Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 11562023

Roll No. 24077000697
Total Mark 55/75.00

Exam M.SC-III_ODD_EXAM_NOV_2025
Subject B010902T - Quantum Mechanics-II

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5						
1B	4/5						
1C	4/5						
1D	4/5						
1E	4/5						
1F	4/5						
1G	4/5						
1H	4/5						
1I	3/5						
2	9/15						
3	0/15						
4	0/15						
5	0/15						
6	0/15						
7	11/15						
8	0/15						
9	0/15						

PART-I

Date of Exam 05/12/25 Shift III Room No. 25

Paper Code B010902T Subject QUANTUM MECHANICS Year/Sem II/III

Name of Candidate FARHEEN RAHMAN

Roll No. 2 4 0 7 7 0 0 0 6 9 7

Signature of Candidate Farheen Rahman

Signature of Invigilator

COE Facsimile

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures	Max. Marks
Total Marks in Words	

B 0 1 0 9 0 2 T

Paper Code

Signature of Evaluator

PART-III

Course M.Sc.

Session 2025-26 Year/Semester II/III

Subject QUANTUM MECHANICS

Paper Code

B	0	1	0	9	0	2	T

Exam Date

0	5	/	1	2	/	2	5

Name of Candidate

F A R H E E N R A H M A N

Father's Name

H A B I B U R R A H M A N

महाविद्यालय का कोड
College Code

K	N	0	4	
Ⓐ	Ⓐ	●	⓪	⓪
Ⓔ	Ⓑ	①	①	①
Ⓕ	Ⓓ	②	②	②
Ⓗ	Ⓙ	③	③	③
●	Ⓚ	④	●	④
Ⓛ	Ⓛ	⑤	⑤	⑤
Ⓝ	Ⓜ	⑥	⑥	⑥
Ⓢ	●	⑦	⑦	⑦
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	Ⓤ	⑨	⑨	⑨
	Ⓦ			

परीक्षा केन्द्र का कोड
Exam Centre Code

K	N	0	4	
Ⓐ	Ⓐ	●	⓪	⓪
Ⓔ	Ⓑ	①	①	①
Ⓕ	Ⓓ	②	②	②
Ⓗ	Ⓙ	③	③	③
●	Ⓚ	④	●	④
Ⓛ	Ⓛ	⑤	⑤	⑤
Ⓝ	Ⓜ	⑥	⑥	⑥
Ⓢ	●	⑦	⑦	⑦
Ⓤ	Ⓣ	⑧	⑧	⑧
	Ⓤ	⑨	⑨	⑨
	Ⓦ			

परीक्षा का प्रकार
Type of Exam

संस्थागत Regular ● भूतपूर्व छात्र Ex Student ○

व्यक्तिगत Private ○ बैक पेपर परीक्षा Back paper Exam ○

ANSWER BOOKLET NO.

11562023

B 0 1 0 9 0 2 T

Paper Code

PART-IV

नामांकन संख्या Enrollment Number C S J M A 2 4 0 0 0 1 3 0 9 5 4

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number

2	4	0	7	7	0	0	0	6	9	7
⓪	⓪	●	⓪	⓪	●	●	●	⓪	⓪	⓪
①	①	①	①	①	①	①	①	①	①	①
●	②	②	②	②	②	②	②	②	②	②
③	③	③	③	③	③	③	③	③	③	③
④	●	④	④	④	④	④	④	④	④	④
⑤	⑤	⑤	⑤	⑤	⑤	⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥	⑥	●	⑥	⑥
⑦	⑦	⑦	●	●	⑦	⑦	⑦	⑦	⑦	●
⑧	⑧	⑧	⑧	⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨	⑨	⑨	●	⑨

