Roll No. ....

Question Booklet Number

O. M. R. Serial No. | | | | | | | |

M. A./M. Sc. (Fourth Semester)
(NEP) EXAMINATION, 2025-26

MATHEMATICS

(Integral Equation And Boundary Value Problems)

Paper Code							
B	0	3	1	0	0	2	T

Questions Booklet Series

D

Time : 1:30 Hours]   [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.

2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.

3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।

2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।

3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. The Fredholm series

$$\phi = f + \lambda K f + \lambda^2 K^2 f^2 + ...$$

is essentially analogous to :

(A) Taylor series

(B) Fourier series

(C) Laurent series

(D) Geometric series

2. Let K(x, t) = 2 on [0, 1] × [0, 1]. Find the range of λ for convergence :

(A) $|\lambda| < 1$

(B) $|\lambda| < 1/2$

(C) $|\lambda| < 2$

(D) All λ

3. The Fredholm series converges, when :

(A) $\lambda = 0$

(B) λ is an eigen value of the homogeneous equation

(C) Kernel is continuous

(D) $f(x) = 0$

4. Let K(x, t) = 1, $0 \le x, t \le 1$. The Fredholm series converges for :

(A) $|\lambda| < 1$

(B) $|\lambda| < 2$

(C) $|\lambda| < 1/2$

(D) $|\lambda| > 0$

5. If the kernel K(x, t) is continuous on a finite square [a, b] × [a, b], then the Fredholm series :

(A) Converges absolutely and uniformly for sufficiently small $|\lambda|$

(B) Converges only pointwise

(C) Never converges uniformly

(D) Converges only if K is symmetric

6. Consider the Fredholm integral equation of second kind

$$\phi(x) = f(x) + \lambda \int_a^b K(x, t)\, \phi(t) dt$$

Then associated Fredholm series converges if :

(A) $|\lambda| < \frac{1}{\sup|K(x, t)|}$

(B) It always converges for every λ

(C) $|\lambda| < \frac{1}{M(b-a)}$, M = max | K (x, t) |

(D) None of the above

7. The solution of integral equation

$u(x) = \sec^2 x + \lambda \int_0^1 u(t) dt$ is :

(A) $\tan^2 x + \frac{\lambda}{1-\lambda} \tan 1$

(B) $\sec^2 x + \frac{\lambda}{1-\lambda} \tan 1$

(C) $\sec^2 x - \frac{\lambda}{1-\lambda} \tan 1$

(D) None of the above

8. If $u(x) = f(x) + \lambda \int_0^\pi \sin x\, u(t) dt$, then the value of $D(\lambda)$ is :

(A) $1 + \lambda$

(B) $1 - \lambda$

(C) $1 + 2\lambda$

(D) $1 - 2\lambda$

9. If $u(x) = f(x) + \lambda \int_0^\pi \sin x\, u(t) dt$, then the value of $D(x, t, \lambda)$ is :

(A) e^{x-t}

(B) e^{x+t}

(C) $\sin x$

(D) None of the above

10. If $u(x) = f(x) + \lambda \int_0^1 u(t) dt$, then the value of $D(x, t, \lambda)$ is :

(A) 1

(B) 0

(C) – 1

(D) $x - t$

11. If $u(x) = f(x) + \lambda \int_0^1 u(t)dt$, then the value of D(λ) is given by :

(A) D(λ) = λ

(B) D(λ) = 1 – λ

(C) D(λ) = 1 + λ

(D) None of the above

12. Solution $u(x)$ of integral equation $u(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\, \phi(t)dt$ is given by :

