

Roll No. ....

Question Booklet Number

O. M. R. Serial No.

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Question Booklet Number
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**M. A./M. Sc. (Fourth Semester)**  
**(NEP) EXAMINATION, 2025-26**  
**MATHEMATICS**  
**(Advanced Abstract Algebra)**

Paper Code							
B	0	3	1	0	0	1	T

Questions Booklet Series
<b>C</b>

Time : 1:30 Hours ]

[ Maximum Marks : 75

**Instructions to the Examinee :**

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

**परीक्षार्थियों के लिए निर्देश :**

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

***(Only for Rough Work)***

1. An unital R-module M is said to be free module over R if :
  - (A) It admits a basis
  - (B)  $M = R$
  - (C)  $M = R \times R$
  - (D) None of the above
2. Let R be a commutative ring with identity. Then the polynomial ring  $R[x]$  is Noetherian if and only if R is Noetherian. This is known as :
  - (A) Hilbert Basis Theorem
  - (B) Noether-Laskar Theorem
  - (C) Schur's Lemma
  - (D) Wedderburn-Artin Theorem
3. Let M be a Noetherian module. Then any non-zero submodule contains a :
  - (A) Uniform module
  - (B) Vector space
  - (C) Field
  - (D) None of the above
4. Let M and N be simple R-modules. Then a non-zero homomorphism from M to N is an isomorphism is known as :
  - (A) Schur's Lemma
  - (B) Wedderburn-Artin Theorem
  - (C) Hilbert Basis Theorem
  - (D) None of the above
5. An R-module M is completely reducible if :
  - (A)  $M = \prod_{\alpha \in \Lambda} M_\alpha$ , where  $M_\alpha$  are simple R-submodules
  - (B)  $M = M_1 \times M_2$ ,  $M_1$  and  $M_2$  are simple R-submodules
  - (C)  $M = \sum_{\alpha \in \Lambda} M_\alpha$ ,  $M_\alpha$  are simple R-submodules
  - (D) None of the above
6. Let M be a R-module and N a submodule such that  $M/N$  is free. Then M is isomorphic to :
  - (A)  $N \oplus \frac{M}{N}$
  - (B)  $N \oplus M$
  - (C)  $M \cap N$
  - (D)  $M \cup N$
7. Let M be a free R-module with basis  $\{e_1, e_2, \dots, e_n\}$ . Then :
  - (A)  $M \cong R$
  - (B)  $M \cong R^{n-1}$
  - (C)  $M \cong R^n$
  - (D)  $M = R^{n+1}$
8. A module M is direct sum of its two submodules  $M_1$  and  $M_2$  if :
  - (A)  $M = M_1 + M_2, M_1 \cap M_2 = \{1\}$
  - (B)  $M = M_1 + M_2, M_1 \cap M_2 = \{0\}$
  - (C)  $M = M_1 + M_2, M_1 \cap M_2 = \{0, 1\}$
  - (D) None of the above

