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O. M. R. Serial No. | | | | | | |

Question Booklet Number

M. A./M. Sc. (Second Semester) (NEP) EXAMINATION, 2025-26

MATHEMATICS

(Advanced Real Analysis)

Paper Code							
B	0	3	0	8	0	1	T

Questions Booklet Series
C

Time : 1:30 Hours] [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(Remaining instructions on the last page)

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. Which statement is not true ?

(A) If E is a measurable set, then $L^4(E) \subset L^2(E)$.

(B) Let E be a measurable set with finite measure. Then $L^{\infty}(E) \subset L^p(E)$, for $1 \le p < \infty$.

(C) If $f \in L^p \cap L^q$ then $f \in L^r$ for all $q < r < p$.

(D) L^2 space is not complete.

2. Which space is complete ?

(A) $L^{1/2}$

(B) $L^{1/4}$

(C) L^2

(D) None of the above

3. If f and g are in L^p where $1 \le p \le \infty$ then $\| f + g \|_p \le \| f \|_p + \| g \|_q$. This result is known is :

(A) Holder inequality

(B) Minkowski inequality

(C) Cauchy-Schwarz inequality

(D) None of the above

4. In Holder inequality when $p = q = 2$, then it is called :

(A) Minkowski inequality

(B) Fatou's Lemma

(C) Cauchy-Schwarz inequality

(D) None of the above

5. Let X = [0, 4] and $f : X \to R$ defined as $f(x) = x^{-1/2}$. Then :

(A) $f \in L^1(X)$

(B) $f \in L^2(X)$

(C) $2f \in L^2(X)$

(D) None of the above

6. If f is a convex function on $(-\infty, \infty)$ and g an integrable function on [0, 1], then which is true ?

(A) $\int_0^1 f(g(t))dt = f\left(\int_0^1 g(t)dt\right)$

(B) $\int_0^1 f(g(t))dt \geq f\left(\int_0^1 g(t)dt\right)$

(C) $\int_0^1 f(g(t))dt \leq f\left[\int_0^1 g(t)dt\right]$

(D) None of the above

7. A function f defined on an open interval (a, b) is convex function and if $0 \leq \lambda \leq 1$ and for $x, y \in (a, b)$, then :

(A) $f(\lambda x + (1-\lambda)y)$

$\lambda f(x) + (1-\lambda) f(y)$

(B) $f(\lambda x + (1-\lambda)y) \leq$

$\lambda f(x) + (1-\lambda) f(y)$

(C) $f(\lambda x) = \lambda f(x) + \lambda f(y)$

(D) $f(\lambda x + (1-\lambda)y) \geq \lambda f(x)$

$+(1-\lambda) f(y)$

8. If f is integrable over E and $\{E_i\}$ is a sequence of disjoint measurable sets such that $E = \bigcup_{i=1}^{\infty} E_i$, then :

(A) $\int_E f = \sum_{i=1}^{\infty} \int_{E_i} f$

(B) $\int_E f \leq \sum_{i=1}^{\infty} \int_{E_i} f$

(C) $\int_E f \geq \sum_{i=1}^{\infty} \int_{E_i} f$

(D) None of the above

9. Let $\{f_n\}$ be a sequence of integrable functions which converges almost everywhere to an integrable function f and if $\lim_{n\to\infty} \int |f - f_n| \to 0$. Then :

(A) $\lim_{n\to\infty} \int |f_n| \to |f|$

(B) $\lim_{n\to\infty} |\int f_n| \to |f|$

(C) $\lim_{n\to\infty} |f_n| \to 0$

(D) $\lim_{n\to\infty} \int f_n \to \int f_n$

10. Which statement is not true ?

(A) In Rieman integration $|f|$ may be integrable while f may not be.

(B) f is Lebesgue integrable if and only if $|f|$ is Lebesgue integrable.

(C) A measurable function f is integrable over E while f^{+} cannot be integrable.

