Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 11552282

Roll No. 24077000697
Total Mark 49/75.00

Exam M.SC-III_ODD_EXAM_NOV_2025
Subject B010901T - Classical Electodynamics and Plasma Physic

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5						
1B	3/5						
1C	2/5						
1D	4/5						
1E	3/5						
1F	4/5						
1G	4/5						
1H	3/5						
1I	4/5						
2	9/15						
3	0/15						
4	0/15						
5	0/15						
6	9/15						
7	0/15						
8	0/15						
9	0/15						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-I

Date of Exam : 03/12/25 Shift : III Room No. : 25

Paper Code B010901T Subject PLASMA PHYSICS CLASSICAL ELECTRODYNAMICS AND Year/Sem II/III

Name of Candidate : FARHEEN RAHMAN

Roll No. 2 4 0 7 7 0 0 0 6 9 7

Signature of Candidate

Signature of Invigilator

COE Facsimile

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures | Max. Marks

Total Marks in Words

Paper Code

Signature of Evaluator

PART-III

Course M.Sc.

Session 2025-26 Year/Semester II/III

Subject CLASSICAL ELECTRODYNAMICS AND PLASMA PHYSICS

Paper Code: B 0 1 0 9 0 1 T

Exam Date: 0 3 / 1 2 / 2 5

Name of Candidate: F A R H E E N R A H M A N

Father's Name: H A B I B U R R A H M A N

College Code: K N 0 4

Exam Centre Code: K N 0 4

Type of Exam

- Regular ✓
- Ex. Student
- Private
- Back paper Exam

ANSWER BOOKLET NO.

11552282

B 0 1 0 9 0 1 T

Paper Code

PART-IV

Enrollment Number: C S J M A 2 4 0 0 0 1 3 0 9 5 4

Candidate's Roll Number: 2 4 0 7 7 0 0 0 6 9 7

Paper Code: B 0 1 0 9 0 1 T

Signature of Candidate

Signature of Invigilator

C.S Facsimile

COE Facsimile

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.

Correct method	Incorrect method
① ● ③ ④	

4. Make no Stray marks on this sheet.
5. **DO NOT WRITE OR MARK ON THE BAR CODE.**

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1–24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले ले।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करे, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पैंसिल का प्रयोग न करें।
10. B कॉपी या अतिरिक्त पन्ना नहीं दिया जायेगा।

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

INSTRUCTIONS TO THE CANDIDATE

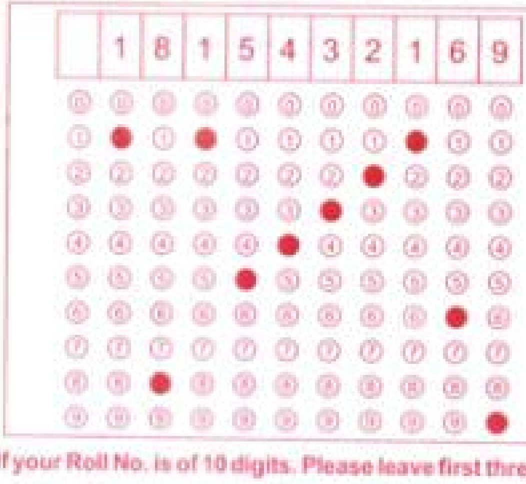
1. Read the instructions carefully given on the Question Paper, Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subject Name and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in status as Carry Over. Those appearing as Ex-Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखे तथा कोई भी चिन्ह न बनावें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लैस साइंटफिक कैल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रूपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numberals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.



Note - If your Roll No. is of 10 digits. Please leave first three columns.

Section - A

Short Answer Type Questions

Answer no. 1 (a)

For part (a)

To show

$$\Lambda^T g \Lambda = g$$

means Lorentz Transformation preserve the spacetime Interval.

spacetime interval in original coordinates

$$(ds)^2 = x^\alpha g_{\alpha\beta} x^\beta \quad -①$$

and spacetime in new coordinates

$$(ds')^2 = x'^\mu g_{\mu\nu} x'^\nu \quad -②$$

Apply Lorentz Transformation

$$x'_\mu = \Lambda_{\mu\alpha} x_\alpha \quad -③$$

also

$$x'_\nu = \Lambda_{\nu\beta} x_\nu \quad -④$$

Substituting eq. ③ and ④ in equation ②

$$(ds')^2 = (\Lambda_{\mu\alpha} x_\alpha) g_{\mu\nu} (\Lambda_{\nu\beta} x_\nu) \quad -⑤$$