(A) $u(x) = 1$

(B) $u(x) = x^2$

(C) $u(x) = x$

(D) $u(x) = -x$

13. Solution $u(x)$ of Integral equation $u(x) = 1 + \int_0^x (x-t)\, u(t)dt$ with $u_0(x) = 0$ is :

(A) $u(x) = \sinh x$

(B) $u(x) = \cosh x$

(C) $u(x) = e^x + e^{-x}$

(D) None of the above

14. Iterated kernel $k_n(x, t)$ for the function $k(x, t) = \sin\,(x - 2t)$, $0 \le x \le 2\pi$, $0 \le t \le 2\pi$ is :

(A) 0

(B) $\sin x$

(C) $\sin\,(x - 2t)$

(D) $\frac{\pi}{2} - \sin\,(x - 2t)$

15. Let $k(x, t) = (t - x)$ be the kernel of a Volterra Integral equation and λ = 1, then resolvent kernel is :

(A) e^{x-t}

(B) $-\sin\,(x - t)$

(C) $\cos\,(x - t)$

(D) $\sin\,(x - t)$

16. Which of the following is the resolvent kernel for the integral equation

$$\phi(x) = x + \int_{-1}^{0} (1 + x)(1 - t)\, \phi(t)dt \ ?$$

(A) $\frac{2}{3 - \lambda}(1 + x)(1 - t)$

(B) $(3 + \lambda)(1 + x)(1 - t)$

(C) $\frac{3}{3 - 2\lambda}(1 + x)(1 - t)$

(D) None of the above

17. The resolvent kernel for the integral equation

$$\phi(x) = x^2 + \int_0^x e^{t-x}\,\phi(t)dt$$

is :

(A) 1

(B) e^{t-x}

(C) e^{x-t}

(D) $x^2 + e^{x-t}$

18. Find the resolvent kernel of the kernel $k(x, t) = 2x$.

(A) $xe^{x^2-t^2}$

(B) $e^{x^2-t^2}$

(C) x

(D) $2xe^{x^2-t^2}$

19. If $u(x) = x - \int_0^x (x-t)\,u(t)dt$ with $u_0(x) = 0$, then the value of $u(x)$ is :

(A) $\cos x$

(B) e^x

(C) $-\cos x$

(D) $\sin x$

20. If

$$\phi(x) = (1+x^2) + \int_0^x \left(\frac{1+x^2}{1+t^2}\right)\phi(t)dt$$

then $R(x, t, \lambda)$:

(A) $\phi(x) = (1+x^2)e^x$

(B) $\phi(x) = x^2e^x$

(C) $\phi(x) = e^x$

(D) None of the above

21. If $\phi(x) = 1 + \int_0^x \phi(t)dt$, then $R(x, t, \lambda)$ is :

(A) e^x

(B) $x - t$

(C) e^{x-t}

(D) $e^{(x-t)^2}$

22. If $\phi(x) = F(x) + \lambda\int_0^x k(x,t)\,\phi(t)dt$ then Resolvent Kernel is :

(A) $R(x,t,\lambda) = \sum_{n=1}^{\infty} \lambda^n\, k_{n+1}(x,t)$

(B) $R(x,t,\lambda) = \sum_{n=1}^{\infty} \lambda\, k_{n+1}(x,t)$

(C) $R(x,t,\lambda) = k(x,t) + \sum_{n=1}^{\infty} \lambda^n k_{n+1}(x,t)$

(D) None of the above

23. $\phi(x) = \lambda\int_0^1 \phi(t)dt$

Eigen values are :

(A) 1

(B) 0

(C) – 1

(D) infinite

24. If $\phi(x) = \lambda\int_1^x (x-t)^2\,\phi(t)dt$ Eigen values are :

(A) $\lambda > 0$

(B) $\lambda = 3$

(C) $\lambda < 0$

(D) $\lambda = 0$ only

25. Consider $\phi(x) = \lambda\int_0^x \phi(t)dt$ Eigen values are :

(A) $\lambda = 1$

(B) $\lambda = -1$

(C) $\lambda = 0$ only

(D) Real value

26. Eigen value of integral equation

$$u(x) = \lambda\int_0^1 e^x e^t u(t)dt$$

is :

(A) 1

(B) 0

(C) $\dfrac{1}{e^2 - 1}$

(D) $\dfrac{2}{e^2 - 1}$

27. Eigen values of a symmetric kernel are :

(A) Real

(B) May be real on complex

(C) Only positive value

(D) None of the above

28. For degenerate kernel, successive approximation reduces to :

(A) Differential equation

(B) Infinite system

(C) Finite linear system

(D) No solution

29. The n[th] iterate of the kernel is defined by :

(A) $k_n(x, t) = k(x, t)^n$

(B) $k_n(x, t) = \int_a^b k(x, s)\, k_{n-1}(s, t) ds$

(C) $k_n = \lambda^n k$

(D) $k_n = f(x)\, k(x, t)$

30. The resolvent kernel R(x, t, λ) corresponding to the Neumann series is :

(A) $k(x, t)$

(B) $\sum_{n=1}^{\infty} \lambda^{n-1} k_n(x, t)$

(C) $f(x)$

(D) $\lambda k(x, t)$

31. Solution of integral equation

$u(x) + \int_0^1 x(e^{xt} - 1)\, u(t)dt = e^x - x$

is :