9. Let  $T$  be a module homomorphism. Then  $T$  is an isomorphism if :
- (A) kernel  $T = \{1\}$   
 (B) kernel  $T = \{0\}$   
 (C) kernel  $T = \{0, 1\}$   
 (D) kernel  $T = \{0, 1, 2\}$
10. If  $T$  is a homomorphism of an  $R$ -module  $M$  into an  $R$ -module  $N$ , then which is not true ?
- (A)  $T(0) = 0$   
 (B)  $T(-m) = -T(m), m \in M$   
 (C)  $T(m_1 - m_2) = T(m_1) - T(m_2),$   
 $m_1, m_2 \in M$   
 (D)  $T(0) = 4$
11. Suppose  $R$  is a ring with unity and  $M$  is a module over  $R$  and is not unital. Then there exists  $m \in M$  such that :
- (A)  $rm = 0, \forall r \in R$   
 (B)  $rm = 1, \forall r \in R$   
 (C)  $rm = 2, \forall r \in R$   
 (D)  $rm = 4, \forall r \in R$
12. If  $A$  and  $B$  are submodules of  $M$ , then which is true ?
- (A)  $\frac{A+B}{B} \cong A$   
 (B)  $\frac{A+B}{B} \cong \frac{A}{A \cap B}$   
 (C)  $\frac{A+B}{B} \cong A \cap B$   
 (D)  $\frac{A+B}{B} \cong B$
13. Let  $N$  be an  $R$ -module. Then  $\text{Hom}_R(N, N)$  is :
- (A) A subring of  $\text{Hom}(N, N)$   
 (B) A subfield of  $\text{Hom}(N, N)$   
 (C) A vector space of  $\text{Hom}(N, N)$   
 (D) None of the above
14. A module is generalization of :
- (A) Group  
 (B) Ring  
 (C) Field  
 (D) Vector space
15. The submodule  $A + B$  is generated by :
- (A)  $A \cup B$   
 (B)  $A \cap B$   
 (C)  $A^C \cap B$   
 (D)  $A \cup B^C$
16. If a finite field  $F$  has 27 elements, then  $a \in F$  satisfies the relation :
- (A)  $a^{27} = a^4$   
 (B)  $a^{27} = a^3$   
 (C)  $a^{27} = a^2$   
 (D)  $a^{27} = a$
17. If  $F$  is a finite field of characteristic 7, then  $F$  can have :
- (A) 8 elements  
 (B) 49 elements  
 (C) 9 elements  
 (D) 2 elements

18. If  $F$  is a finite field of characteristic  $p$ , then  $F$  can have :
- (A)  $p^n$  elements  
 (B) 2 elements  
 (C) 4 elements  
 (D) 8 elements
19. Which statement is not true ?
- (A)  $\mathbb{Q}(2^{1/3}, 3^{1/2})$  is a radical extension of  $\mathbb{Q}$ .  
 (B) Angle  $\theta$  is constructible if  $\sin \theta$  is constructible.  
 (C) Regular pentagon is constructible.  
 (D) We can trisect an angle by ruler and compass only.
20. If characteristic  $f=p$  and  $a, b \in F$ , then :
- (A)  $(a+b)^{p^2} = a+b$   
 (B)  $(a+b)^{p^2} = a^{p^2} + b^{p^2}$   
 (C)  $(a+b)^{p^2} = a^{p^2}$   
 (D)  $(a+b)^{p^2} = a$
21. The set of rational numbers form :
- (A) Not a field  
 (B) Prime field  
 (C) Not a group with respect to addition  
 (D) None of the above
22.  $x^m - 1$  divides  $x^n - 1$  over a field  $F$  if :
- (A)  $m > n$   
 (B)  $m = 8$   
 (C)  $m \mid n$   
 (D)  $m = 2n$
23. If  $K$  is a finite extension of  $F$ , then :
- (A)  $\alpha(G(K, F)) \geq [K : F]$   
 (B)  $\alpha(G(K, F)) = 0$   
 (C)  $\alpha(G(K, F)) \leq [K : F]$   
 (D) None of the above
24. Let  $k = \mathbb{Q}(2^{1/3})$ . Then  $O(G(K, \mathbb{Q}))$  is :
- (A) 1  
 (B) 2  
 (C) 3  
 (D) 4
25.  $O(G(K, \mathbb{Q}))$  where  $k = \frac{\mathbb{Q}[x]}{\langle x^2 - 2 \rangle}$  is :
- (A) 2  
 (B) 4  
 (C) 6  
 (D) 8
26.  $\mathbb{C}$  is normal extension of  $\mathbb{C}$  ( $\mathbb{C}$  denotes the field of complex numbers) :
- (A)  $\mathbb{Q}$   
 (B)  $\mathbb{R}$   
 (C)  $\mathbb{N}$   
 (D)  $\mathbb{Z}$
27.  $O(G(\mathbb{C}, \mathbb{R}))$  is, where  $\mathbb{C}$  denotes the field of complex numbers and  $\mathbb{R}$  the field of real numbers :
- (A) 2  
 (B) 4  
 (C) 6  
 (D) 8
28. Which statement is not true ?
- (A) Let  $\alpha, \beta$  be separable over field  $F$ . Then  $F(\alpha, \beta)$  is a simple extension of  $F$ .  
 (B) Galois group of a polynomial is a group of permutations of its roots.  
 (C) Every field with a non-zero characteristic is perfect.  
 (D) Every finite field is perfect.