Lorentz invariance condition so,

$$(ds')^2 = (ds)^2$$

putting the value of $(ds)^2$

$$x'^{\alpha} g_{\alpha\beta} x'^{\beta} = \Lambda_{\mu\alpha} g_{\mu\nu} \Lambda_{\nu\beta} x'^{\alpha} x'^{\beta}$$

or

$$g_{\alpha\beta} = \Lambda_{\mu\alpha} g_{\mu\nu} \Lambda_{\nu\beta} \quad -⑥$$

or

$$\boxed{\Lambda^T g \Lambda = g} \quad -⑦$$

Hence proved

For part (b)

To show,

$$\Lambda^{-1} = g \Lambda^T g$$

using equation ⑦

$$\Lambda^T g \Lambda = g$$

On multiplying both sides Λ^{-1}, we get

$$\Lambda^T g \Lambda \Lambda^{-1} = g \Lambda^{-1}$$

$\therefore \Lambda \Lambda^{-1} = I_4$

$$\Lambda^T g I = g \Lambda^{-1}$$

On multiplying g^{-1} from left we get.

$$g^{-1} \Lambda^T g = g^{-1} g \Lambda^{-1}$$

$\therefore g g^{-1} = I_4$

$$\boxed{g^{-1} \Lambda^T g = \Lambda^{-1}}$$

Hence proved

Do Not Write anything in this Portion

Answer no. 1 (b)

Types of Lorentz Groups - Lorentz groups classified based on the two ways:

1- $\det(\Lambda) = \pm 1$ and
2- sign of time component

1) **Proper orthochronous Lorentz Group** -

$\det(\Lambda) = +1$ (which is positive)
$\Lambda^0_0 \geq 1$ (preserve the direction of time
i.e, Boost and Rotation 'Λ'

2) **Improper Orthochronous Lorentz Group** -

$\det(\Lambda) = -1$ (which is negative)
$\Lambda^0_0 \geq 1$ (preserve the direction of time)
i.e, parity
$P = \Lambda_P : x \to -x$

3) **Proper Non-Orthochronous Lorentz Group** -

$\det(\Lambda) = +1$ determinant is positive
$\Lambda^0_0 \leq 1$ (Reverse the orientation

i.e, Time reversal
$T = \Lambda_T : t \to -t$

4) **Improper Non-orthochronous Lorentz Group** -

Paper Code B010901T

04

$\det(\Lambda) = -1$ (determinant is negative)
$\Lambda_0^0 \leq 1$ (Reverse the orientation)

Answer 1(c)

Continuity Equation in Covarient Form-

Continuity Equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \quad -①$$

where,
$\vec{J}$ = current density
equation ① relates to ~~non~~ relatovistically
($v \ll c$)
position four vector
$$x_\mu = (\vec{x}, ict) \quad -②$$
current density four vector

$$J_\mu = (\vec{J}, ic\rho) \quad -③$$

J in 4D space
$$J = J_x, J_y, J_z, ic\rho$$
or $$J = J_1, J_2, J_3, J_4$$
where, $J_x = J_1$, $J_y = J_2$, $J_z = J_3$
and $J_4 = ic\rho$

differential operator in 4-D space

$$\delta_\mu = \frac{\partial}{\partial x}, \frac{i}{c}\frac{\partial}{\partial t} \quad -④$$

B010901T 05

$$\delta_\mu \delta_\mu = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \quad -(5)$$

equation (6) can write as

$$\nabla\cdot(J_x + J_y + J_z + ic\rho) + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial}{\partial x_1} J_1 + \frac{\partial}{\partial x_2} J_2 + \frac{\partial}{\partial x_3} J_3 + \frac{\partial}{\partial t}\left(\frac{i}{c}\rho\right) = 0$$

using equation (5)

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial x_\mu} J_\mu = 0 \quad -(7)$$

or

$$\boxed{\delta_\mu J_\mu = 0}$$ Tensor form of continuity equation

Lorentz condition in covarient form

Lorentz condition is

$$\nabla\cdot A + \mu\varepsilon\frac{\partial\phi}{\partial t} = 0$$

or

$$\nabla\cdot A + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0 \quad -(8)$$

where

A = potential four vector

equation (8) relates relativistically

$$A_\mu = A_1 + A_2 + A_3 + A_4$$

$$A_\mu = A_x, A_y, A_z, \frac{i}{c}E$$

$$\frac{\partial}{\partial x_1}A_1 + \frac{\partial}{\partial x_2}A_2 + \frac{\partial}{\partial x_3}A_3 + \frac{\partial(\phi)}{\partial t}.\frac{1}{c^2} = 0$$

using eq. ⑤ and ⑥

$$\sum_{i=1}^{4} \frac{\partial}{\partial x_i} A_i = 0$$

$$\boxed{S_\mu A_\mu = 0}$$

This is covariant form of Lorentz condition.