(D) If a measurable function f is integrable over E, then f^{+} and f^{-} will be integrable over E.

11. Let $f : [0, 1] \to \mathrm{R}$ defined as :

$$f(x) = \begin{cases} \dfrac{1}{x^{2/3}} & , \ 0 < x < 1 \\ 0 & , \ x = 0 \end{cases}$$

Then $\int_0^1 f \, dx$ will be :

(A) 1

(B) 2

(C) 3

(D) 4

12. Let f be a measurable function over a set E and g be an integrable function such that $|f| \le g$. Then which is true ?

(A) f will be integrable over E

(B) f will not be integrable over E

(C) f will be differentiable over E

(D) f will be constant

13. Let f be an integrable function defined over a measurable set E. Then which is always true ?

(A) $\int_E |f| \le |\int_E f|$

(B) $|\int_E f| \le \int_E |f|$

(C) $\int_E |f| = \int_E |f|$

(D) None of the above

14. Let $\{f_n\}$ be a sequence of integrable function such that $f_n \to f$ a.e., and f is integrable such that $\lim_{n\to\infty} \int |f - f_n| \to 0$. Then which is true ?

(A) $\lim_{n\to\infty} \int |f_n| \to \int |f|$

(B) $\lim_{n\to\infty} \int |f_n| \to 0$

(C) $\lim_{n\to\infty} \int |f_n| \to f$

(D) None of the above

15. If $\int_E f = 0$ and $f(x) \geq 0$ on E, then :

(A) $f > 0$ a.e.

(B) $f = 0$ a.e.

(C) $f > 1$ a.e.

(D) $f < -2$

16. Let

$$f(x) = \begin{cases} 2 \text{ when } x \text{ is an irrational number in } [-2, 2] \\ -2 \text{ when } x \text{ is a rational number in } [-2, 2] \end{cases}$$

Then the value of $\int_{-2}^{2} f(x)dx$ will be :

(A) 8

(B) 16

(C) 0

(D) – 1

17. Which statement is not true ?

(A) Monotone convergence theorem does not hold good for a decreasing sequence of functions.

(B) Non-negativity of functions is necessary for Fatou's lemma.

(C) A measurable function f is integrable over E if and only if $|f|$ is integrable over E.

(D) Let f be a measurable function and $|f|$ is integrable over E. Then f is not integrable over E.

18. Let $\{f_n\}$ be an increasing sequence of non-negative measurable functions and $f = \lim f_n$ a.e., then :

(A) $\int f \leq \lim \int f_n$

(B) $\int f \geq \lim \int f_n$

(C) $\int f = \lim \int f_n$

(D) None of the above

19. If $\{f_n\}$ is a sequence of non-negative measurable functions and $f_n(x) \to f(x)$ a.e., on a set E, then $\int_E f \leq \underline{\lim} \int_E f_n$, this result is known as :

(A) Bounded convergence theorem

(B) Fatou's lemma

(C) Monotone convergence theorem

(D) Dominated convergence theorem

20. Which statement is not true ?

(A) Every signed measure can be expressed as the difference of two measures.

(B) Hahn decomposition is unique.

(C) Labesgue measure is not a finite measure.

(D) Labesgue measure is not totally finite.

21. If μ is a signed measure and E be a measurable set, then which is true ?

(A) $\mu(E) = \mu^+(E) - \mu^-(E)$

(B) $\mu(E) = \mu^+(E) + \mu^-(E)$

(C) $\mu(E) = \mu^+(E)$

(D) $\mu(E) = \mu^-(E)$

22. $(X, \mathbf{A}, \mu)$ be a measure space and $E \in \mathbf{A}$, then total variation of μ is defined as :

(A) $\mu^{+}(E)$

(B) $\mu^{-}(E)$

(C) $|\mu|(E) = \mu^{+}(E) + \mu^{-}(E)$

(D) None of the above

23. If μ is a signed measure then there exist two disjoint sets A and B whose union is X such that A is positive and B is negative with respect to μ, then the sets A and B are called :