पेपर कोड Paper Code

B	0	1	0	9	0	2	T
Ⓐ	●	⓪	●	⓪	●	⓪	Ⓝ
●	①	●	①	①	①	①	Ⓟ
Ⓒ	②	②	②	②	②	●	Ⓡ
Ⓔ	③	③	③	③	③	③	●
Ⓕ	④	④	④	④	④	④	
Ⓖ	⑤	⑤	⑤	⑤	⑤	⑤	
Ⓩ	⑥	⑥	⑥	⑥	⑥	⑥	
Ⓜ	⑦	⑦	⑦	⑦	⑦	⑦	
Ⓦ	⑧	⑧	⑧	⑧	⑧	⑧	
	⑨	⑨	⑨	●	⑨	⑨	

Farheen Rahman

Signature of Candidate

Signature of Invigilator

C S Facsimile

COE Facsimile

नोट : 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पत्र के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. बॉक्स में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.

Correct method
① ● ③ ④

Incorrect method
✓ ✗ ◉ ●

4. Make no Stray marks on this sheet.
5. **DO NOT WRITE OR MARK ON THE BAR CODE.**

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखे तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लाये, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लैस साइंटिफिक कैल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रूपये न रखे न ही उत्तर पुस्तिका में छिपायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1–24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्त पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नही की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पैंसिल का प्रयोग न करें।
10. B कॉपी या अतिरिक्त ग्राफ नहीं दिया जायेगा।

INSTRUCTIONS TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damag in your answer script, if found than change the answer scrip immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g Subject Code, Subject Name and Question of the Questior Paper during first THIRTY MINUTES of the commencement o the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over paper should fill in status as Carry Over. Those appearing as Ex Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numberals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9
⓪	⓪	⓪	⓪	⓪	⓪	⓪	⓪	⓪	⓪	⓪
①	●	①	●	①	①	①	①	●	①	①
②	②	②	②	②	②	②	●	②	②	②
③	③	③	③	③	③	●	③	③	③	③
④	④	④	④	④	●	④	④	④	④	④
⑤	⑤	⑤	⑤	●	⑤	⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥	⑥	⑥	⑥	●	⑥
⑦	⑦	⑦	⑦	⑦	⑦	⑦	⑦	⑦	⑦	⑦
⑧	⑧	●	⑧	⑧	⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨	⑨	⑨	⑨	⑨	●

Note - If your Roll No. is of 10 digits. Please leave first three columns

Section - A

Short Answer Type Question

Answer no. 1 (A)

given.

an harmonic oscillator of mass = m

angular frequency = ω

potential $V(x) = \frac{1}{2} m\omega^2 x^2 + bx^4$

ground state wave function $\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(\frac{-m\omega x^2}{2\hbar}\right)$

First order energy correction $E_n' = \langle \psi_n^0 | H' | \psi_n^0 \rangle$ ①

to the groundstate $E_0' = \langle \psi_0 | H' | \psi_0 \rangle$

$E_0' = \langle \psi_0 | bx^4 | \psi_0 \rangle$ ②

here,

b is independent of x

so, $E' \neq 0$

we know

$$\boxed{\langle 0 | x^4 | 0 \rangle = 3\left(\frac{e}{2m\omega}\right)^2}$$ ③

use ground state wave function in eq. ②, we get

$$E_0' = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}} \frac{b}{m^2\omega^2} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$ ④

on solving equation ④
we get

$$E_0' = \sqrt{\frac{m\omega}{\pi\hbar}} \; \frac{3e^2b}{4m^2\omega^2} \, e^{-\{\frac{m\omega x^2}{2\hbar}}$$

$$\boxed{E_0^1 = \frac{3}{4}\sqrt{\frac{m\omega}{\pi\hbar}} \; \frac{e^2b}{m^2\omega^2} \, e^{-\frac{m\omega x^2}{\hbar}}}$$

also

$$b\langle 0|x^4|0\rangle = b \cdot 3\left(\frac{e}{2m\omega}\right)^2$$

Answer no. 1 (B)

For H-atom

To show, $E^1 = 0$

Let intendity of electric field E and corresponding energy in the form of ~~H~~ work done.

