Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 11289177

Roll No. 24079000007
Total Mark 61/75.00

Exam M.SC-III_ODD_EXAM_NOV_2025
Subject B060903T - Statistical Inference

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5	9	12/15				
1B	4/5						
1C	4/5						
1D	4/5						
1E	5/5						
1F	4/5						
1G	4/5						
1H	4/5						
1I	4/5						
2	0/15						
3	12/15						
4	0/15						
5	0/15						
6	0/15						
7	0/15						
8A	0/7						
8B	0/7						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-I

Date of Exam: 6/12/25 Shift: 3 Room No.: 9

Paper Code: B060903T Subject: Statistics Year/Sem.: 3

Name of Candidate: Divya Dwivedi

Roll No.: 2 4 0 7 9 0 0 0 0 0 7

Signature of Candidate: Divya

Signature of Invigilator

COE Facsimile

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures — Max. Marks

Total Marks in Words

B0609O3T

Paper Code

Signature of Evaluator

Course: MSC

Session: 2025-26 Year/Semester: 3

Subject: Statistics (statistical Inference)

Paper Code: B 0 6 0 9 0 3 T

Exam Date: 0 6 1 2 2 0 2 5

Name of Candidate: D I V Y A D W I V E D I

Father's Name: S A N T R A M D W I V E D I

महाविद्यालय का कोड College Code: K N O 3

परीक्षा केन्द्र का कोड Exam Centre Code: K N O 3

परीक्षा का प्रकार Type of Exam: Regular ● / Ex. Student ○; Private ○ / Back paper Exam ○

ANSWER BOOKLET NO.: 11289177

B0609O3T

Paper Code

नामांकन संख्या Enrollment Number: C S J M A 2 4 0 0 0 0 0 3 0 2 1

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number: 2 4 0 7 9 0 0 0 0 0 7

पेपर कोड Paper Code: B 0 6 0 9 0 3 T

Signature of Candidate: Divya

Signature of Invigilator

C S Facsimile

COE Facsimile

नोट : 1. परीक्षार्थी को निर्देशित किया जाता है कि कार्यरत पन्ने में मुख्य भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. बॉक्स में नहीं आने वाली प्रविष्टियों वाली उत्तर से शुरू की जाएं। 3. पेंसिल को काले या नीले बॉलपेन से गहरा जाएं।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.

4. Make no Stray marks on this sheet.
5. **DO NOT WRITE OR MARK ON THE BAR CODE.**

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1--24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेंसिल का प्रयोग न करें।
10. B कोपी या अतिरिक्त ग्राफ नहीं दिया जायेगा।

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

INSTRUCTIONS TO THE CANDIDATE

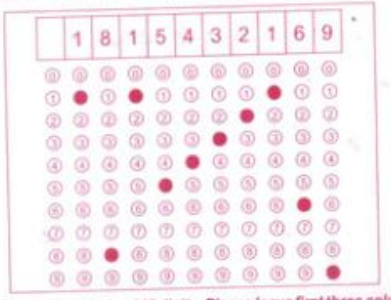
1. Read the instructions carefully given on the Question Paper Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damag in your answer script, if found than change the answer scrip immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g Subject Code. Subject Name and Question of the Questio Paper during first THIRTY MINUTES of the commencement the exam, so that it can be corrected in TIME. After that n corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over pape should fill in status as Carry Over. Those appearing as E Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्दिष्ट स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखें तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, कोपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही संभोगी जैसे साइंटिफिक कैल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numberals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.



Note - If your Roll No. is of 10 digits. Please leave first three colum

Paper Code B 0 6 0 9 0 3 T 01

SHORT ANSWER TYPE QUESTION

1(A)-

Principle of Maximum liklihood Estimation-

Maximum liklihood estimation is used to obtain parameter (θ) from the ~~probabili~~ population or sample data.

Suppose from an example—

You have to diagonised the diseases and you are a doctor.

So there are number of patients as sample data having different symptoms.

The patients considering different diseases (parameter)

to the maximum no of symptoms of disease is obtain as liklihood function.

Definition—

Suppose, sample $(x_1, x_2 - - - - x_n)$ be

a random sample of density function $f(x, \theta)$ then the likihood function is —

$$L\theta = \prod_{i=1}^{n} f(x; \theta)$$

MLEs preferred in large samples because likelihood functions is general. most powerful test which ensures optical, structure and rejecting false H_0 and give fixed level of significance.

(B) The principle of least square is used to fit a curve of form

$$g = (a_0, a_1 - - - - a_k)$$

where a is unknown parameter.
i.e set of n observation, $i = 1, 2 - - n$

form bivariate population consistent in minimising sum of square of residuals

$$e = \Sigma \{ y_i - f(x \cdot a_0 a_1 - - a_k) \}^2$$

subject to variation in
$a_0, a_1, a_2 - - - - - a_k$

The normal equation for estimating $a_0, a_1 \dots a_K$

$$\frac{dE}{da_i} = 0$$

(c)

Minimum chi square =

In minimum chi square statistics the parameter is obtained by minimizing chi square statistics —.

$$\chi^2 = \sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i}$$

Where,

O_i = Observed frequency
E_i = Expected frequency

Modified Minimum chi square —

In modified minimum chi square the parameter is obtained by modifying the minimum chi square statistic.