(A) Hahn-Decomposition of X with respect to μ

(B) Jordan-Decomposition of X

(C) Randon-Nikodym form

(D) None of the above

24. Which statement is not true ?

(A) Lebesgue measure is not a finite measure

(B) Lebesgue measure is σ-finite measure

(C) Lebesgue measure is not totally finite measure

(D) Lebesgue measure of real line is finite

25. Suppose $X = [0, 2\pi]$ and $\mathbf{A}$ be the class of all Lebesgue measurable subsets of X and $(X, \mathbf{A}, \mu)$ be measure space then. The positive set of signed measure μ defined by $\mu(X) = \int_X \sin x \, dx$ is :

(A) $[0, \pi]$

(B) $\left[\pi, \frac{3\pi}{2}\right]$

(C) $\left[\frac{3\pi}{2}, 2\pi\right]$

(D) $[\pi, 2\pi]$

26. If f is a bounded measurable function defined on a set E of finite measure and if A and B are disjoint measurable subsets of E, then which is true ?

(A) $\int_{A\cup B} f \geq \int_A f + \int_B f$

(B) $\int_{A\cup B} f = \int_A f + \int_B f$

(C) $\int_{A\cup B} f \leq \int_A f + \int_B f$

(D) None of the above

27. If f is a bounded measurable function defined on a set E of finite measure, then :

(A) $\int_E |f| \leq |\int_E f|$

(B) $|\int_E f| \leq \int_E |f|$

(C) $\int_E |f| = |\int_E f|$

(D) None of the above

28. Let E be a measurable set of finite measure and $\{f_n\}$ be a sequence of measurable functions which converge to f a.e. on E. Then given $\alpha > 0$, there is a subset $A \subset E$ with $m(A) < \alpha$ such that $\{f_n\}$ converges to f uniformly on E – A, this theorem is known as :

(A) Egoroff's theorem

(B) Lusin theorem

(C) Riesz Fischer theorem

(D) Jensen inequality

29. The function f defined on E = [0, 1] as

$$f(x) = \begin{cases} 3, & \text{if } x = 0 \\ \dfrac{1}{x}, & \text{if } 0 < x < 1 \\ 5, & \text{if } x = 1 \end{cases}$$

Then $E(f > \alpha)$ where $1 < \alpha < 3$ will be :

(A) [0, 1]

(B) [0, 2]

(C) $\left(0, \frac{1}{\alpha}\right) \cup [0,1]$

(D) $\left(0, \frac{1}{\alpha}\right) \cup \{0,1\}$

30. The function f defined on R by :

$$f(x)=\begin{cases} x+5, & \text{if } x<-1 \\ 2, & \text{if } -1\leq x<0 \\ x^2, & \text{if } 0\leq x \end{cases}$$

Then $R(f\leq 4)$ will be :

(A) $(\infty, 2]$

(B) $[0, 1]$

(C) $[0,\infty)$

(D) $(-\infty, 5)$

31. If α is any real number, then which statement is true ?

(A) $E(f>\alpha)=f^{-1}(-\infty,\alpha)$

(B) $E(f>\alpha)=f^{-1}(\alpha,\infty)$

(C) $E(f<\alpha)=f^{-1}(\alpha,\infty)$

(D) $E(f=\alpha)=f^{-1}(\alpha,\infty)$

32. Let g be a measurable real-valued function defined on a set E and f a continuous function defined on the range of g, then f o g will be a :