$$W = Fd \quad -①$$

$$W = qE.z$$

charge in direction of

$$W = -eEz\cos\theta$$

for n=1

$$W = eEz \quad -②$$

Hamiltonian of system is H

$$H = \frac{p^2}{2u} + V(r) \quad -③$$

for the ground stat

$n = 1$
$l = n-1 = 0$
$m = -l \text{ to } +l = 0$

By using the selection rule,

$\Delta m = 0$ and
$\Delta l = \pm 1 \quad -④$

are not followed the energy correction of first order for H-atom

$$E^1 = \langle \psi^\circ | H' | \psi^\circ \rangle$$

$\Delta m \neq 0$ for first order energy correction for ground state

Hence, $\boxed{E^1 = 0}$ proved

There is no first order energy correction to the ground state of Hydrogen atom due to symmetry and parity

Answer no. 1 (C)

Validity condition of WKB approximation

There are some condition that tells the accurate approximation

1) The potential of scattered waves are small compared to the kinetic energy of incident wave.

$$\left|\frac{1}{4\pi}\int e^{ikr'} V(r')dr'\right| << 1$$

2) The magnitude of intensity of scattered wave is small compared to incident wave

$$\left|\frac{2m}{\hbar^2 k}\int r V(r)\sin(kr)dr\right| << 1$$

3) This approximation applied only for low potential energy scattering.

4) Transition probability must occur when $E_f - E_i = \hbar\omega$

$$T_{i\to f} = \frac{2\pi}{\hbar}|V_{fi}|^2 \rho(E)$$

5) It must follow energy conservation

$$E_f = E_i + \hbar\omega$$

6) Amplitude must small

$$C_f(t) = -\frac{i}{\hbar}\int e^{i\omega t} V(r')dr'$$

Answer no. 1 (D)

given,

H-atom with atomic number z

$$Ze \rightarrow (Z-1)e \quad \text{nuclear charge}$$

wave function

$$\psi_0 = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \exp\left(\frac{-Zr}{a_0}\right) \quad -①$$

so,

probability for new Hamiltonian after change nuclear charge

$$P(r,t) = \int \psi^* H \psi \, d\tau \quad -②$$

$$= \int \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{(-Zr/a_0)} H \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{(-Zr/a_0)}$$

after nuclear charge changes

probability will be

$$P(r,t) = \int \frac{1}{\sqrt{\pi}}\left(\frac{Z-1}{a_0}\right)^{3/2} . \, e^{\left(-\frac{Zr}{a}+e\right)} H \frac{1}{\sqrt{\pi}}\left(\frac{Z-1}{a_0}\right)^{3/2} e^{\left(-\frac{Zr}{a}+e\right)}$$

$$P(r,t) = \frac{1}{\sqrt{\pi}} \int \frac{(Z-1)^{3/2}}{a_0} e^{\left(\frac{-Zr}{a}\right)} . e^{1} d\tau$$

when nuclear charges are change then probability also changes from z to (Z-1)e

Answer no.1(E)

Einstein Coefficients — The Albert Einstein gives the some absorption and emission coefficient to describe the transition between two energy levels or state.

Absorption coefficient (B_{12})

when atom in ground energy state after absorbing radiation it jumps to higher energy state with emitting photon of frequency ν.

Stimulated emission (B_{21})

Atom is placed in higher energy state and fall down during absorption of radiation and emit a photon of energy $h\nu$.

Spontaneous emission (A_{21})

Atom in higher energy state falls down to lower energy state with taking the incident energy and emits a photon of energy same $h\nu$.

Relation between coefficients

There are two relation associates with

absorption coefficient and stimulated emission of radiation

$$\boxed{B_{12} = B_{21}}$$

and

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}}$$

absorption coefficient and stimulated emission are relates above expression for the radiation

where,

ν = frequency of incident Photon
c = Speed of light
B_{21} = stimulated emission coefficient
h = plank's constant

Answer no, 1(F)

Optical Theorem → The optical theorem relates the scattering amplitude's imaginary part and total cross-section.