Answer no. 1 (d)

Lagrangian density → Lagrangian density describes the field equation in field theory when non-massive vector field and source are present with the help of euler-lagrange's equation.

Lagrange density (L)

when Field is present

$$\boxed{L = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu}} \quad -①$$

and corresponding euler's equation is

$$\frac{\partial}{\partial x^\nu}\left(\frac{\partial L}{\partial(\partial_\nu A_\mu)}\right) - \frac{\partial L}{\partial(\partial A_\mu)} = 0$$

or

$$\boxed{\partial_\nu F^{\mu\nu} = 0} \quad -②$$

when source is present

$$\boxed{L = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu} \quad -③$$

then corresponding Euler equation is

$$\frac{\partial}{\partial x^\nu}\left(\frac{\partial L}{\partial(\partial_\nu A_\mu)}\right) - \frac{\partial L}{\partial(\partial_\nu A_\mu)} = 0$$

or

$$\cancel{\partial_\nu A_\mu} \quad \boxed{\partial_\nu F^{\mu\nu} = \mu_0 J^\mu} \quad ④$$

when field and source both are present then Lagrangian density becomes write as

$$\boxed{L = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} + \frac{m_0^2}{2\mu} A^\mu A_\mu - \mu_0 J^\mu} \quad -⑤$$

and the Euler equation for this case is

$$\partial_\nu F^{\mu\nu} + m_0 A^\mu = \mu_0 J^\mu$$

also the field Equation is

$$(\Box^2 + m^2) A^\mu = \mu_0 J^\mu \quad -⑥$$

This is inhomogeneous equation.

$$(\Box^2 + m^2) A^\mu = 0 \quad ⑦$$

This is homogeneous equation for lagrange density

Answer no. 1(e)

Lienard and Wiechert Potential

Relativistically accelerated potential is termed as Lienard-Wiechert potential.

Solution of electromagnetic wave equation in terms of ϕ & A is

$$\Box\phi = -\frac{\rho}{\varepsilon_0} \quad \text{or} \quad \phi = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r', t)d\tau'}{[r]} \quad -①$$

$$\Box A = -\mu_0 J_\mu \quad \text{or} \quad A = \frac{\mu_0}{4\pi}\int \frac{J(r', t)d\tau'}{[r]} \quad -②$$

Let a moving charge moves with acceleration along a line with velocity then

$$r' = r + [illegible]t \quad -③$$
$$t' = [illegible]\frac{r}{c}$$

and

$$[r] = (r - \vec{\beta}\cdot\vec{r}) \quad -④$$

$$\int \rho(r', t)d\tau' = q \quad -⑤$$

$$\int J(r', t)d\tau' = [illegible] \quad -⑥$$

using equation ⑤ and ⑥ in ① and ② respectively we get

$$\phi = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{[r]}$$

$$\boxed{\phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{[r - \vec{\beta}.\vec{r}]}} \quad -⑦$$

and

$$A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qv}{[r - \vec{\beta}.\vec{r}]}$$

or

$$\boxed{A = \frac{1}{4\pi\varepsilon_0 c} \cdot \frac{q\beta}{[r - \vec{\beta}.\vec{r}]}} \quad -⑧$$

In eq. ⑦ & ⑧ A and ϕ are the potentials for an accelerated point charge.

Answer no. 1 (F)

Lienard Formula – This is for relativistically accelerated charge particle which is given by

$$\boxed{P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e^2\gamma^6}{3c}\left[\dot{\vec{\beta}}^2 + |\vec{\beta} \times \dot{\vec{\beta}}|^2\right]} \quad ①$$

This is the required Lienard formula.

For Synchrotron Radiation – When the charge is circularly accelerated then total power radiated per second is given by following way.

here acceleration is transverse

$$\vec{\beta} \times \dot{\vec{\beta}} = \vec{\beta} \cdot \dot{\vec{\beta}} \quad \text{and} \quad \dot{\beta} \neq 0$$

and

$$\gamma \neq 1$$

so, equation ① becomes write as

$$P = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c} [\dot{\beta}^2 + (\beta \cdot \dot{\beta})^2]$$

$$\Rightarrow P = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c} [\dot{\beta}^2 + \beta^2 \dot{\beta}^2 \cos^2\theta]$$

$$\Rightarrow P = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c} [\dot{\beta}^2 + (1+\cos^2\theta)]$$

$$\boxed{P_t = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c} \gamma^4}$$

This is the total power radiated per second.