(A) measurable function

(B) differentiable function

(C) constant function

(D) None of the above

33. If f is a measurable function and O is an open set, then the set $A=\{x: f(x)\in 0\}$ is a :

(A) Null set

(B) Measurable set

(C) Non-measurable set

(D) Finite set

34. If $\{f_n\}$ is a sequence of measurable functions converging to f on E, then f will be :

(A) Zero

(B) Constant

(C) Measurable

(D) None of the above

35. Consider the function $f:[0,1] \to \mathrm{R}$ defined as :

$$f(x) = \begin{cases} 0, x \in \mathrm{Q} \text{ (set of rational members} \\ 1, x \in \mathrm{Q}' \text{ (set of irrational members} \end{cases}$$

Then f will be :

(A) Constant

(B) Zero

(C) Measurable

(D) None of the above

36. If f is a measurable function, then :

(A) f^2 will be measurable

(B) $-f$ is not measurable

(C) f^2 is not measurable

(D) None of the above

37. Let E = [0, 1) and E_1 be a non-measurable subset of E. Then the function :

$f : \mathrm{E} \to \mathrm{R}$ defined as

$$f(x) = \begin{cases} 1, x \in \mathrm{E}_1 \\ -1, x \notin \mathrm{E}_1 \end{cases}$$

will be :

(A) Constant

(B) Measurable

(C) Non-measurable

(D) None of the above

38. Let f and g be two measurable functions. Then $f + g$ will be :

(A) zero

(B) finite

(C) measurable

(D) None of the above

39. Let E be a measurable set and $A \subset E$ and χ_A is the characteristic function of A and α is any real number. Then $E(\chi_A > \alpha)$ will be :

(A) α

(B) ϕ

(C) A, if $0 \leq \alpha < 1$

(D) α^2

40. Let E be a measurable set and $E_1 \subset E$ and if characteristic function χ_{E_1} is measurable, then :

(A) E_1 is zero

(B) E_1 is finite

(C) E_1 is measurable

(D) None of the above

41. If χ_A is a characteristic function of A, then :

(A) $\chi_{A^c} = \chi_A$

(B) $\chi_{A^c} = 1 - \chi_A$

(C) $\chi_A = 1$

(D) $\chi_A = 0$

42. If $A = \bigcup_n A_n$, where $\{A_n\}$ is a sequence consisting of disjoint subsets of A, then characteristic function χ_A is defined as :

(A) $\chi_A = \sum_{n=1}^{\infty} \chi_{A_n}$

(B) $\chi_A = \chi_{A_1} \cdot \chi_{A_2}$

(C) $\chi_A = \chi_{A_1} - \chi_{A_2}$

(D) $\chi_A = \chi_{A_1} \cap \chi_{A_2} \cap \ldots \cap \chi_{A_n}$

43. If χ is a characteristic function, then :

(A) $\chi_\phi = 0$

(B) $\chi_\phi = 1$

(C) $\chi_\phi = 2$

(D) $\chi_\phi = 3$

44. The characteristic functions of a set A is defined as :

(A) $\chi_A(x) = \begin{cases} 1, & x \notin A \\ 0, & x \in A \end{cases}$

(B) $\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$

(C) $\chi_A(x) = \begin{cases} 1, & x \in A \\ 2, & x \notin A \end{cases}$

(D) $\chi_A(x) = 0, \forall x \in A$

45. If f and g are measurable functions defined on a set E, then the set $A = \{x \in E \mid f(x) < g(x)\}$ will be a :

(A) Zero set

(B) Finite set

(C) Measurable set

(D) Countable set

46. Cantor set is a/an :

(A) Zero set

(B) Finite set

(C) Constant set

(D) Uncountable set

47. Let f be a measurable function defined on a set E. If for any real number α, the set $\{x : f(x) = \alpha\}$ is measurable, then :