29. Which statement is not true ?
- (A) If  $L$  is a normal extension of  $F$  and  $K$  is an intermediate field then  $L$  is also normal extension over  $K$ .
- (B) If  $L$  is a normal extension of  $F$  and  $K$  is an intermediate field then  $K$  is a normal extension of  $F$ .
- (C)  $K$  is a normal extension of field  $F$  of characteristic zero if and only if  $K$  is the splitting field of some polynomial over  $F$ .
- (D) Every field of characteristic zero is perfect.
30. Which statement is not true ?
- (A) A field is called perfect if all finite extensions of  $F$  are separable
- (B) Every field of characteristic zero is perfect
- (C) Every finite field is perfect
- (D) Every field with a non-zero characteristic is perfect
31. A finite extension  $k$  of a field  $F$  is said to be a normal extension of  $F$  if the fixed field of  $G(k, F)$  is :
- (A)  $F$
- (B)  $Q$
- (C)  $k$
- (D) None of the above
32. The Galois group of the equation  $x^3 - 2 = 0$  over  $Q$  has :
- (A) 1 element
- (B) 2 elements
- (C) 4 elements
- (D) 6 elements
33. Let  $G$  be a subset of  $A(k)$ , where  $A(k)$  denotes the set of all automorphism of  $k$ , then fixed field of  $G$  will be :
- (A)  $k$
- (B)  $G$
- (C) Subfield of  $k$
- (D) None of the above
34. Let  $A(k)$  denotes the set of all automorphism of  $k$ , then  $A(k)$  will form a group with respect to :
- (A) Addition
- (B) Multiplication
- (C) Composition of mapping
- (D) None of the above
35. Let  $k$  be a field. Then  $\sigma$  will be an automorphism of  $k$  if : where  $a, b \in k$ .
- (A)  $\sigma(a + b) = \sigma(a) + \sigma(b)$
- (B)  $\sigma(ab) = \sigma(a) \sigma(b)$
- (C)  $\sigma(a + b) = \sigma(a) + \sigma(b)$ ,  
 $\sigma(ab) = \sigma(a) \sigma(b)$
- (D) None of the above

36. If  $m$  is an integer which is not a perfect square and if  $\alpha + \beta\sqrt{m}$  ( $\alpha, \beta$  are rationals) is the root of a polynomial  $p(x)$  having rational coefficients, then other root will be :
- (A)  $\alpha - \beta$   
 (B)  $\alpha - \beta\sqrt{m}$   
 (C)  $\alpha$   
 (D)  $\beta$
37. Let  $F$  be a field of characteristic 7. Then any algebraic extension of  $F$  will be separable, if :
- (A)  $F = F^7$   
 (B)  $F = F^5$   
 (C)  $F = F^{49}$   
 (D)  $F = F^2$
38. Let  $F$  be a field of characteristic  $p$  and  $k$  be its extension and  $a$  be an algebraic element of  $k$  over  $F$ . Then  $a$  will be separable over  $F$  if :
- (A)  $F(a) = F$   
 (B)  $F(a^p) = F(a)$   
 (C)  $F(a^p) = F$   
 (D) None of the above
39. Which statement is not true ?
- (A) Any finite extension of a field of characteristic zero is a simple extension  
 (B)  $[Q(\sqrt{3}, \sqrt{5}) : Q] = 4$   
 (C)  $[Q(\sqrt{3}, \sqrt{5}) : Q] = 8$   
 (D)  $Q(\sqrt{3}, \sqrt{5}) = Q(\sqrt{3} + \sqrt{5})$
40. Primitive element for  $Q(i, 2^{1/2})$  over  $Q$  is :
- (A)  $\sqrt{2}$   
 (B)  $\sqrt{2} + i$   
 (C)  $i$   
 (D) 2
41. Which statement is true ?
- (A)  $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$   
 (B)  $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{5})$   
 (C)  $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2})$   
 (D)  $Q(\sqrt{2}) = Q(\sqrt{3})$
42. Let  $F$  be a field of characteristics 11. Then the polynomial  $x^{121} - x \in F[x]$  has :
- (A) All zero roots  
 (B) All distinct roots  
 (C) No root  
 (D) None of the above

