_{ij} is a symmetric covariant tensor of rank 2
- (B) g_{ij} is a non-symmetric covariant tensor of rank 2
- (C) g_{ij} is a symmetric covariant tensor of rank 3
- (D) None of the above

37. If g_{ij} is a contravariant tensor of rank 2 and if $A_{ij} = \frac{1}{2} (g_{ij} + g_{ji})$, then :

- (A) A_{ij} is a negative tensor
- (B) A_{ij} is a asymmetric tensor
- (C) A_{ij} is a symmetric tensor
- (D) None of the above

38. The distance between two adjacent points (x, y, z) and $(x + dx, y + dy, z + dz)$ in Cartesian Coordinate is given by:

- (A) $ds^2 = dx^2 - dy^2 + dz^2$
- (B) $ds^2 = dx^2 + dy^2 + dz^2$
- (C) $ds^2 = dx^2 + dy^2 - dz^2$
- (D) None of the above

39. In n -dimensional space, the distance between two neighbouring points with coordinates x^μ and $x^\mu + dx^\mu$ is given by :

- (A) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n g_{\mu\nu} x^\mu x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (B) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n x^\mu / x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (C) $ds^2 = \sum_{\mu=1}^n \sum_{\nu=1}^n x^{-\mu} x^\nu$,
 $|g_{\mu\nu}| \neq 0$
- (D) None of the above

40. If $\bar{T}_\alpha = \frac{\partial x_l}{\partial \bar{x}_\alpha} T_l$ and

$$\bar{S}^{\alpha\beta} = \frac{\partial \bar{x}^\alpha}{\partial x^m} \frac{\partial \bar{x}^\beta}{\partial x^n} S^{mn}, \text{ then :}$$

- (A) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial \bar{x}^\beta}{\partial x^n} T_l S^{ln}$
- (B) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial \bar{x}_\beta}{\partial x^n} T_l S^{ln}$
- (C) $\bar{T}_\alpha \bar{S}^{\alpha\beta} = \frac{\partial x_\beta}{\partial x^n} T_l S^{ln}$
- (D) None of the above

41. Outer product of two tensors is a tensor whose rank is :
- (A) the sum of the ranks of the given tensors
- (B) the product of the ranks of the given tensors
- (C) the difference of the ranks of the given tensors
- (D) None of the above
42. If $A_{\sigma}^{\alpha\beta}$ and B_{ρ}^{γ} are two tensors of rank 3 and 2 respectively, then the rank of $A_{\sigma}^{\alpha\beta} \cdot B_{\rho}^{\gamma}$ is equal to :
- (A) 1
- (B) 6
- (C) 5
- (D) 2
43. If A_i and B_j be two vectors such that $K_{ij} A_i B_j$ is invariant, then K_{ij} is a :
- (A) First order tensor
- (B) Second order tensor
- (C) Third order tensor
- (D) Fourth order tensor
44. Kronecker delta is a :
- (A) mixed tensor of rank 2
- (B) mixed tensor of rank 3
- (C) mixed tensor of rank 1
- (D) None of the above
45. If A^{μ} and B_{μ} are any two vectors, then $A^{\mu} \cdot B_{\mu}$ is the :
- (A) rank two tensor
- (B) invariant
- (C) rank one tensor
- (D) None of the above

46. If $x^1 = r$ and $x^2 = \theta$, then the metric tensor $g_{\mu\nu}$ in the matrix form is :

(A) $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 1 & r^2 \end{bmatrix}$

(B) $g_{\mu\nu} = \begin{bmatrix} 1 & 1 \\ 0 & r^2 \end{bmatrix}$

(C) $g_{\mu\nu} = \begin{bmatrix} 0 & 1 \\ 0 & r^2 \end{bmatrix}$

(D) $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}$

47. The metric tensor for two dimensional plane in polar coordinates is given by :

(A) $ds^2 = dr^2 + r^2 d\theta^2$

(B) $ds^2 = dr^2 - r^2 d\theta^2$

(C) $ds^2 = dr^2 + 2r^2 d\theta^2$

(D) $ds^2 = dr^2 - 2r^2 d\theta^2$

48. The length L of a tensor A^i is given by :

(A) $L = \sqrt[3]{A^i A_j}$

(B) $L = \sqrt[i]{A^i A_j}$

(C) $L = \sqrt{A^i A_j}$

(D) None of the above

49. If $\vec{\omega}$ is any arbitrary contravariant

vector and $A_{ij}\vec{\omega}$ is a covariant vector,

then :