(A) f is always measurable

(B) f may not be measurable

(C) f is constant

(D) f is not defined

48. If f is a measurable function, then for any real number α, which statement is not true ?

(A) $\{x : f(x) > \alpha\}$ is measurable

(B) $\{x : f(x) \geq \alpha\}$ is measurable

(C) $\{x : f(x) < \alpha\}$ is measurable

(D) $\{x : f(x) > \alpha\}$ cannot be measurable

49. If f is a measurable function defined on E, then for any real number α :

(A) $\{x : f(x) = \alpha\}$ is empty

(B) $\{x : f(x) = \alpha\}$ is finite

(C) $\{x : f(x) = \alpha\}$ is measurable

(D) None of the above

50. If f is a measurable function defined on E, then $E(f \geq \alpha)$ is :

(A) $\{x \in E \mid f(x) \geq \alpha\}$

(B) $\{x \in E \mid f(x) > \alpha\}$

(C) $\{x \in E \mid f(x) < \alpha\}$

(D) $\{x \in E \mid f(x) = \alpha\}$

51. If f is a measurable function defined on E, then which is not true ?

(A) $\{x : f(x) > \alpha\} = \{x : f(x) \leq \alpha\}^C$

(B) $\{x : f(x) < \alpha\} = \{x : f(x) \leq \alpha\}^C$

(C) $\{x : f(x) \geq \alpha\} =$

$$\bigcap_{n=1}^{\infty} \left\{x : f(x) > \alpha - \frac{1}{n}\right\}$$

(D) $\{x : f(x) > \alpha\} = \{x : f(x) < \alpha\}$

52. If f is a measurable function on the set E and $E_1 \subset E$ is a measurable set, then :

(A) $E_1(f > \alpha) = E(f > \alpha) \cap E$

(B) $E_1(f > \alpha) = E(f > \alpha) \cap E_1$

(C) $E_1(f > \alpha) = E$

(D) $E_1(f > \alpha) = E(f > \alpha)$

53. Which statement is not true ?

(A) If r is any rational number and f is a measurable function then the set $\{x : f(x) < r\}$ is a measurable set.

(B) If f and g are measurable functions on a set E, then the set $\{x \in E \,|\, f(x) < g(x)\}$ is measurable.

(C) A constant function, with a measurable domain is measurable.

(D) If f is a measurable function then for any real number α, the set $\{x : f(x) > \alpha\}$ cannot be measurable.

54. Which statement is not true ?

(A) A step function is a measurable function.

(B) Constant function with a measurable domain is measurable.

(C) It f is measurable function on the set E and $E_1 \subset E$ is a measurable set, then f is a measurable function on E_1.

(D) An integrable function cannot be measurable.

55. Which statement is not true ?

(A) Every function defined on a set of measure zero is measurable.

(B) If f and g are measurable functions, then $f + g$ is measurable.

(C) Continuous function defined on a measurable set is measurable.

(D) Constant function with a measurable domain cannot be measurable.

56. Which statement is not true ?

(A) If f is measurable then $|f|$ is measurable.

(B) If f is measurable then f^+ is measurable.

(C) If f is measurable then f^- is measurable

(D) If domain of a function f is not measurable, then f will be measurable.

57. Let f be a function defined on a measurable set E, f is measurable and G is an open set in R. Then $f^{-1}(G)$ is a/an :

(A) measurable set

(B) empty set

(C) closed set

(D) None of the above

58. Which statement is not true ?

(A) Every Riemann integrable function is Lebesgue integrable

(B) Every Lebesgue integrable function is Riemann integrable.

(C) Continuous function defined on a measurable set is measurable.

(D) If f is a measurable function defined on the set E and $E_1 \subset E$, a measurable set, then f is a measurable function on E_1.

59. A bounded function f defined on a measurable set E of finite measure is integrable when :

(A) f is measurable

(B) f is not measurable

(C) f is unbounded

(D) None of the above

60. Let f be defined on a measurable set E with $m(E)$ finite and

$$\inf_{f \le \Psi} \int_E \psi(x)dx = \sup_{f \ge \phi} \int_E \phi(x)dx$$

for all simple functions ϕ and ψ. Then f will be :

(A) zero

(B) constant

(C) measurable

(D) None of the above

61. Let ψ be a simple function and takes the values $\alpha_1, ..., \alpha_n$ such that $\psi = \sum_{i=1}^{n} \alpha_i \chi_{A_i}$ where $A_i = \{x : \psi(n) = \alpha_i\}$.