(D) Properties of maximum likelihood

- Consistency — An estimator is consistent if it gives close and close value to the true parameter as sample size increases —

$$\hat{\theta}_n \to \theta \text{ as } n \to \infty$$

- Unbiasedness — An estimator is said to be unbiased if its expected value is equal to the true parameter value

$$E[T(x)] = \theta$$

- Sufficiency — An estimator is said to be sufficient for a parameter (θ) if its all value is known.

- Efficiency — An efficiency of an estimator is said to be efficient if its variance is minimum.

Asymptotic efficiency →

If comparer two or more estimators and the estimator is the asymptotic efficient. Assymptotically efficient for parameter θ iff —

It is consistent, its asymptotic variance is equal and follows Cramer Rao lower Bound

(E) Consistency — An estimator is consistent if it gives close and close value to the true parameter as sample size increases.

formally;

$\theta_n \longrightarrow \theta$ as $n \longrightarrow \infty$

→ In consistency

- Variance $\longrightarrow 0$

$\Sigma(\theta)^2 = 0$

- Mean square Error = 0

→ There are two type of consistency —

Strong consistency — Converges almost surely

Weak consistency — It do converges probability

(F) Unbiasedness — An estimator is said to be unbiased if its expected

Value is equal to the true parameter.

$$E(T(x)) = 0$$

→ for an unbiasedness,

$$Bias = 0$$

- It gives correct average value of an estimator.
- Most classical theory use shows unbiasedness.
- Unbiasedness shows no systematic error.

(ii) Efficiency — An efficiency of an estimator is said to be efficient if its variance is minimum.

the estimator whose variance is minimum shows efficiency

$$E(T) = \frac{Var(T_2)}{Var(T_1)}$$

Eff ≤ 1

After unbiasedness, the next goal is an efficiency of the estimator.

Relative efficiency,

$$R(e) = \frac{Var(T_2)}{Var(T_1)}$$

$R(E)$ = equality (shows)

$R(E) > 1$ — Better T_1

let
estimate 1 — Median
estimate 2 — Sample Mean

On estimating μ, if the variance of the sample mean is less than the median than the sample mean is more efficient.

(F) Factorization theorem —

In sufficiency, on estimating the parameter (Θ), it knows all the information about Θ.

there is no need to obtain more information after sufficiency.

In sufficiency, the parameter Θ and the density function $f(x;\Theta)$ then, the

$$f(x;\Theta) = g(t(x)\cdot\Theta)\cdot h(x)$$

this is the factorization function.

These are used to compare estimators and helps in obtaining UMVU.

Example

In Bernaulli distribution →

$$p^{\Sigma x_i} (1-p)^{n-\Sigma x_i}$$

$$p^{\Sigma x_i} (1-p)^{n-\Sigma x_i}$$

Replacing $\Sigma x_i = T$ to make it sufficient.

$$\boxed{p^T (1-p)^{n-T}}$$

This shows the use of factorization theorem throug sufficiency to make estimator sufficient.

(Q1)

Exponential family distribution :-

Exponential family distribution on a function is defined as -

$$\boxed{f(x) = h(x)\, c(\theta)\, E[g_i(x)\, w_i(\theta)]}$$

- Exponential family contains all -

→ Normal distribution

Poission distribution, Exponential distribution.

The test UniFormally Most powerful and Uniformally Most powerful Unbiased test is also satisfy with the help of Exponential family distributions.

Properties of Exponential family —.

- It is used for estimating GLM
- It is used for Inference
- It is also used for regression like ANOVA, ACNOVA
- It also provide Optimal structure.

(H) Loss function —.

In statistics decesion Making, when we estimate a parameter there is an error.

Error' occur.

that error produces loss.

loss function is a ... of penalty by choosing the estimator which is not true for the parameter.

loss function L defined for a parameter Θ as a penalty such that

$$L(\Theta, \delta) \geq 0$$

→ In Bayesian estimation, it plays a
- vital role by proving inaccuracy
- It also helps in decesion making
- It is also useful for ~~Solving~~ deceding uncertainty of Θ.

Risk function — Whenever loss occur the risk function is a practice

need of expecting loss when the sample values.

Statistician Provide Risk function to minimize the maximum through the minimax estimator at the worst case scenario.

The risk function

$$R(\theta, s) = E_\theta [L(\theta, s) \cdot x]$$

where,

$$R(\theta, s) \geq 0$$

- It is used as safety tool.
- It is used to compare estimates
- It is used to provide accuracy

(I) Uniformly Most Power Test —

For the best test,
there should be
Probability of Refecting H_0
and,
Fixed level of Significance (α)
Uniformly Most powerful test provide both.
for ~~unif~~ UMP, the size (α) should be same. , there are maximum high power test of size α at one parameter is not enough. Uniformally Most powerful test gives estimates of high power and size α at all the parameter.