This also called synchotron radiation.

Answer no. 1 (G)

Magnetic pressure — when plasma placed in external magnetic field then magnetic lines of forces freeze along the direction of motion. Due to this a stress tensor is generated along the length of plasma then a pressure act on the plasma which is isotropic pressure

Due to this plasma confined along the direction of motion in region of column that's pressure is known as magnetic pressure ($\vec{P}$)

$$\boxed{\rho_m \frac{d\vec{u}}{dt} = \vec{J} \times \vec{B} - (\vec{\nabla} \cdot \vec{P})}$$

where,

$\vec{P}$ = magnetic pressure

and final equation is

$$\boxed{\vec{\nabla} \cdot (\overline{\overline{P}}_m - \overline{\overline{T}}_m) = 0}$$

where,

$\overline{\overline{T}}_m$ = stress tensor

Velocity of Alfven wave – Alfven wave generated when the oscillations are transverse oscillation. Its velocity can be expressed as term of magnetic field

$$\boxed{V_A = \frac{B}{\sqrt{\mu_0 \rho}}}$$

where,

ρ = density of particle

B = magnetic field

$\mu_0 = 4\pi \times 10^{-7}$

Do Not Write anything in this Portion

Answer no. 1 (H)

Pinch Effect - Pinch effect defined as " confinement of plasma due to its own generated external magnetic field is term as pinch effect" in the region of column.

For pinch current

Kinetic pressure = Magnetic Pressure

$$P^2 = \frac{B^2}{2\mu_0}$$

$$P > 2nKT$$

So
pinch current will be

$$\boxed{I = 2\times10^{7} A\,nKT}$$

or

$$I \approx 10^6 \text{ ampere}$$

highest current is required for the confinement / binding of plasma in the certain region.

Answer no. 1 (I)

Plasma Frequency- Under the effect of external field, the frequency by which light particles of plasma re-distribute for maintaining the neutrality is called plasma frequency.

Let plasma consist uniform slab of no. of electron with electron density

by gauss law, $\nabla . E = \frac{\rho}{\varepsilon_0}$ —①

$\nabla . E = \frac{qn}{\varepsilon_0}$

for one dimention $\frac{dE}{dx} = \frac{qn}{\varepsilon_0}$ —②

on integrating, we get

$$\int dE = \int \frac{qn}{\varepsilon_0} dx$$

$$E = \frac{qnx}{\varepsilon_0}$$

we know, force

$$F = ma \quad -③$$

$$-qE = ma$$

$$-\frac{q^2 nx}{\varepsilon_0} + m\frac{d^2x}{dt^2} = 0 \quad -④$$

for Harmonic oscillator

Do Not Write anything in this Portion

$$\frac{d^2x}{dt} + \omega^2 x = 0$$

Compair with eq. 4, we get

$$\omega^2 = \left(\frac{ne^2}{m\varepsilon_0}\right)$$

Or

$$\boxed{f = \sqrt{\frac{ne^2}{m\varepsilon_0}}}$$

where,

n_0 = electron density

e = charge of electrom

m = 9.1×10^{-31} kg

ε_0 = permittivity.

Section - B

Long Answer Type Question

Maxwell Equation in Covariant Form

The Maxwell equation are given as.

$$\nabla^2 \phi - \mu\varepsilon \frac{\partial^2\phi}{\partial t^2} - \frac{\partial}{\partial t}\left[\nabla \cdot A + \mu\varepsilon \frac{\partial \phi}{\partial t}\right] = -\rho/\varepsilon_0 \quad -①$$

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 \phi}{\partial t^2} + \nabla\left[\nabla \cdot A + \mu\varepsilon \frac{\partial \phi}{\partial t}\right] = -\mu_0 J \quad -②$$

Apply Lorentz condition

$$\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial \phi}{\partial t} = 0 \quad \text{we get}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi - 0 = -\frac{\rho}{\varepsilon_0} \quad -③$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A + 0 = -\mu_0 J \quad -④$$

D'Alembert operator defined as

$$\delta_\mu \delta_\mu = \Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

Then equations we get as

$$\Box\phi = -\frac{\rho}{\varepsilon_0} \quad -⑥ \qquad \text{and} \qquad \Box A = -\mu_0 J \quad -⑥$$