Then $\int \psi$ is defined as :

(A) $\sum_{i=1}^{n} \alpha_i A_i$

(B) $\sum_{i=1}^{n} \alpha_i m(A_i)$

(C) $\alpha_1 A_1$

(D) $\alpha_1 A_1 + \alpha_2 A_2$

62. A simple function :

(A) is always zero

(B) is a constant function

(C) assumes only a finite number of values

(D) assumes infinite number of values

63. If f is a measurable function defined on a measurable set E, then for every $\in > 0$, there exists a closed set $F \subset E$ such that $m(E - F) < \in$ and f is continuous on E. This theorem is known as :

(A) Monotone convergence theorem

(B) Lusin theorem

(C) Riesz Fischer theorem

(D) Dominated convergence theorem

64. Which statement is true ?

(A) Constant function with a measurable domain is measurable

(B) Constant functions are not measurable

(C) Measurable functions are constant

(D) None of the above

65. If A and B are disjoint measurable sets, then :

(A) $m(A \cup B) = m(A)$

(B) $m(A \cup B) = m(A \cap B)$

(C) $m(A \cup B) = m(A) + m(B)$

(D) $m(A) = m(B)$

66. Lebesgue measure of the set

A = $\{x \in R : 0 < x < 1$ and x has a decimal expansion not containing the digit 3} is :

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) 0

67. If $\{E_i\}$ is an infinite increasing sequence of sets then which statement is true ?

(A) $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m^*(E_i)$

(B) $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{i \to \infty} m^*(E_i)$

(C) $m^*\left(\bigcap_{i=1}^{\infty} E_i\right) = m^*(E_1)$

(D) $m^*(E_1 + E_2) = m^*(E_1)$

68. If E be a measurable set, then we can find a G_δ set G such that :

(A) $E \subset G$ and $m^*(G - E) = 0$

(B) $G \subset E$ and $m^*(G - E) = 1$

(C) $G \subset E$ and $m^*(G - E) < 0$

(D) None of the above

69. If $\{E_n\}$ be an infinite increasing sequence of measurable set, then :

(A) $m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} m(E_n)$

(B) $m\left(\bigcup_{n=1}^{\infty} E_n\right) = m(E_1)$

(C) $m\bigcup_{n=1}^{\infty}(E_n) = m(E_n)$

(D) None of the above

70. Let $\{E_n\}$ be an infinite decreasing sequence of measurable sets and if $m(E_1)$ is finite, then :

(A) $m\left(\bigcap_{n=1}^{\infty} E_n\right) = m(E_1)$

(B) $m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} m(E_n)$

(C) $m\left(\bigcap_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} m(E_n)$

(D) None of the above

71. Which statement is true ?

(A) If E is a measurable set, then $m(\mathrm{E}+y)<m(\mathrm{E})$, y is any real number.

(B) If E is a measurable set, then $m(\mathrm{E}-x)>m(x)$, x is any real number.

(C) If E is a measurable set and x is any real number $m(\mathrm{E}+x)=m(\mathrm{E})$.

(D) None of the above

72. If E_1 and E_2 are measurable sets such that $\mathrm{E}_2 \subset \mathrm{E}_1$ and $m(\mathrm{E}_2)<\infty$, then which is true ?

(A) $m(\mathrm{E}_1-\mathrm{E}_2)+m(\mathrm{E}_2)=m(\mathrm{E}_1)$

(B) $m(\mathrm{E}_1+\mathrm{E}_2)=m(\mathrm{E}_1)+m(\mathrm{E}_2)$

(C) $m(\mathrm{E}_1-\mathrm{E}_2)=m(\mathrm{E}_1)$

(D) $m(\mathrm{E}_1+\mathrm{E}_2)=m(\mathrm{E}_1)$

73. If outer measure is restricted to the set of measurable sets then it is called :

(A) Lebesgue measure

(B) Borel measure

(C) Finite measure

(D) None of the above

74. Which statement is not true ?

(A) If E_1 and E_2 are measurable, then $\mathrm{E}_1 \cap \mathrm{E}_2$ is measurable.