43. Let  $F$  be a field of characteristic 7. Then the polynomial  $x^{49} - x \in F[x]$  has :
- (A) No roots  
 (B) No multiple roots  
 (C) All roots are zero  
 (D) None of the above
44. Which statement is true ?
- (A)  $\mathbb{Q}(2^{1/2})$  is a simple extension of  $\mathbb{Q}$   
 (B)  $\sin^{-1} 1$  is algebraic over  $\mathbb{Q}$   
 (C)  $\mathbb{Q}(\sqrt{2} + \sqrt{3}) = \mathbb{Q}(\sqrt{5})$   
 (D)  $[\mathbb{Q}(\sqrt{2}) : \mathbb{Q}] = 5$
45. Let  $f(x) = g(x)(x-a)^{10}$ , where  $g(x)$  is a polynomial in  $F[x]$  and  $a \in F$ , where  $F$  is a field, then :
- (A)  $f(x) = 0$   
 (B)  $g(x) = 0$   
 (C)  $f(x)$  has multiple roots  
 (D) None of the above
46. Let  $F$  be a field and let  $f(x) \in F[x]$  such that  $f'(x) = 0$  and characteristic of  $F$  is  $p$ , then which is true ?
- (A)  $f(x) = 0$   
 (B)  $f'(x) = 0$   
 (C)  $f(x) = g(x^p)$  where  $g(x)$  is a polynomial in  $F[x]$   
 (D) None of the above
47. Let  $f(x) \in F[x]$  where  $F$  is a field of characteristic zero and if  $f'(x) = 0$ , then which is true ?
- (A)  $f(x) = 0$   
 (B)  $f(x)$  is a constant polynomial  
 (C)  $\deg f(x) < \deg f'(x)$   
 (D)  $\deg f(x) = 0$
48. If  $p(x) \in F[x]$  is irreducible and if  $a, b$  are two roots of  $p(x)$ , then which statement is not true ?
- (A)  $F(a)$  is isomorphic to  $F(b)$   
 (B)  $F(a)$  is a simple extension of  $F$   
 (C)  $F(b)$  is a simple extension of  $F$   
 (D)  $F(a)$  is a subfield of  $F$
49. Let  $f(x) \in F[x]$  be an irreducible polynomial and if  $E_1$  and  $E_2$  are its two splitting fields, then :
- (A)  $E_1$  is isomorphic to  $E_2$   
 (B)  $E_1 > E_2$   
 (C)  $E_1 < E_2$   
 (D)  $E_1$  and  $E_2$  are not related
50. Degree of splitting field of  $x^4 + 1 \in \mathbb{Q}[x]$  will be :
- (A) 2  
 (B) 4  
 (C) 6  
 (D) 8