(A) – 1

(B) – 3

(C) 0

(D) 1

32. Solution of Integral equation

$\int_0^x (x-t)^2\, u(t)dt = x^3 =$

is :

(A) 2

(B) 3

(C) 0

(D) 1

33. The value of $u(x)$ which satisfies integral equation :

$u(x) = \sin x + 2\int_0^x \cos(x-t)\, u(t)dt$

is :

(A) $u(x) = x$

(B) $u(x) = -x$

(C) $u(x) = e^x$

(D) $u(x) = xe^x$

34. Integral equation corresponding to the differential equation $y'' + y = \cos x$ with $y(0) = 0, y'(0) = 1$ is :

(A) $u(x) = \cos x - x - \int_0^x (x - t) u(t)dt$

(B) $u(x) = \sin x - x - \int_0^x (x - t) u(t)dt$

(C) $u(x) = \cos x - \int_0^x (x - t) u(t)dt$

(D) $u(x) = \cos x - x + \int_0^x (x - t) u(t)dt$

35. Integral equation corresponding to the differential equation $y'' - 5y' + 6y = 0$ with $y(0) = 0, y'(0) = -1$ is :

(A) $u(x) = 1 + \int_0^x (2x - t)\, u(t)dt$

(B) $u(x) = 1 - \int_0^x (2x - t)\, u(t)dt$

(C) $u(x) = \int_0^x (2x - t)\, u(t)dt$

(D) $u(x) = -1 - \int_0^x (2x - t)\, u(t)dt$

36. Convolution type kernel is defined by :

(A) $k(x, t) = k(x - t)$

(B) $k(x, t) = x + t$

(C) $k(x, t) = xt^2$

(D) $k(x, t) = \sin(xt)$

37. An integral equation is :

(A) An equation containing only derivatives of an unknown function

(B) An equation in which the known function appears under an integral sign

(C) An Algebraic equation involving constants

(D) None of the above

38. The value of $y(x)$ which satisfies integral equation

$$y(x) = e^x - \int_0^x e^{x-t}\, y(t)\, dt \text{ is}$$

(A) e^x

(B) 0

(C) $e^x - xe^x$

(D) xe^x

39. $y'' = f(x)$, $y(0) = 0$, $y(1) = 0$ is equivalent to :

(A) Volterra equation of first kind

(B) Fredholm equation of second kind

(C) Fredholm equation of first kind

(D) Non-linear integral equation

40. The differential equation corresponding to integral equation

$$y(x) = 1 + \int_0^x (x-t)\, y(t)\, dt$$

(A) $y'' = y$ with $y(0) = 1$, $y'(0) = 0$

(B) $y'' = y$ with $y(0) = 0$, $y'(0) = 1$

(C) $y'' = y + 1$

(D) $y'' = 1$

41. Corresponding integral equation of

$$\frac{d^2y}{dx^2} + y = 0, \quad \begin{matrix} y(0) = 0 \\ y'(0) = 1 \end{matrix}$$

(A) $\phi(x) = x + \int_1^x (x-t)\,\phi(t)\, dt$

(B) $\phi(x) = x + \int_0^x (x-t)\,\phi(t)\, dt$

(C) $\phi(x) = -x - \int_0^x (x-t)\,\phi(t)\, dt$

(D) None of the above

42. The boundary value problem,

$$y'' + \lambda y = 0, y(0) = 0, y(1) = 0$$

can be converted into :