(B) (a,∞) is measurable.

(C) Cantor set is measurable.

(D) Borel set in R is not measurable.

75. If $m^*(\mathrm{A})=0$ and if $\mathrm{B} \subset \mathrm{A}$, then which statement is true ?

(A) $m^*(\mathrm{A} \cup \mathrm{B})=m^*(\mathrm{A})$

(B) $m^*(\mathrm{A} \cap \mathrm{B})=m^*(\mathrm{A}) + m^*(\mathrm{B}) + 1$

(C) $m^*(\mathrm{B})=0$

(D) $m^*(\mathrm{A} \cup \mathrm{B})=m^*(\mathrm{A} \cap \mathrm{B})$

76. Which statement is not true ?

(A) If $m^*(A) = 0$, then A is measurable

(B) $m^*(N) = 0$, N is the set of natural numbers

(C) $m^*(1, 2) = 1$

(D) $m^*(1, 2) = 2$

77. A set E is said to be measurable. Then for any set A, which is true ?

(A) $m^*(A) = m^*(A \cap E)$

(B) $m^*(A) = m^*(A \cap E) + m^*(A \cap E^c)$

(C) $m^*(A) = m^*(A \cap E^c)$

(D) $m^*(E) = m^*(A)$

78. If $m^*(A) = 0$, then which statement is true ?

(A) $m^*(A \cup B) = m^*(B)$

(B) $m^*(A \cup B) = m^*(A \cap B)$

(C) $m^*(A \cup B) = m^*(A)$

(D) $m^*(A \cup B) = m^*(A).m^*(B)$

79. Which statement is not true ?

(A) If A is a countable set then $m^*(A) = 0$

(B) N is the set of natural numbers, $m^*(N) = 0$

(C) Z is the set of integers $m^*(Z) = 0$

(D) Cantor set is a countable set

80. Which statement is not true ?

(A) Outer measure satisfies countable sub-additive property

(B) Outer measure satisfies countable additive property

(C) Outer measure of cantor set is zero

(D) Cantor set is an uncountable set

81. The outer measure of Z (the set of integers) is :

(A) 0

(B) m

(C) ∞

(D) None of the above

82. Which statement is true ?

(A) $m^*[0, 4] = 3$

(B) $m^*(0, 4) = 5$

(C) $m^*(0, 4] = 4$

(D) None of the above

83. Which statement is not true ?

(A) If $A \subset B \subset R$, then $m^*(A) \leq m^*(B)$

(B) $m^*(A + x) = m^*(A)$, where $A \subset R$ and $x \in R$

(C) Outer measure of an interval is its length

(D) Any countable set is not measurable

84. Power set of a set X forms :

(A) a σ-algebra on X

(B) A vector space

(C) A field

(D) None of the above

85. Let A be a set of real numbers and let $\{I_n\}$ be a countable collection of open intervals that cover A. Then outer measure of A will be :

(A) $\inf \sum_n l(I_n)$

(B) $\sup \sum_n l(I_n)$

(C) $\prod_{r=1}^{n} l(I_n)$

(D) None of the above

86. Let (X, **A**, m) be a complete measurable space. Then which is true ?

(A) If $A \in \mathbf{A}$ and $m(A) = 0$ and $B \subset A$ then $B \notin \mathbf{A}$

(B) If $A \in \mathbf{A}$ and $m(A) = 0$ and $B \subset A$ then $B \in \mathbf{A}$