51. Degree of splitting field of  $x^3 - 7 \in \mathbb{Q}[x]$  will be :  
 (A) 2  
 (B) 4  
 (C) 6  
 (D) 8
52. Splitting field of  $x^5 - 2 \in \mathbb{Q}[x]$  will be :  
 (A)  $\mathbb{Q}(2^{1/2})$   
 (B)  $\mathbb{Q}(2^{1/3})$   
 (C)  $\mathbb{Q}(2^{1/5})$   
 (D)  $\mathbb{Q}(2^{1/7})$
53. Degree of extension of the splitting field of  $x^n - 1 \in \mathbb{Q}[x]$  will be :  
 (A)  $n$   
 (B)  $n - 1$   
 (C)  $\phi(n)$  (Euler's function)  
 (D)  $n^2$
54. Degree of splitting field of  $f(x) = x^4 - 2 \in \mathbb{Q}[x]$  will be :  
 (A) 2  
 (B) 4  
 (C) 6  
 (D) 8
55. Splitting field of  $x^{13} - 1 \in \mathbb{Q}[x]$  will be of degree :  
 (A) 12  
 (B) 13  
 (C) 14  
 (D) 15
56. Basis of  $\mathbb{Q}(\sqrt{7}, \sqrt{11})$  over  $\mathbb{Q}$  is :  
 (A)  $\{1, \sqrt{7}\}$   
 (B)  $\{1, \sqrt{11}\}$   
 (C)  $\{1, \sqrt{77}\}$   
 (D)  $\{1, \sqrt{7}, \sqrt{11}, \sqrt{7}\sqrt{11}\}$
57. Degree of splitting field for  $f(x) = x^{11} - 2$  over  $\mathbb{Q}$  is :  
 (A) 11  
 (B) 110  
 (C) 120  
 (D) 140
58. The degree of splitting field for  $f(x) = x^7 - 2 \in \mathbb{Q}[x]$  is :  
 (A) 7  
 (B) 14  
 (C) 28  
 (D) 42
59. Let  $F$  be the splitting field of  $x^7 - 2 \in \mathbb{Q}[x]$  and  $\alpha = e^{2\pi i/7}$ , a primitive seventh root of unity, then :  
 (A)  $[F : \mathbb{Q}(\alpha)] = 9$   
 (B)  $[F : \mathbb{Q}(\alpha)] = 7$   
 (C)  $[F : \mathbb{Q}] = 4$   
 (D)  $[F : \mathbb{Q}] = 5$
60. Let  $F$  be a field of characteristic 7 and let  $b$  be a root of  $f(x) = x^7 - x - a \in F[x]$ . Then the splitting field of  $f(x)$ , over  $F$  is :  
 (A)  $F$   
 (B)  $F(b)$   
 (C)  $F(a)$   
 (D) None of the above

61. Splitting field of the polynomial  $x^7 - 1 \in \mathbb{Q}[x]$  is of degree :
- (A) 2  
(B) 4  
(C) 6  
(D) 8
62. Degree of splitting field of  $x^4 + 2$  over  $\mathbb{Q}$  is :
- (A) 2  
(B) 4  
(C) 6  
(D) 8
63.  $[\mathbb{Q}(\sqrt{11}, i) : \mathbb{Q}]$  is :
- (A) 4  
(B) 6  
(C) 8  
(D) 10
64. Consider the polynomial  $x^2 + 2x + 2 \in \mathbb{Z}_3[x]$ . Then its splitting field will be :
- (A)  $\mathbb{Z}_3[x]$   
(B)  $\frac{\mathbb{Z}_3[x]}{\langle x^2 + x + 1 \rangle}$   
(C)  $\frac{\mathbb{Z}_3[x]}{\langle x^2 + 2x + 2 \rangle}$   
(D)  $\frac{\mathbb{Z}_3[x]}{\langle x^2 + 5x + 5 \rangle}$
65. Consider the polynomial  $x^2 - 2x + 2 \in \mathbb{Z}_3[x]$  and if  $\mathbb{Z}_3(\alpha)$  be its splitting field, then which is true ?
- (A)  $[\mathbb{Z}_3(\alpha) : \mathbb{Z}_3] = 2$   
(B)  $[\mathbb{Z}_3(\alpha) : \mathbb{Z}_3] = 4$   
(C)  $[\mathbb{Z}_3(\alpha) : \mathbb{Z}_3] = 6$   
(D) None of the above
66. Splitting field of  $x^2 + x + 2 \in \mathbb{Z}_3[x]$  is :
- (A)  $\mathbb{Z}_3$   
(B)  $\mathbb{Z}_3[i]$   
(C)  $\mathbb{Z}$   
(D) None of the above
67. Splitting field of  $x^4 - x^2 - 2 \in \mathbb{Q}[x]$  is :
- (A)  $\mathbb{Q}(\sqrt{2})$   
(B)  $\mathbb{Q}(\sqrt{2}, \sqrt{3})$   
(C)  $\mathbb{Q}(\sqrt{2}, i)$   
(D)  $\mathbb{Q}(i)$
68. Let  $f(x) \in \mathbb{F}[x]$  be a irreducible polynomial. Then the finite extension  $k$  of  $\mathbb{F}$  in which  $f(x)$  has all roots and if  $\deg f(x) = 5$  then  $[k : \mathbb{F}]$  will be at most :
- (A) 1  
(B) 2!  
(C) 5!  
(D) 7!