Proof – scattering amplitude for partial wave analysis approximation is given by

$$f(\theta,\phi) = \sum\left(\frac{2l+1}{k}\right) e^{i\delta_l} \sin\delta_l (P_l)\cos\theta \quad —①$$

for $\theta = 0$

scattering amplitude becomes.

$$f(0) = \sum \left(\frac{2l+1}{K}\right) e^{i\delta_l} \sin\delta_l \, P_l(\cos 0) \quad \text{—②}$$

for low energy scattering $\quad P_l(1) = 1 \qquad \therefore \cos 0^\circ = 1$

expand $e^{i\delta_l}$ then equation ② becomes

$$f(0) = (2l+1)(i\sin\delta_l + \cos\delta_l)\sin\delta_l$$

taking only imaginary part

$$\text{Img.} f(0) = \frac{(2l+1)}{K}\sin^2\delta_l \quad \text{—③}$$

The differential cross-section for partial wave ($l=0$)

$$\sigma_T = \frac{4\pi}{K^2}(2l+1)\sin^2\delta_l \quad \text{—④}$$

substituting equation ④ in equation ③ we get

$$\boxed{\sigma_T = \frac{4}{K}\ ng[f(0)]}$$

This is the expression for optical Theorem which relates imaginary part of scattering and total differential cross-section.

Answer no. 1 (G)

Phase Shift - In Quantom scattering the phase shift describes that how the scattered wave delayed in phase compared to the incident wave to due to the [illegible] potential.

If scattering occur then how the scattered wave is shift due to potential then corresponding wave function ψ is

$$\boxed{\psi(r) \sim \left(\sin kr - \frac{l\pi}{2} + \delta_l\right)} \quad -①$$

where,

l = angular quantom number
ψ = wave function
δ_l = Phase shift by scattered wave

Nature of phase shift - This is based upon the given potential in scattering

for low scattering $l=0$
so,

$$\boxed{\frac{\delta_l}{k} = \frac{\tan(K_0a) - K_0a}{K^2}} \quad -②$$

when repulsive potential applied then nature of phase shift changes

from equation ②

due to potential

differential cross-section is expressed as.

$$\boxed{\sigma_T = \frac{4\pi}{K}\left[\frac{\tan(K_0a) - K_0a}{K}\right]}$$

This is the expression for cross-section due to phase shift of attractive and repulsive potential

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Answer no. 1 (H)

Klein- Gordon Equation

For non-relativistically case.

The Klein- Gordon equation is given by

$$\left(\Box^2 - \frac{m^2c^2}{\hbar^2}\right)\psi = 0 \quad -①$$

where,

$\Box^2$ = D·Alembert operator

Schrondinger equation for this case

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = - \;\nabla\cdot\alpha\psi + \beta\cdot mc^2\psi$$

$-②$

For non-relativistic case

position probability densi

$p(\vec{r},t) = |$ [illegible] $\quad -③$

Probability current density

$$J(\vec{r},t) = \frac{i\hbar}{2m}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) \quad -④$$

on solving the schrodinger relativistic equation which is equation ②

we get, after compairing with continuity equation

$$\frac{\partial}{\partial t}P(r,t) + \nabla\cdot\vec{J}(r,t) = 0 \quad -⑤$$

we get

$$P(r,t) = \frac{i\hbar}{2mc^2}\frac{\partial}{\partial t}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial}{\partial t}\psi\right) \quad -⑥$$

$$J(r,t) = \frac{\text{[illegible]}}{2m}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) \quad -⑦$$

again we get

$$P(r,t) \neq |\psi|^2$$
$$J(r,t) = J(r,t)$$

now take

$$P(r,t) = \frac{i\hbar}{2mc^2}\frac{\partial}{\partial t}\left(\psi\frac{\partial}{\partial t}\psi^* - \psi^*\frac{\partial}{\partial t}\psi\right)$$

put $\vec{E} = i\hbar\frac{\partial}{\partial t}$

$$P = \frac{E}{2mc^2}\left(2\psi^*\psi\right) \qquad \therefore \psi\psi^* = 1$$

$$\frac{E}{mc^2}$$

means

E is positive then probability is +ve value

if E is -ve then [illegible] is -vely value of [illegible]