(A) Fredholm equation

(B) Volterra equation

(C) Non-Linear equation

(D) Algebraic equation

43. The integral equation

$$y(x) = 2 + \int_0^x (x-t)\, y(t)\, dt \; :$$

(A) $y(0) = 0, y'(0) = 0$

(B) $y(0) = 2, y'(0) = 2$

(C) $y(0) = 2, y'(0) = 0$

(D) $y(0) = 0, y'(0) = 2$

44. Initial value problem $y' = f(x, y)$, $y(0) = y_0$ is equivalent to :

(A) $y(x) = \int_0^1 f(t, y(t))dt$

(B) $y'' = f(x, y)$

(C) $y(x) = f(x) + \lambda\, y(x)$

(D) $y(x) = y_0 + \int_0^x f(t, y(t))\, dt$

45. The Kernel $k(x, t) = \delta(x - t)$ (where δ is Dirac delta function) is classified as :

(A) Singular kernel

(B) Regular kernel

(C) Degenerate kernel

(D) Symmetric kernel

46. A Kernel of the form

$$K(x, t) = \sum_{i=1}^{n} a_i(x)\, b_i(t)$$

is called :

(A) Singular

(B) Symmetric

(C) Degenerate

(D) Non-homogeneous

47. The equation

$$\phi(x) = \int_0^x \sin(x + t)\, \phi(t)\, dt$$

is :

(A) Homogeneous Fredholm equation of first kind

(B) Homogenous Volterra Equation of second kind

(C) Volterra equation of first kind

(D) None-linear Equation

48. The equation

$$\phi(x) - \lambda \int_0^1 (x + t)\, \phi(t)\, dt = x^2$$

is :

(A) Fredholm equation of first kind

(B) Fredholm equation of second kind

(C) Volterra equation of first kind

(D) None of the above

49. If $\int_0^x e^{x-t} u(t)\, dt = x$, then the value of $u(x) = ?$

(A) x

(B) $1 + x$

(C) $1 - x$

(D) $-x$

50. If $\int_0^x \frac{\phi(t)\, dt}{\sqrt{x-t}} = \sqrt{x}$, then the value of $\phi(x) = ?$

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $-\frac{1}{2}$

51. The Fredholm, determinant is used to determine :

(A) Eigen values

(B) Eigen functions

(C) Solution existence

(D) Both (A) and (C)

52. For a second order linear differential operator, the Green's function must satisfy which property at $x = \xi$?

(A) G is discontinuous

(B) G is continuous

(C) G' is continuous

(D) None of the above

53. The solution of the integral equation $y(x) = 1 + \int_0^x y(t)dt$ is :

(A) $y = e$

(B) $y = 1 + x$

(C) $y = x$

(D) $y = e^x$

54. The Green's function representation of the solution of a B.V.P. $Ly = f(x)$ is :

(A) $y(x) = \int G(x, t)\, f(t)dt$

(B) $y(x) = G(x, t) + f(x)$

(C) $y(x) = \int f(x)dx$

(D) $y(x) = G(x, t)\, f(x)$

55. The integral equation corresponding to the I.V.P. $y''(x) = f(x, y)$, $y(0) = a$, $y'(0) = b$ is :

(A) $y(x) = a + bx + \int_0^x (x - t) f(t, y(t))dt$

(B) $y(x) = a + \int_0^x (x - t)\ f(t, y(t))dt$

(C) $y(x) = bx + \int_0^x f(t, y(t))dt$

(D) None of the above

56. In constructing Green's function for second order B.V.P., the function must satisfy :

(A) Continuity at $x = t$

(B) Discontinuity in derivative

(C) Homogeneous Boundary Conditions

(D) All of the above

57. For self-adjoint Boundary value problems the Green's function satisfies :

(A) $G(x, t) = -G(t, x)$

(B) $G(x, t) = G(t, x)$

(C) $G(x, t) = 0$

(D) $G(x, t) = 1$

58. For the B.V.P. $y'' = f(x)$, $y(0) = 0$, $y(1) = 0$ the Green's function must satisfy :

(A) $G(0, t) = 0$

(B) $G(1, t) = 0$

(C) Both (A) and (B)

(D) None of the above

59. For a second-order differential operator $Ly = \frac{d^2y}{dx^2}$, the Green's function is constructed from two solutions $u(x)$ and $v(x)$ satisfying boundary conditions, the denominator in Green's function expression involves :