(A) A_{ij} is a contravariant tensor of rank 2

(B) A_{ij} is a covariant tensor of rank 2

(C) A_{ij} is a covariant tensor of rank 1

(D) None of the above

50. If $A_{\alpha\beta} = A_{\beta\alpha}$, then :

(A) $A_{\alpha\beta}$ is an anti-symmetric tensor

(B) $A_{\alpha\beta}$ is an asymmetric tensor

(C) $A_{\alpha\beta}$ is a symmetric tensor

(D) None of the above

51. If $v(\alpha, \gamma)$ and $w(\beta, \delta)$ are two vectors in \mathbb{R}^{n+1} at α and β respectively, then $v + w = :$
- (A) $(\alpha + \beta, \gamma + \delta)$
- (B) $(\alpha, \gamma + \delta)$
- (C) $(\beta, \gamma + \delta)$
- (D) Not defined
52. If $f(x_1, x_2) = x_1^2 + x_2^2$ then the level set of f at height 0 is :
- (A) empty
- (B) a point
- (C) a circle of radius 1
- (D) None of the above
53. If $f(x_1, x_2) = x_1^2 - x_2^2$. Then the level set of f at height 0 is :
- (A) a point
- (B) empty
- (C) a pair of straight lines
- (D) a rectangular hyperbola
54. The graph of a smooth real valued function in \mathbb{R}^n is :
- (A) an $n - 1$ – surface in \mathbb{R}^{n+1}
- (B) an 1 – surface in \mathbb{R}^n
- (C) an $(n - 1) -$ surface in \mathbb{R}^{n+1}
- (D) Not a surface
55. The graph of $f(x_1, x_2) = x_1^2 + x_2^2$ is :
- (A) circle
- (B) paraboloid
- (C) cylinder
- (D) sphere
56. The graph of a real valued function in \mathbb{R}^n is a level set for some function in \mathbb{R}^{n+1} , which is :
- (A) always true
- (B) always false
- (C) true only if $n = 2$
- (D) false only if $n = 2$

57. If α and β are two integral curves of a given vector field through a point p ,

then :

- (A) $\alpha = \beta$
- (B) α is the restriction of β
- (C) β is the restriction of α
- (D) Both (B) and (C)

58. Integral curve of the vector field $X(x_1, x_2) = (-x_2, x_1)$ through

$(1, 0)$ is :

- (A) $\alpha(t) = (\sin t, \cos t)$
- (B) $\alpha(t) = (\cos t, \sin 2t)$
- (C) $\alpha(t) = (\cos t, \sin t)$
- (D) $\alpha(t) = (\cos 2t, \sin t)$

59. Integral curve of the vector field

$X(x_1, x_2) = (-x_2, x_1)$ through (a, b) is :

- (A) $\alpha(t) = (a \cos t - b \sin t, a \sin t + b \cos t)$
- (B) $\alpha(t) = (a \cos t + b \sin t, a \sin t + b \cos t)$
- (C) $\alpha(t) = (a \cos t - b \sin t, a \sin t - b \cos t)$
- (D) $\alpha(t) = (a \cos t + b \sin t, a \sin t - b \cos t)$

60. Integral curve of the vector field

$X(x_1, x_2) = (1, 0)$ through $(0, 0)$ is :

- (A) the line $x_1 = 0$
- (B) the line $x_2 = 0$
- (C) the line $x_1 = 1$
- (D) the line $x_2 = 1$

61. Integral curve of the vector field $X(x_1, x_2) = (0, 1)$ through $(0, 0)$ is :
- (A) the line $x_1 = 0$
- (B) the line $x_1 = 1$
- (C) the line $x_2 = 0$
- (D) the line $x_2 = 1$
62. If $(x_1, x_2) = x_1^2 - x_2^2$, then the level set of f at height 1 is :
- (A) a point
- (B) empty
- (C) a rectangular hyperbola
- (D) a pair of straight lines
63. If $(x_1, x_2) = x_1^2 + x_2^2$, then the level set of f at height -1 is :
- (A) a pair of straight lines
- (B) a point
- (C) a circle of radius 1
- (D) empty
64. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ be smooth, then the tangent space to $f^{-1}(p)$, $p \in U$ is :
- (A) $\{\nabla f(p)\}$
- (B) $\text{span}\{\nabla f(p)\}$
- (C) $[\nabla f(p)]^\perp$
- (D) \mathbb{R}_p^{n+1}
65. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ and $\alpha : I \rightarrow U$, then, $\frac{d}{dt}(f \circ \alpha)(t)$ is :
- (A) $f'(\alpha(t)) \alpha'(t)$
- (B) $\Delta f(\alpha(t)) \dot{\alpha}(t)$
- (C) $f'(\alpha(t)) \dot{\alpha}(t)$
- (D) $\Delta f(\alpha(t)) \cdot \alpha'(t)$
66. The dimension of the tangent space to an n -surface in \mathbb{R}^{n+1} at a point is :
- (A) 1
- (B) $n - 1$
- (C) n
- (D) $n + 1$