69. If  $f(x) \in F[x]$ , then the finite extension  $k$  of  $F$  in which  $f(x)$  has a root, then which is true ?
- (A)  $[k : F] \leq \deg f(x)$   
 (B)  $[k : F] \geq \deg f(x)$   
 (C)  $2[k : F] = \deg f(x)$   
 (D) None of the above
70. If  $p(x)$  is a polynomial in  $F[x]$  of degree 7 and it is irreducible over  $F$ , then the extension field  $k$  of  $F$  in which  $p(x)$  has a root, then which is true ?
- (A)  $[k : F] = 2$   
 (B)  $[k : F] = 3$   
 (C)  $[k : F] = 7$   
 (D) None of the above
71. It  $p(x) = x^2 - 2x + 2 \in z_3[x]$  and if  $\alpha$  be a root of  $p(x)$  then the field  $z_3(\alpha)$  has :
- (A) 7 elements  
 (B) 8 elements  
 (C) 9 elements  
 (D) 10 elements
72. If  $p(x)$  is the minimal polynomial for  $a$  over  $F$ , then which statement is true :
- (A)  $[F(a) : F] > \deg p(x)$   
 (B)  $[F(a) : F] < \deg p(x)$   
 (C)  $[F(a) : F] = \deg p(x)$   
 (D) None of the above
73. Which statement is true ?
- (A)  $\sqrt{2} + \sqrt{\pi}$  is algebraic over  $Q(\pi)$   
 (B)  $\sin^{-1} 1$  is algebraic over  $Q$   
 (C)  $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{5})$   
 (D) Basis of  $Q(\sqrt{3}, \sqrt{7})$  over  $Q$  is  $\{1, \sqrt{3}, \sqrt{7}\}$
74. Basis of  $Q(\sqrt{11})$  over  $Q$  will be :
- (A)  $\{1, 11\}$   
 (B)  $\{1, 2\}$   
 (C)  $\{1, \sqrt{11}\}$   
 (D) None of the above
75. Which statement is not true ?
- (A) Let  $F[x]$  be the ring of polynomials in  $x$  over the field  $F$  and let  $g(x) \in F[x]$  of degree  $n$  and let  $\langle g(x) \rangle$  be the ideal generated by  $g(x)$ , then  $F[x] / \langle g(x) \rangle$  is an  $n$ -dimensional vector space over  $F$ .  
 (B) Let  $k$  be an extension of a field  $F$  and let  $a \in k$  be algebraic over  $F$ . Then  $F(a)$  is isomorphic to  $F[x]/V$ , where  $V$  is the ideal of  $F[x]$  generated by the minimal polynomial for  $a$  over  $F$ .  
 (C)  $\mathbb{C}$  is a finite extension of  $Q$ .  
 (D) Every finite extension is algebraic.