(A) Determinant of boundary conditions

(B) Wronskian $W(u, v)$

(C) Forrier coefficient

(D) Eigen value

60. For the B.V.P. $y'' - y = f(x)$, $0 < x < 1$, with $y(0) = 0$, $y(1) = 0$ the Green function satisfies :

(A) $G'' - G = 0$ for $x \neq \xi$

(B) $G'' - G = \delta(x - \xi)$

(C) $G'' = \delta(x - \xi)$

(D) $G'' + G = \delta(x - \xi)$

61. For the modified Green's function $\int_0^1 G_m(x, t)dt$ equals :

(A) 0

(B) x

(C) 1

(D) x^2

62. Consider boundary value problem $y'' = f(x)$, $0 < x < 1$ with boundary conditions $y'(0) = 0$, $y'(1) = 0$. Then eigen function of the homogeneous problem is :

(A) $\sin x$

(B) $\cos x$

(C) 1

(D) e^x

63. If L[Y] = $f(x)$ has boundary conditions for which the homogeneous equation has eigen function $\phi\ (x)$, then $G_m\ (x, t)$ satisfies :

(A) $L[G_m] = \delta(x - t)$

(B) $L[G_m] = 0$

(C) $L[G_m] = \phi(x)$

(D) $L[G_m] = \delta(x - t) - \phi(x)\ \phi(t)$

64. The modified Green's function is generally used when :

(A) The differential operator is non-linear

(B) The homogeneous problem has non-trivial solutions

(C) Boundary conditions are not given

(D) None of the above

65. If the eigen function of the homogeneous problem is, $\phi\ (x) = 1$, $0 < x < 1$, then the modified Green's function satisfies $\int_0^1 G_M(x, t)\, dt = ?$

(A) 0

(B) 1

(C) x

(D) t

66. For the boundary value problem $y'' = f(x)$, $0 < x < 1$, $y' = 0$, $y'(1) = 0$ the ordinary Green's function does not exist because :

(A) Operator is non-linear

(B) Boundary conditions are inconsistent

(C) Homogenous equation has constant solution

(D) Interval in finite

67. If $L(u) = p_0(x)u'' + p_1(x)u' + p_2(x)$, then jump condition for Green's function :

(A) $\left(\frac{\partial G}{\partial x}\right)_{x=t+0} + \left(\frac{\partial G}{\partial x}\right)_{x=t-0} = \frac{-1}{p_0(t)}$

(B) $\left(\frac{\partial G}{\partial x}\right)_{x=t+0} - \left(\frac{\partial G}{\partial x}\right)_{x=t-0} = \frac{-1}{p_0(t)}$

(C) $\left(\frac{\partial G}{\partial x}\right)_{x=t+0} - \left(\frac{\partial G}{\partial x}\right)_{x=t-0} = \frac{1}{p_0(t)}$

(D) None of the above

68. For a self-adjoint boundary value problem, the Green's function satisfies :

(A) $G(x, t) = G(t, x)$

(B) $G(x, t) = -G(t, x)$

(C) $G(x, t) = 0$

(D) $G(x, t) = 1$

69. Green's function $G(x, t)$ for a second order self-adjoint operator satisfies :

(A) $L[G] = \delta(x - t)$

(B) $L[G] = 0$

(C) $L[G] = 1$

(D) $L[G] = f(x)$

70. Consider the boundary value problem $y'' = f(x)$, $0 < x < 1$, $y(0) = 0$, $y(1) = 0$. Then Green's function $G(x, t)$ is :

(A) $x(1 - t)$ for $x < t$, $t(1 - x)$ for $x > t$

(B) $t(1 - x)$ for $x < t$, $x(1 - t)$ for $x > t$

(C) xt

(D) $1 - xt$

71. If $y(x) = x + \lambda\int_0^1 t\,\delta(x - t)\,y(t)\,dt$:

(A) $y(x) = \frac{x}{1 + \lambda x}$

(B) $y(x) = \frac{x}{1 - \lambda x}$

(C) $y(x) = \frac{x}{1 - \lambda}$

(D) $y(x) = x$

72. Solution of the equation :

$$y(x) = \sin x + \lambda \int_0^{\pi} \delta(x-t) \sin t \, dt$$

is :