This is the big drawback of Klein-Gorden equation

Answer no. 1 (I)

given

$$\sigma' = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \quad -①$$

Dirac matrices are

$$\alpha_x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad -②$$

and

pauli matrix is

$$\sigma_x = \begin{bmatrix} \text{[illegible]} & 1 \\ 1 & 0 \end{bmatrix}$$

$$\alpha_y = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ \text{[illegible]} & i & 0 & 0 \\ \text{[illegible]} & 0 & 0 & 0 \end{bmatrix} \quad -③$$

To find

$$[A]\,\sigma x^2\,[B].\,[\sigma x, \alpha x]$$

$$A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 B\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & & 1 & 0 \\ 0 & & 0 & 0 \\ & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad ④$$

pauli matrix is order of 2X2
which is soluable
but Dirac matrix is order of 4X4

on solving equation ④,
we get

$$[A]\begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}[B]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 0$$

we get the zero.

Section - C

Long Answer Type Question

Answer no. 7

part (a) →

For scattering amplitude use Schrondinger time independent equation

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi \qquad -①$$

For incident wave (r → -∞)

potential is zero V(r) = 0

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi$$

$$\nabla^2\psi + K^2\psi = E\psi \qquad -②$$

has a solution

$$\psi_{inc} = e^{ikz}$$

For scattering region (r = 0) Schrondinger equation becomes

$$\nabla^2\psi + K^2\psi = 0 \qquad ③$$

For a particle at a large distance (r → +∞)

$$\nabla^2\psi + K^2\psi = V\psi \qquad -④$$

Scattered wave is spherical so it depends on θ and ϕ

total wave function is

$$\psi = \psi_{inc.} + \psi_{scattered}$$

$$\psi = e^{ikz} + \frac{e^{ikr}}{r} f(\theta,\phi) \quad ⑤$$

Apply perturbation approaches.

$$\psi(r) = e^{ik[illegible]} + V(r) \quad ⑥$$

eq. ⑥ put in ④ we get

$$\nabla^2 e^{ikz} + \nabla^2 V(r) + k^2 e^{ikr} + k^2 V(r) = V.V(r) + V.e^{ikz} \quad ⑦$$

potential is small so $V(r) \to 0$

inhomogenous solution of eq. ⑦ is

$$V(r) = \int G_k(r, r') F(r') dr'$$

use Green function

$$\therefore G_k(r, r') = -\frac{1}{4\pi} \int \frac{e^{ik|r-r'|}}{r-r'}$$

then equation ⑦ [illegible]t as

$$V(r) = -\frac{1}{4\pi} \int \frac{e^{ik|r'-r'|}}{[illegible]} V(r') dr' \quad ⑧$$

$\therefore |r - r'| = r$

On comparing eq. ⑤ and ⑧ we get

$$\psi(r) = e^{ikz} + \frac{e^{ikr}}{r} f(\theta,\phi)$$

$V(r) = -\frac{1}{4\pi}\int \frac{e^{ik}}{}$ [illegible] $\psi(r')dr'$

we get

$$f(\theta,\phi) = -\frac{1}{4\pi}\int e^{ik(z'-\omega r')} V(r')dr'$$

or $\qquad z' - \omega r' = k_0 - k = K$

$\boxed{f(\theta,\phi) = -\frac{1}{4\pi}\int}$ [illegible] $\boxed{{}^{r'} V(r')dr'}$ —⑨

This is the scattering amplitude for Born approximation

eq. ⑨ also wri[illegible] as.

$$\boxed{f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int e^{ikr'} V(r)\, d^3r}$$ —⑩