Define,

$H_0 : \Theta = \Theta_0$

$H_1 : \Theta = \Theta_1$

For a Uniformally Most powerful test, the size (α) is significant

Maximum power among all the test.

Critical region of uniformally most powerful test is –

$$\beta C(\theta) \geq \beta C'(\theta)$$

- UMP test provide Optica optimal structure.
- If UMP does not occur then UMPU occur.
- It provide maximum power of test with size α at every parameter.
- Uniformally most powerful test is significant for simple hypothesis
- For two sided alternative hypothesis uniformally most powerful Unbiased test is used.

Monotone likelihood compares—
Maximum likelihood under null hypotheses
Maximum likelihood under parameter space Θ

such that

$$\sup_{\theta \in \Theta} L(\theta) \geq \sup_{\theta \in \Theta_0} L(\theta)$$

Section B

(3) LONG ANSWER

(3) for a the desirable properties of a good estimator—

the estimator follows following properties.

Consistency — An estimator is said to be consistent if it gives close and close value to the true parameter as the sample size increases.

formally

$$\hat{\theta}_n \longrightarrow \theta \quad \text{as} \quad n \longrightarrow \infty$$

In consistency →

- Variance → 0
- Mean square Error → 0
- Sample Mean consistent to population Mean similarly,
- Sample Variance consistent to population variance

~~There are two types of consistency~~

- Strong Consistency — Converges ~~probability~~ almost surely
- Weak Consistency — Weak Consistency Converges probability

Efficiency — An efficiency of an estimator is said to be efficient if its variance is

minimum.
The estimator whose Variance is minimum shows efficiency.

$$E(T) = \frac{Var(T_2)}{Var(T_1)}$$ eff < 1

After Unbiasedness, the next goal is efficiency
e
Relative efficiency —.

$$R(E) = \frac{Var(T_2)}{Var(T_1)}$$

R(E) > 1 Better (T_2)

for Example —

let,
estimate 1 — Median
estimate 2 — Sample Mean

On estimating μ, if the Variance of the sample Mean is less than the Median than the sample mean is more efficient.

Normality — The normality of function is used to estimate

by the likelihood function

the Normal distribution

is - $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Asymptotic efficiency -

Asymptotic efficiency is used to compare two or more estimators.

Assymptotic efficient is p efficient for the parameter θ

If

- It is consistent
- It is asymptotic variance is equal
- Cramer Rao lower Bound.

$$\frac{[\psi'(\theta)]^2}{I(\theta)}$$

$$V \geq \frac{\frac{d}{d\theta}[\psi'(\theta)]^2}{\frac{d}{d\theta}[L(\theta)]}$$

Section C

(9)

Uniformaly Most Powerful Test —

for the best test.
there should be Probability of rejecting H_0.
Fixed Significance level (α)

Uniformaly Most powerful test provide maximum power among all the estimators

Only on one parameter maximum power is not Enough.

Uniformaly Most Powerful test provide on all the parameters.

Define —:

$H_0 : \theta_0 = \theta_0$

$H_1 : \theta = \theta_1$

For the Uniformaly Most powerful

→ the size (α) should be significant

→ Maximum power among all the test. Critical region — $\beta^*(\theta) \geq \beta(\theta)$

• **Uniformally Most Powerful Unbiased test.**

for testing hypothesis,

Uniformally Most powerful for unbiased test provide —

Sign~~t~~ fixed significance level (α)

and the high probability of rejecting H_0.

Uniformally most powerful Unbiased test provide the maximum power of size α in Both

simple as well as alternative hypothesis.

$H_0 : \theta = \theta_0$

$H_0 : \theta = \theta_1$

Critical region of Uniformally Most Powerful Unbiased test is -

$$\boxed{E_{\theta_0}[\phi(x)] = \alpha}$$

$$\frac{d}{d\theta} E_{\theta_0}[\phi(x)] = 0$$

- If on estimating UMP does not work, than UMPU used.
- It provide unbiasedness and it helps in estimating Both sided hypotheses.
- Uniformally Most powerful test is the among all the unbiased estimator of maximum power of size α.

LRT Statistic in large sample.

liklihood Ratio Test -

let the random Variable $x_1, x_2 \ldots x_n$

of pdf $f(x, \theta)$

Its statistic is

$$L(\lambda) = \frac{\sup_{\theta \in \Theta_0} L(\theta)}{\sup_{\theta \in \Theta_0} L(\theta)}$$

Likelihood ratio test

Compares—

- Maximum likelihood under null hypothesis
- Maximum likelihood under parameter space = Θ —

Where parameter space is larger so,

$$\sup_{\theta \in \Theta} L(\theta) \geq \sup_{\theta \in \Theta_0} L(\theta)$$

Paper Code

23

X

diagonse diagons
diagonise

$E\,\Theta.\,[\phi(x)]$

$f(x;\Theta) = g(t(x)\cdot\Theta)\,h(x)$

$p^{\Sigma x_i}(1-p)^{n-\Sigma x_i}$

$\Sigma x_i \ln p + (n-\Sigma x_i)\ln(1-p)$

$h(x)\,c(\Theta)\,E[g(t(x))\,\omega(\Theta)]$