(A) $(1+\lambda) \sin x$

(B) $(1-\lambda) \sin x$

(C) $\sin x$

(D) $\lambda \sin x$

73. $\int_{-\infty}^{\infty} \sinh 2t \, \delta(2-t) dt = ?$

(A) $e^{-2^{-\infty}}$

(B) e^2

(C) $\sinh t$

(D) $\sinh^4$

74. $\int_{-\infty}^{\infty} e^{-t^2} \delta(t-2) \, dt = ?$

(A) e^{-t}

(B) e^{-t^2}

(C) e^{-4}

(D) None of the above

75. Solution $y(x)$ of

$$y(x) = x^2 + \int_0^1 \delta(x-t) \, t^2 dt$$

is :

(A) $1 + x^2$

(B) $x(1+x)$

(C) $2x^2$

(D) x^2

76. Value of $\int_0^2 (3x+1) \, \delta(x-1) dx$ is :

(A) 3

(B) 4

(C) 2

(D) 1

77. Non-trivial solution of Cauchy type equation exists when :

(A) Kernel is bounded

(B) Parameter satisfies eigen value condition

(C) Interval is infinite

(D) Function is continuous

78. Which of the following is a Hilbert kernel ?

(A) $K(x, t) = \cos\frac{(x-t)}{2}$

(B) $K(x, t) = \sin\frac{(t-x)}{2}$

(C) $K(x, t) = \cot\frac{(t-x)}{2}$

(D) None of the above

79. If H(H*f*) (x) = – f (x), then the Hilbert transform operator satisfies :

(A) $H^2 = -I$

(B) $H^2 = I$

(C) $H = 0$

(D) $H^{-1} = H$

80. Hilbert transform of constant $f(x) = 1$ is :

(A) 1

(B) 0

(C) x

(D) $\ln x$

81. If $f(x) = \int_0^x \frac{\phi(t)dt}{\sqrt{x-t}}$, $f(x) = \sqrt{x}$, then $\phi(x)$ =

(A) Constant

(B) Proportional to $\frac{1}{\sqrt{x}}$

(C) Zero

(D) x

82. The Abel equation is a special case of :

(A) Fredholm equation

(B) Volterra equation of first kind

(C) Non-linear equation

(D) Cauchy equation

83. Solving $x = \int_0^x \frac{\phi(t)dt}{\sqrt{x-t}}$ the solution is :

(A) $\phi(x) = \frac{2}{\pi}\sqrt{x}$

(B) $\phi(x) = 1$

(C) $\phi(x) = x$

(D) $\phi(x) = \frac{1}{x}$

84. The standard Abel integral equation of first kind is :

(A) $f(x) = \int_0^x (x - t)\, \phi(t)\, dt$

(B) $f(x) = \int_0^x \mathrm{K}(x, t)\, \phi(t)\, dt$

(C) $f(x) = \int_0^x \frac{\phi(t)}{\sqrt{x - t}}\, dt$

(D) None of the above

85. Fourier transform method is preferred over Laplace transform when :

(A) Interval is finite

(B) Equation is non-linear

(C) Kernel is separable

(D) Kernel depends on $(x - t)$ over entire real line

86. Consider

$$\phi(x) = f(x) + \lambda \int_{-\infty}^{\infty} e^{-a|x-t|}\, \phi(t)\, dt, \text{ then}$$

$\hat{\phi}(\omega) =$

(A) $\hat{f}(\omega)\,(a^2 + \omega^2)$

(B) $\dfrac{\hat{f}(\omega)}{1 - \lambda\left(\dfrac{2a}{a^2 + \omega^2}\right)}$

(C) $\hat{f}(\omega)\,(1 - \lambda)$

(D) None of the above

87. The Fourier transform of

$$k(x) = e^{-a|x|}$$

is :

(A) $\dfrac{2a}{a^2 + \omega^2}$

(B) $\dfrac{a}{a^2 - \omega^2}$

(C) $\dfrac{1}{a + \omega}$

(D) $e^{-a\omega}$

88. $\phi(x) = e^{-x^2} + \lambda \int_{-\infty}^{\infty} e^{-(x-t)^2} \phi(t) dt,$

the Fourier transform of e^{-x^2} is proportional to :

(A) $e^{\frac{-\omega^2}{4}}$

(B) $\frac{1}{1+\omega^2}$

(C) $\delta(\omega)$

(D) $\sin \omega$

89. If $\phi(x) = f(x) + \lambda \int_{-\infty}^{\infty} K(x-t)\phi(t)\, dt$

then after taking Fourier transform :