76. Which statement is not true ?

- (A)  $\frac{\mathbb{R}[x]}{\langle x^2 + 1 \rangle} \cong \mathbb{C}$
- (B) There exists a field of 8 elements
- (C)  $[\mathbb{Q}(2^{1/3}, 2^{1/2}) : \mathbb{Q}] = 6$
- (D)  $\mathbb{R}$  is a finite extension of  $\mathbb{Q}$

77. Consider the polynomial

$$p(x) = x^2 + x + 1$$

in  $\mathbb{Z}_2[x]$ . If  $\alpha$  be a root of  $p(x)$  then the extended field containing  $\alpha$  will be :

- (A)  $\mathbb{Z}_2(\alpha)$
- (B)  $\mathbb{Z}_3$
- (C)  $\mathbb{Z}_4$
- (D)  $\mathbb{Z}_5(\alpha)$

78. Let  $k$  be an extension of a field  $F$  and let  $a \in k$  be algebraic of degree 7 over  $F$ . Then  $F(a)$  will be of the form :

- (A)  $\{\alpha_0 + \alpha_1 a + \alpha_2 a^2 : \alpha_i \in F\}$
- (B)  $\{\alpha_0 + \alpha_1 a : \alpha_2 a^2 + \alpha_3 a^3 + \alpha_4 a^4 + \alpha_5 a^5 + \alpha_6 a^6 : \alpha_i \in F\}$
- (C)  $\{\alpha_0 + \alpha_1 a : \alpha_i \in F\}$
- (D)  $\{\alpha_0 + \alpha_1 a + \alpha_2 a^2 + \alpha_3 a^3 : \alpha_i \in F\}$

79. Inverse of  $1 - 2^{1/3} + 4^{1/3} \in \mathbb{Q}(2^{1/3})$  is :

- (A)  $\frac{(2^{1/3} + 1)}{3}$
- (B)  $2^{1/3}$
- (C)  $\frac{2^{1/3} - 1}{2}$
- (D)  $\frac{2^{1/3} + 2^{1/2}}{2}$

80. Inverse of  $3 - \sqrt{2} \in \mathbb{Q}(\sqrt{2})$  is :

- (A)  $\frac{1}{7}(\sqrt{2} + 3)$
- (B)  $\sqrt{2} + 3$
- (C)  $\sqrt{2}$
- (D)  $\sqrt{3}$

81.  $[\mathbb{Q}(2^{1/3}, 2^{1/2}) : \mathbb{Q}(2^{1/2})]$  is :

- (A) 2
- (B) 3
- (C) 4
- (D) 5

82. The monic, minimal polynomial corresponding to  $\sqrt{2}$  is :

- (A)  $x^2 - 4$
- (B)  $x^2 - 2$
- (C)  $x^3 - 2$
- (D)  $x^4 - 2$

83. Which statement is not true ?
- (A)  $\mathbb{R}$  is a transcendental extension of  $\mathbb{Q}$
- (B)  $\mathbb{C}$  is an algebraic extension of  $\mathbb{R}$
- (C)  $[\mathbb{C} : \mathbb{R}] = 2$
- (D)  $\{1, i\}$  does not form a basis of  $\mathbb{C}$  over  $\mathbb{R}$
84. The element  $a \in k$  is a root of  $p(x) \in F[x]$  where  $F \subseteq k$  of multiplicity 5, if :
- (A)  $\frac{(x-a)^5}{p(x)}$  whereas  $(x-a)^6 \nmid p(x)$
- (B)  $\frac{(x-a)^6}{p(x)}$  whereas  $(x-a)^7 \nmid p(x)$
- (C)  $\frac{(x-a)^7}{p(x)}$  whereas  $(x-a)^8 \nmid p(x)$
- (D) None of the above
85. A polynomial of degree 5 over a field can have at most :
- (A) 5 roots in any extension field
- (B) 6 roots in any extension field
- (C) 7 roots in any extension field
- (D) No roots
86. If  $a \in k$  is a root of  $p(x) \in F[x]$  where  $F \subseteq k$  then in  $k[x]$  :
- (A)  $\frac{(x-a)}{p(x)}$
- (B)  $\frac{(x-a)^2}{p(x)}$
- (C)  $\frac{(x-a)^3}{p(x)}$
- (D)  $(x-a)$  does not divide  $p(x)$
87. If  $p(x) \in F[x]$ , then an element  $a$  lying in some extension field of  $F$  is a root of  $p(x)$  if :
- (A)  $p(a) = 0$
- (B)  $p(a) = 1$
- (C)  $p(a) = 2$
- (D)  $p(a) = 4$
88. Which statement is true ?
- (A) There exists a field of 24 elements
- (B) There exists a field of 48 elements
- (C) There exists a field of 92 elements
- (D) There exists a field of 25 elements
89. Which statement is true ?
- (A) If  $a_1, a_2, \dots, a_n \in k$  are algebraic over  $F$  then  $F(a_1, a_2, \dots, a_n)$  is a finite extension of  $F$ .
- (B) Any finite extension is not algebraic.
- (C) If  $F(a)$  is a finite extension of  $F$  then  $a$  is not algebraic over  $F$ .
- (D) If  $a \in k$  is algebraic over  $F$  then  $F(a)$  is not a finite extension of  $F$ .
90. If  $L$  is a finite extension of  $k$  and  $k$  is a finite extension of  $F$  then :
- (A)  $[L : F] = [L : k]$
- (B)  $[L : F] = [L : k][k : F]$
- (C)  $[L : F] = [L : \mathbb{Q}]$
- (D) None of the above