(C) If $A \in \mathbf{A}$ and $A \subset B$ then $B \in \mathbf{A}$

(D) None of the above

87. If (X, **A**, m) be a measurable space such that $m(X) < +\infty$, then m is :

(A) A finite measure

(B) Not a finite measure

(C) Not a countable measure

(D) None of the above

88. If (X, **A**, m) be a measurable space and $\{E_i\}$ be an infinite sequence of disjoint sets of A, then :

(A) $m\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} m(E_i)$

(B) $m\left(\bigcup_{i=1}^{\infty} E_i\right) \ge \sum_{i=1}^{\infty} m(E_i)$

(C) $m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i)$

(D) None of the above

89. If (X, **A**, m) is a measurable space, then :

(A) $m(\phi) = 0$

(B) $m(\phi) = 1$

(C) $m(\phi) = 2$

(D) $m(\phi) = 3$

90. Complement of a G_δ set is :

(A) G_δ set

(B) Open interval

(C) F_σ-set

(D) None of the above

91. Let X be a set and let **A** be a σ-algebra on X. Then (X, **A**) is called :

(A) Measurable space

(B) Measure space

(C) Vector space

(D) Euclidean space

92. $[a, b]$ can be written as :

(A) $(a, b]$

(B) $\bigcap_{n=1}^{\infty}\left(a-\frac{1}{n}, b+\frac{1}{n}\right)$

(C) $\bigcap_{n=1}^{\infty}\left(a+\frac{1}{n}, b-\frac{1}{n}\right)$

(D) $\bigcap_{n=1}^{\infty}\left(a-\frac{1}{n}, b-\frac{1}{n}\right)$

93. The σ-algebra generated by the family of all sets of R is called :

(A) Borel σ-algebra

(B) Finite σ-algebra

(C) Countable σ-algebra

(D) None of the above

94. A set which is a countable intersection of open sets is called :

(A) Closed set

(B) G_δ-set

(C) F_σ set

(D) Closed interval

95. A set which is a countable union of closed sets is called :

(A) Open set

(B) Open interval

(C) F_σ-set

(D) None of the above

96. Let X be an infinite set and let **A** be the collection of all finite subsets of X. Then :

(A) **A** is an algebra on X

(B) **A** is not an algebra on X

(C) **A** is a group

(D) None of the above

97. Let X be an infinite set and let **A** be the collection of all subsets A of X such that either A or A^C is finite. Then :

(A) **A** is a group

(B) **A** is an algebra on X

(C) **A** is an σ-algebra on X

(D) **A** is a field

98. Let X be a set and let **A** be the collection of all subsets of X. Then **A** will form :

(A) a group

(B) a field

(C) a vector space

(D) a σ-algebra on X

99. An algebra **A** of sets is called a σ-algebra if :

(A) **A** is closed under countable union.

(B) **A** is closed with respect to product of sets.

(C) **A** is closed with respect to product of complement of sets.

(D) None of the above

100. If **A**, the collection of subsets of X is an algebra on X then which is not true ?

(A) $X \in A$

(B) If $A \in \mathbf{A}$ then $A^C \in \mathbf{A}$

(C) If $A_1, A_2 ...., A_n \in \mathbf{A}$, then

$$\bigcup_{i=1}^{n} A_i \in \mathbf{A}$$

(D) If $A_1, A_2 \in \mathbf{A}$, then $A_1 \times A_2 \in \mathbf{A}$

(Only for Rough Work)

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 (A) ● (C) (D)

Q. 2 (A) (B) ● (D)

Q. 3 (A) ● (C) (D)

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.
6. All answers are to be given on OMR Answer Sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.
7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.
8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.
9. There will be no negative marking.
10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.
11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.
12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is any discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर–A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 (A) ● (C) (D)

प्रश्न 2 (A) (B) ● (D)

प्रश्न 3 (A) ● (C) (D)

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।
6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।
7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।
8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।
9. निगेटिव मार्किंग नहीं है।
10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।
11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।
12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमत: जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।