Do Not Write anything in this Portion

we know

$$\delta_\mu \delta_\mu = \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial t(ct)}$$

and

~~potion~~ potential four vector is

$$A_\mu = (A_1, A_2, A_3, A_4)$$
$$A_\mu = (A_x, A_y, A_z, A_t)$$
$$A_\mu = (A_x, A_y, A_z, \frac{i}{c}\Phi)$$

So maxwell equations are-

$$\Box A_1 = -\mu_0 J_1$$

similarly

$$\Box A_2 = -\mu_0 J_2$$
$$\Box A_3 = -\mu_0 J_3$$

from equation (6),

$$\Box \Phi = -\frac{\rho}{\epsilon_0}$$

$$\Box \frac{i}{c}\Phi = -\frac{ic\rho}{\epsilon_0}$$

$$\therefore c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow \quad \Box A_4 = -\mu_0 J_4$$

Hence,

$$\boxed{\Box A_\mu = -\mu_0 J_\mu}$$

This is the general equation of covariant form of maxwell's equations.

Maxwell Equations are Invariant

If we move from frame S to frame S' then Lorentz transformation also change

$$S \longrightarrow S'$$
$$A_\mu \longrightarrow A'_\mu$$
$$J_\mu \longrightarrow J'_\mu$$

then potential four vector and current density four vector invariant under Lorentz transformation.

so,

$$A_\mu = (A_x, [illegible], A_z, \tfrac{i}{c}\phi) \quad -①$$

and

$$J_\mu = ([illegible] J_y, J_z, ic\rho) \quad -②$$

$$J'_\mu = \Lambda_{\mu\nu} J_\nu \quad -③$$

and $A'_\mu = \Lambda_{\mu\nu} A_\nu$ ④ as the Lorentz transformation

$$x'_\mu = \Lambda_{\mu\nu} x_\nu$$

where,

$$\Lambda_{\mu\nu} = \begin{bmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$\Lambda_{\mu\nu}$ operate on A'_μ, we get

$$\left.\begin{aligned} A'_1 &= \gamma A_1 + 0\cdot A_2 + 0\cdot A_3 + i\beta\gamma A_4 \\ A'_2 &= 0\cdot A_1 + A_2\cdot 1 + 0\cdot A_3 + 0\cdot A_4 \\ A'_3 &= 0\cdot A_1 + 0\cdot A_2 + 1\cdot A_3 + 0\cdot A_4 \end{aligned}\right\} ⑤$$

$$A_4' = -i\beta\gamma A_1 + 0A_2 + 0A_3 + \gamma A_4$$

$\Box$ operate equation ⑤ we get

$$\left.\begin{aligned} \Box A_1' &= \Box(\gamma A_1 + i\beta\gamma A_4) \\ \Box A_2' &= \Box\gamma(A_2) \\ \Box A_3' &= \Box A_3 \\ \Box A_4' &= \Box(-\gamma i\beta A_1 + \gamma A_4) \end{aligned}\right\} ⑥$$

Similarly

$$\left.\begin{aligned} \Box J_1' &= \Box(\gamma J_1 + i\beta\gamma J_4) \\ \Box J_2' &= \Box J_2 \\ \Box J_3' &= \Box J_3 \\ \Box J_4' &= \Box(-i\beta\gamma J_1 + \gamma J_4) \end{aligned}\right\} ⑦$$

from eq. ⑥

$$\Box A_1' = \gamma[\Box A_1 + i\beta A_4]$$

put $\Box A_4 = -\mu_0 J_4$

$$\Box A_1' = \gamma[-\mu_0 J_4]$$

Similarly

$$\Box A_2' = -\mu_0 J_2'$$

$$\Box A_3' = -\mu_0 J_3'$$

$$\Box A_4' = -\mu_0 J_4'$$

It's proved that $\Box A_\mu = -\mu_0 J_\mu$
also exist
and same for all inertial frame of reference

$$\boxed{\Box A'_\mu = -\mu_0 J'_\mu}$$ hence proved.

potential and current density both are invariant under Lorentz transformation for Maxwell's equation

Section-C

Long Answer Type Question

Power radiated by non-relativistically accelerated charge -

An accelerated charge particle radiates the electromagnetic radiations.
for the non-relativistically charge particle q