67. The dimension of the normal space to an n -surface in \mathbb{R}^{n+1} at a point is :
- (A) $n - 1$
- (B) 1
- (C) n
- (D) $n + 1$
68. If $f : U \rightarrow \mathbb{R}$, $U \in \mathbb{R}^{n+1}$ be smooth and $\alpha : I \rightarrow U$ be a parameterized curve, then, which is a necessary and sufficient condition for the image of α is contained in a level set of ?
- (A) $\alpha(t)$ is constant
- (B) $f(\alpha(t)) = 0$
- (C) $\dot{\alpha}(t) = k \Delta f(\alpha(t))$
- (D) $\dot{\alpha}(t) \perp k \Delta f(\alpha(t))$
69. Let f be a smooth real valued function on \mathbb{R}^{n+1} . A point $p \in \mathbb{R}^{n+1}$ such that $\nabla f(p) \neq 0$ is called :
- (A) Critical point
- (B) Regular point
- (C) Terminal point
- (D) Point of inflection
70. An n -surface in \mathbb{R}^{n+1} is called a hypersurface when :
- (A) $n = 1$
- (B) $n > 1$
- (C) $n > 2$
- (D) $n > 3$
71. What is the surface of revolution obtained by rotating the plane curve $x_1^2 + (x - 2)^2 = 1$ about x_1 -axis ?
- (A) Plane
- (B) Sphere
- (C) Cylinder
- (D) Torus

72. What is the surface of revolution obtained by rotation the plane curve $x_1^2 + (x-2)^2 = 1$ about x_2 -axis ?
- (A) Sphere
(B) Plane
(C) Cylinder
(D) Torus
73. What is the surface of revolution obtained by rotation the plane curve $x_2 = 1$ about x_1 -axis ?
- (A) Plane
(B) Sphere
(C) Cylinder
(D) Torus
74. What is the surface of revolution obtained by rotation the plane curve $x_2 = 1$ about x_2 -axis ?
- (A) Plane
(B) Sphere
(C) Cylinder
(D) Torus
75. What is the surface of revolution obtained by rotation the plane curve $-x^2 + y^2 = 1, y > 0$ about x -axis ?
- (A) Paraboloid
(B) 1-sheeted hyperboloid
(C) 2-sheeted hyperboloid
(D) Torus
76. The set of all unit vectors at all points of \mathbb{R}^2 is a :
- (A) 1-surface in \mathbb{R}^2
(B) 2-surface in \mathbb{R}^3
(C) 3-surface in \mathbb{R}^4
(D) 4-surface in \mathbb{R}^5

77. The cylinder $x_1^2 + x_2^2 = 1$ in \mathbb{R}^3 is obtained by rotating :

- (A) The circle $x_1^2 + x_2^2 = 1$ about x_1 -axis
- (B) The line $x_1 = 1$ in the plane x_1x_3 about x_1 -axis
- (C) The line $x_1 = 1$ in the plane x_1x_2 about x_2 -axis
- (D) The line $x_2 = 1$ in the plane x_2x_3 about x_3 -axis

78. The maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $x_1^2 + 2x_2^2 = 1$ are :

- (A) 2 and 1 respectively
- (B) 1 and 1/2 respectively
- (C) 3 and 2 respectively
- (D) 1 and 0 respectively

79. A disconnected 2-surface in \mathbb{R}^3 having two components has exactly how many choices for orientation ?

- (A) 2
- (B) 5
- (C) 3
- (D) 4

80. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be consistent for the condition :

- (A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$
- (B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$
- (C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$
- (D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

81. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be inconsistent for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

82. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be right handed for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

83. If S be an n -surface in \mathbb{R}^{n+1} , then the basis $\{v_1, \dots, v_n\}$ for the tangent space S_p is said to be left handed for the condition :