Part (b) →

$f(\theta,\phi) = ?$

given, $V(r) = -\left(\frac{Ze^2}{r}\right)\exp(-\alpha r)$

by using fourier transform

$$\int e^{ikr} e^{-\alpha r} = \frac{4\pi}{k^2+\alpha^2} \quad -⑪$$

putting this value in equation ⑩

$$= \int e^{ikr} V(r) d^3r$$

$$= \int e^{ikr} \left(-\frac{Ze^2}{r}\right) e^{-\alpha r} d^3r$$

by using eq.n

$$= -Ze^2 \int e^{ikr} e^{-\alpha r} d^3r$$

$$= -Ze^2 \cdot \frac{4\pi}{k^2+\alpha^2}$$

then $f(\theta,\phi)$ becomes

$$f(\theta,\phi) = \frac{-m}{2\pi\hbar^2}\left[-Ze^2 \cdot \frac{4\pi}{k^2+\alpha^2}\right]$$

$$\boxed{f(\theta,\phi) = \left[\frac{-\text{[illegible]}\,m}{\hbar^2} \cdot \frac{1}{k^2+\alpha^2}\right]} \quad -⑬$$

ans.

This is the required result for given potential V(r) which is Coulomb potential.

part (c) →

Rutherford's Formula - It is used for alpha scattering for coulomb potential.

which is defined as-

$$\frac{d\sigma}{d\Omega} = \left| \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{4E} \cdot \frac{1}{\sin^2(\theta/2)} \right|^2 \quad -⑭$$

where,

Z_1 and Z_2 = charges
E is electric field
e is charge of particle
θ is angle between scattered and incident beam
ε_0 is permittivity

Equation ⑭ is the expression for Rutherford scattering.

Section - B

Long Answer Type Question

Answer no. 2.

Variational Principle — Variation principle is used for system of schrondinger equation which is not solved by approximation method.

It states that " For a given trial wave function the expectation value of the energy state is always greater than the energy of ground state level." $\langle E \rangle \geq E_0$

This method is used to calculate ground state energy of the system

$$\langle E \rangle = \langle \Psi | H' \Psi \rangle$$

$$\langle E \rangle = \frac{\int \Psi^* H' \Psi d\tau}{\langle \Psi | \Psi \rangle} \geq E_0 \quad -①$$

Steps involving variation method

1) Take the trial wave function Ψ

2) minize the Hamiltonian with respect to variable parameter say λ

$$\frac{d\langle H\rangle}{dA} = 0$$

3) Minimise the Hamiltonian
4) Get the expectation energy is greater than ground state energy

For Ground-state Energy -

particle of upper bound x_0 with mass m and wave funtion ψ

then $E = \langle \psi_n^0 | H | \psi_n^0 \rangle$

$$\langle E\rangle = \int \frac{\psi^* H \psi d\tau}{\langle \psi | \psi \rangle} \quad -②$$

If the wave function does not equal

$$\psi_0 \neq \psi$$

then the

$$\phi = \int a_n \phi_m \quad -③$$

$$\phi = \int a_m a_m^* \psi^* \psi d\tau$$

then for perturbation energy becomes.

$$E = \int \psi^* \psi d\tau \quad -④$$

where, $\psi^* \psi = 1$

from equation ②

$$\langle E \rangle = \sum |a_n|^2 \phi_m E \quad -⑤$$

Subtracting [illegible] on both sides we get

$$\langle E \rangle - E_0 = \Sigma |a_n|^2 E - E_0 \quad ⑥$$

weget the $\langle E \rangle - E_0 \geq 0$

Here an is the positive negligible quantity and for E its value is 1 for the trial wave function so,

we can write it as.

$$\boxed{\langle E \rangle \geq E_0}$$

Hence
It showed that expectation value of energy is always greater than ground state energy.

Eigen values of Hydrogen atom -

Variation is used for upper bound state to the energy so we can write as.

$$\frac{d\langle H\rangle}{d\lambda_1}, \frac{d\langle H\rangle}{d\lambda_2}, \frac{d\langle H\rangle}{d\lambda_3} \; ----$$

for Hydrogen atom upper bound is valid for variation principle method.

Limitation Variational principle-

The variation method used only for upper bound state not the lower bound state of the quantum system.

It is used only for atom, molecules or system where state are bound and system doesnot solve by time independent Schrondinger equation.

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