91.  $[Q(2^{1/2}, 2^{1/4}, 2^{1/8}) : Q]$  is :
- (A) 2  
(B) 4  
(C) 6  
(D) 8
92. Basis of  $Q(\sqrt{3}, \sqrt{5})$  over  $Q$  is :
- (A)  $\{1, 3\}$   
(B)  $\{1, \sqrt{3}, \sqrt{5}, \sqrt{15}\}$   
(C)  $\{1, \sqrt{15}\}$   
(D)  $\{1, \sqrt{5}\}$
93. If  $Q$  is the field of rational numbers, then  $[Q(2^{1/3}, 3^{1/4}) : Q]$  is :
- (A) 12  
(B) 24  
(C) 36  
(D) 6
94. If  $Q$  is the field of rational numbers, then  $[Q(\sqrt{2}, \sqrt{3}) : Q]$  is :
- (A) 2  
(B) 3  
(C) 4  
(D) 8
95. If  $k$  is an extension of a field  $F$  and  $a \in k$  is algebraic over  $F$  then  $F(a)$  is :
- (A) A finite extension of  $F$   
(B) An infinite extension of  $F$   
(C)  $F(a)$  is not a field  
(D) None of the above
96. The field  $Q(\sqrt{3}, \sqrt{5})$  is equal to :
- (A)  $Q$   
(B)  $Q(2)$   
(C)  $Q(\sqrt{3} + \sqrt{5})$   
(D)  $Q(15)$
97.  $(Z, +, \cdot)$  be the ring of integers and  $\langle n \rangle$  be the ideal generated by then  $\frac{z}{\langle n \rangle}$  is isomorphic to :
- (A)  $Z_n$   
(B)  $Q$   
(C)  $R$   
(D)  $\mathbb{C}$
98.  $\frac{z[x]}{\langle x \rangle}$  is isomorphic to :
- (A)  $\phi$   
(B)  $Q$   
(C)  $Z$   
(D)  $R$
99. Number of elements in  $\frac{z[i]}{\langle 1 + 2i \rangle}$  is :
- (A) 5  
(B) 25  
(C) 20  
(D) 21
100. Order of the field  $\frac{z[i]}{\langle 3 \rangle}$  is :
- (A) 49  
(B) 89  
(C) 9  
(D) 4

***(Only for Rough Work)***

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

**Example :**

**Question :**

Q. 1 (A) ● (C) (D)

Q. 2 (A) (B) ● (D)

Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

**Impt. :** On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

**उदाहरण :**

**प्रश्न :**

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

**महत्वपूर्ण :** प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।