(A) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} > 0$

(B) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \neq 0$

(C) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = 0$

(D) $\det \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} < 0$

84. The positive rotation in the tangent space of a 2-surface in \mathbb{R}^3 is given by :

(A) $R_\theta(v) = (\cos \theta)v + (\sin \theta) N(p) \times v$

(B) $R_\theta(v) = (\sin \theta)v - (\cos \theta) N(p) \times v$

(C) $R_\theta(v) = (\cos \theta)v - (\sin \theta) N(p) \times v$

(D) $R_\theta(v) = (\sin \theta)v + (\cos \theta) N(p) \times v$

85. The unit n -sphere is disconnected for the condition :
- (A) $n > 0$
 (B) $n = 1$
 (C) $n > 2$
 (D) $n = 0$
86. The special linear group $SL(2)$ is :
- (A) 1-surface in \mathbb{R}^2
 (B) 2-surface in \mathbb{R}^3
 (C) 3-surface in \mathbb{R}^4
 (D) 4-surface in \mathbb{R}^5
87. What is the spherical image of surface ?
- (A) Image under Weingarten map
 (B) Image under Gauss map
 (C) Image under Lagrange's map
 (D) Image of sphere
88. The relationship between the spherical images of the surface S with orientation N and S with orientation $-N$ is :
- (A) reflection through origin
 (B) reflection about x -axis
 (C) reflection about y -axis
 (D) None of the above
89. If $\alpha : I \rightarrow S$ is a geodesic on an n -surface S in \mathbb{R}^{n+1} , then :
- (A) acceleration of α is tangent to S
 (B) acceleration of α is orthogonal to S
 (C) velocity of α is orthogonal to S
 (D) velocity of α is tangent to S
90. A geodesic in the cylinder $x_1^2 + x_2^2 = 1$ is always of form :
- (A) $(\cos(at + b), \sin(at + b), ct + d)$
 (B) $(\cos t, \sin t, \cos t + \sin t)$
 (C) $(\cos at, \sin bt, 0)$
 (D) $(\cos(at + b), \sin(at + b), at + b)$

91. On the sphere, a maximal geodesic in the 2-sphere is always a part of :

- (A) a circle
- (B) an ellipse
- (C) great circles
- (D) straight lines

92. $\alpha : I \rightarrow S$ is a geodesic if and only if it satisfies the differential equation :

- (A) $\ddot{\alpha} + (\dot{\alpha} \cdot \dot{N}\alpha)(N\alpha) = 0$
- (B) $\ddot{\alpha} + (\dot{\alpha} \cdot N\alpha)(\dot{N}\alpha) = 0$
- (C) $\dot{\alpha} + (\ddot{\alpha} \cdot \dot{N}\alpha)(N\alpha) = 0$
- (D) $\dot{\alpha} + (\ddot{\alpha} \cdot N\alpha)(N\alpha) = 0$

93. $\alpha : I \rightarrow S$ is a geodesic on an n -surface S in \mathbb{R}^{n+1} , then :

- (A) $\dot{\alpha}$ is orthogonal to S
- (B) speed of α is constant
- (C) $\dot{\alpha} \perp \ddot{\alpha}$
- (D) All of the above

94. An n -surface S in \mathbb{R}^{n+1} is said to be geodesically complete, if :

- (A) domain of every maximal geodesic is \mathbb{R}
- (B) image of every maximal geodesic is S
- (C) image of every maximal geodesic is complete subset of S
- (D) domain of every maximal geodesic is complete subset of \mathbb{R}

95. Covariant derivative of a vector field X on an n -surface S is given by :

- (A) $\dot{X}(t) + \{\dot{\alpha}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (B) $\dot{X}(t) - \{\dot{X}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (C) $\dot{X}(t) + \{\dot{X}(t) \cdot N(\alpha(t))\}N(\alpha(t))$
- (D) $\dot{X}(t) - \{\dot{\alpha}(t) \cdot N(\alpha(t))\}N(\alpha(t))$

96. What is the relationship between the ordinary derivative \dot{X} and X' of a vector field X on an n -surface ?
- (A) \dot{X} is the projection of X' to the tangent space
- (B) \dot{X} is the projection of X' to the normal space
- (C) X' is the projection of \dot{X} to the tangent space
- (D) One is a scalar multiple of the other
97. What is the relationship between the acceleration $\ddot{\alpha}$ and the covariant acceleration $\dot{\alpha}'$ of a parameterized curve α on an n -surface :
- (A) one is a scalar multiple of the other
- (B) $\dot{\alpha}'$ is the projection of $\ddot{\alpha}$ to the normal space
- (C) $\ddot{\alpha}$ is the projection of $\dot{\alpha}'$ to the tangent space
- (D) $\dot{\alpha}'$ is the projection of $\ddot{\alpha}$ to the tangent space
98. If X and Y be two vector fields along a parameterized curve α on an n -surface, then $(X.Y)'$ is equal to :
- (A) $\dot{X}.Y + X.\dot{Y}$
- (B) $X'Y + X.Y'$
- (C) $\dot{X}.Y' + X'.\dot{Y}$
- (D) Both (A) and (B)
99. If X is Euclidean parallel, then :
- (A) X is constant
- (B) X is Levi-Civita parallel
- (C) X is oscillatory
- (D) None of the above
100. If α be a geodesic on a 2-surface S , Then a vector field X tangent to S along α is parallel along α if and only if :
- (A) $\|X\|$ and the angle between $\|X\|$ and $\dot{\alpha}$ are constants
- (B) $\|X\|$ is constant
- (C) $\|X\|$ and $\dot{\alpha}$ are constants
- (D) $\|X'\|$ and $\dot{\alpha}'$ are constants

(Only for Rough Work)

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 (A) ● (C) (D)

Q. 2 (A) (B) ● (D)

Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।