(A) $\hat{\phi}(\omega) = \hat{f}(\omega) + \lambda \hat{k}(\omega)$

(B) $\hat{\phi}(\omega) = \hat{f}(\omega) + \lambda \hat{k}(\omega) \hat{\phi}(\omega)$

(C) $\hat{\phi}(\omega) = \hat{f}(\omega) \hat{k}(\omega)$

(D) $\hat{\phi}(\omega) = \frac{\hat{k}(\omega)}{\hat{f}(\omega)}$

90. The Fourier transform method is most suitable for solving :

(A) Fredholm equation on finite interval

(B) Volterra equation with variable limit

(C) Convolution type equation on $(-\infty, \infty)$

(D) Non-linear integral equation

91. Consider

$\phi(x) = f(x) + \int_0^x (x-t)^2 \phi(t) dt,$

The Laplace transform of the kernel is :

(A) $\frac{1}{s^2}$

(B) $\frac{1}{s^3}$

(C) $\frac{2}{s^3}$

(D) $\frac{6}{s^4}$

92. For the equation

$$\phi(x) = e^x + \lambda\int_0^x e^{x-t}\phi(t)dt$$

after Laplace transform :

(A) $\phi(s) = \frac{1}{s-1} + \lambda\ \phi(s)$

(B) $\phi(s) = \frac{1}{s-1} + \frac{\lambda}{s-1}\ \phi(s)$

(C) $\phi(s) = \frac{1}{s+1} + \frac{\lambda}{s-1}\ \phi(s)$

(D) None of the above

93. The Convolution theorem states :

(A) $L\{f + g\} = F(s)\ G(s)$

(B) $L\{fg\} = F(s) + G(s)$

(C) $L\{f * g\} = F(s)\ G(s)$

(D) $L\{f/g\} = F(s)/G(s)$

94. A convolution type kernel always corresponds to :

(A) Fredholm equation

(B) Singular integral equation

(C) Integral equation of first kind only

(D) Volterra integral equation

95. For equation

$$\phi(t) = f(t) + \lambda\int_0^t k(t-s)\ \phi(s)ds,$$

after Laplace transform :

(A) $\phi(s) = F(s) + \lambda\ K(s)\ \phi(s)$

(B) $\phi(s) = F(s) + \lambda\ K(s)$

(C) $\phi(s) = F(s)\ \phi(s)$

(D) $\phi(s) = K(s) + \lambda F(s)$

96. If the Laplace transform of the solution is $\phi(s) = \frac{s+1}{s(s^2+1)}$, then the solution contains :

(A) e^t

(B) $\sin t$

(C) $\cos t$

(D) both $\sin t$ and $\cos t$

97. The Laplace transform method is most suitable for :

(A) Fredholm equation of first kind

(B) Non-linear integral equation

(C) Singular integral equation

(D) Volterra equation with convolution kernel

98. If $\phi(t) = 1 + \int_0^t \phi(s)ds$, then the solution is :

(A) e^t

(B) $1 + t$

(C) e^{2t}

(D) te^t

99. For the equation

$$\phi(t) = e^t + \int_0^t e^{t-s}\phi(s)ds$$

the Laplace transform of the kernel is :

(A) $\frac{1}{s}$

(B) $\frac{1}{s-1}$

(C) $\frac{1}{s+1}$

(D) $\frac{s}{s-1}$

100. Consider the integral equation $\phi(t) = t + \int_0^t (t-s)\,\phi(s)ds$, applying the Laplace transform, the transformed equation becomes :

(A) $\phi(s) = \frac{1}{s^2} + \frac{1}{s^2}\,\phi(s)$

(B) $\phi(s) = \frac{1}{s^2} + \frac{1}{s}\,\phi(s)$

(C) $\phi(s) = \frac{1}{s^2} + \phi(s)$

(D) None of the above

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 Ⓐ ● Ⓒ Ⓓ
Q. 2 Ⓐ Ⓑ ● Ⓓ
Q. 3 Ⓐ ● Ⓒ Ⓓ

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ
प्रश्न 2 Ⓐ Ⓑ ● Ⓓ
प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।