$\Rightarrow \quad \beta = \frac{v}{c} \quad \frac{v}{c} << 1$

Only radiation part of electric field and magnetic field exist

$\vec{E_r}$ & $\vec{B_r}$ are exist

$\vec{E_c}$ & $\vec{B_c}$ are not exist

So electric and magnetic fields are-

$$\vec{E_r} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\vec{r} \times (\vec{r} - \vec{\beta}.r) \times \vec{\dot{\beta}}}{c(r - \vec{\beta}.\vec{r})^3} \quad -(1)$$

$$\vec{B_r} = \frac{\mu_0}{4\pi} \cdot \frac{\vec{r} \times \{\vec{r} \times (\vec{r} - \vec{\beta}.r) \times \vec{\dot{\beta}}\}}{c r (r - \vec{\beta}.\vec{r})^3} \quad -(2)$$

applying non-relativistically conditions.

$$E_r = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\vec{r} \times \vec{r} \times \vec{\beta}}{c\, r^3}$$

$\vec{r} - \vec{\beta}\cdot\vec{r} \approx \vec{r}$ and $r - \vec{\beta}\cdot\vec{r} \approx r$

$$E_r = \frac{q}{4\pi\varepsilon_0 c} \cdot \frac{\vec{n} \times \vec{n} \times \vec{v}}{c\, r}$$

$$E_r = \frac{q}{4\pi\varepsilon_0 c} \cdot \frac{(\vec{n}\cdot\vec{v})\vec{n} - (\vec{n}\cdot\vec{n})\vec{v}}{c\, r}$$

$\therefore \vec{n}\cdot\vec{n} = 1$

$$E_r = \frac{q}{4\pi\varepsilon_0 c} \cdot \frac{(\vec{n}\cdot\vec{v})\vec{n} - \vec{v}}{c\, r} \quad -③$$

and
magnetic field $\vec{B}$

$$\vec{H}\ B = \frac{n \times \vec{E}_r}{\mu_0 c} \quad -④ \qquad \therefore \vec{H} = \frac{\vec{B}}{\mu_0}$$

we introduce poynting vector $\vec{P}$ or $\vec{S}$
which define as

$$\boxed{\vec{S} = \vec{P} = \vec{E} \times \vec{H}} \quad -⑤$$

on putting $\vec{E}_r$ and $\vec{H}_r$ we get

$$\vec{P} = \frac{\vec{n} \times \vec{E}_r^2}{\mu_0 c}$$

Substituting value of E_r in poynting vector we get

Do Not Write anything in this Portion

$$\boxed{P = \frac{n}{4\pi\epsilon_0} \cdot \frac{q^2}{4\pi r^2 c^3} \dot{v}^2 \sin^2\theta} \quad -⑥$$

$\Rightarrow \quad P \propto \sin^2\theta$

Angular power distribution - It is the ratio of power into area.

$$\text{Poynting vector} = \frac{\text{Power}}{\text{Area}} \quad -⑦$$

$$\text{Power} = \vec{P} \times A$$

$$\frac{dP'}{d\Omega} = \frac{\vec{P} \times (4\pi r^2 \hat{n})}{4\pi}$$

putting value of $\vec{P}$ from equation⑥

where,

$d\Omega$ = solid angle

$$\frac{dP'}{d\Omega} = \frac{\vec{n}}{4\pi\epsilon_0} \cdot \frac{q^2}{4\pi r^2 c^3} \dot{v}^2 \sin^2\theta \, (r_0^2 \hat{n})$$

-⑧

Total Power radiated - we get total power by ~~and~~ integrating equation⑧

$$\int \frac{dP'}{d\Omega} = \frac{q^2}{4\pi\varepsilon_0} \cdot \frac{\dot{v}^2 \cancel{\Omega}}{4\pi c^3} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$P' = \frac{q^2}{4\pi\varepsilon_0} \frac{\dot{v}^2}{\cancel{4}\cancel{\pi} c^3} \cdot 2\cancel{\pi} \cdot \frac{\cancel{4}}{3}$$

On solving Integration we get total power

$$\boxed{P_T = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q^2\dot{v}^2}{3c^3}} \quad -(9)$$

Expression for angular power distribution is given write as -

$$\boxed{P = \frac{n}{4\pi\varepsilon_0} \cdot \frac{q^2}{4\pi r^2 c^3} \dot{v}^2 \sin^2\theta}$$

where θ = angle between wave propagation and radiation

and total power radiated by accelerated charge is write as.

$$\boxed{P_T = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e^2\dot{v}^2}{3c^3}}$$

$q = ne$ for $n=1$

These are the required results

Do Not Write anything in this Portion