



Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 11923648

Roll No. 23081000411
Total Mark 56/75.00

Exam BSC-V_ODD_EXAM_NOV_2025
Subject B030501T - Group and Ring TheoryAndLinear Algebra

Question wise Mark Summary

Q.No Mark Q.No Mark Q.No Mark Q.No Mark

1A 4/5

1B 4/5

1C 4/5

1D 4/5

1E 4/5

1F 4/5

1G 4/5

1H 4/5

1I 4/5

2 0/15

3 0/15

4 10/15

5 0/15

6 10/15

7 0/15

8 0/15

9 0/15

Chhatrapati Shahu Ji Maharaj University Kanpur, Uttar Pradesh

Date of Exam : 25/11/25 Shift : II Room No. : 31
 Paper Code : B030501T Subject : Maths Year/Sem : Vth
 Name of Candidate : Sneha Shukla
 Roll No. : 23081000411

Signature of Candidate : Sneha
 Signature of Investigator : Bha
 COE Facsimile : [Signature]

PART-II

MARKS OBTAINED										
Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
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(f)										
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(h)										
(i)										
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Total										
Total Marks in Figures							Max. Marks			
Total Marks in Words										

B030501T

Paper Code

Signature of Evaluator

Course : B.Sc. (maths)
 Session : 2025-26 Year Semester : Vth
 Subject : Mathematics
 Paper Code : B030501T
 Exam Date : 25/11/2025
 Name of Candidate : SNEHA SHUKLA
 Father's Name : ATUL KUMAR

कॉलेज कोड College Code : AU03
 परीक्षा केंद्र का कोड Exam Centre Code : AU03

● A ● 0 ● 0	● A ● 0 ● 0
● B ● 1 ● 1 ● 1	● B ● 1 ● 1 ● 1
● C ● 2 ● 2 ● 2	● C ● 2 ● 2 ● 2
● D ● 3 ● 3 ● 3	● D ● 3 ● 3 ● 3
● E ● 4 ● 4 ● 4	● E ● 4 ● 4 ● 4
● F ● 5 ● 5 ● 5	● F ● 5 ● 5 ● 5
● G ● 6 ● 6 ● 6	● G ● 6 ● 6 ● 6
● H ● 7 ● 7 ● 7	● H ● 7 ● 7 ● 7
● I ● 8 ● 8 ● 8	● I ● 8 ● 8 ● 8
● J ● 9 ● 9 ● 9	● J ● 9 ● 9 ● 9

परीक्षा का प्रकार Type of Exam :
 Regular Ex. Student
 Private Back paper Exam

ANSWER BOOKLET NO. : 11923648

Paper Code : B030501T

नामांकन संख्या Enrollment Number : CSJMA23000003854
 परीक्षार्थी अंकगणक संख्या Candidate's Roll Number : 23081000411
 पेपर कोड Paper Code : B030501T

● 0 ● 0 ● 0 ● 0	● 0 ● 0 ● 0 ● 0
● 1 ● 1 ● 1 ● 1	● 1 ● 1 ● 1 ● 1
● 2 ● 2 ● 2 ● 2	● 2 ● 2 ● 2 ● 2
● 3 ● 3 ● 3 ● 3	● 3 ● 3 ● 3 ● 3
● 4 ● 4 ● 4 ● 4	● 4 ● 4 ● 4 ● 4
● 5 ● 5 ● 5 ● 5	● 5 ● 5 ● 5 ● 5
● 6 ● 6 ● 6 ● 6	● 6 ● 6 ● 6 ● 6
● 7 ● 7 ● 7 ● 7	● 7 ● 7 ● 7 ● 7
● 8 ● 8 ● 8 ● 8	● 8 ● 8 ● 8 ● 8
● 9 ● 9 ● 9 ● 9	● 9 ● 9 ● 9 ● 9

Sneha

Signature of Candidate

Bha

Signature of Investigator

CS Facsimile

COE Facsimile

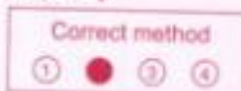
Note : 1. परीक्षार्थी को निर्दिष्ट किया जाता है कि आवरण पन्ने के पृष्ठ भाग पर अंकित सभी निर्देशों को आवरण परीक्षा पूर्व ही पढ़ें।
 2. कोला में भरी जाने वाली प्रतिक्रियाएँ सभी तरफ से शुरू की जाएँ। 3. पोलों को कान्ने या नीले कोलासेन से भरा जाएँ।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.



4. Make no Stray marks on this sheet.
5. DO NOT WRITE OR MARK ON THE BAR CODE.

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखें तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेद करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाइल, डिजिटल डायरी, कोपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती हैं। केंद्र से संबंधित प्रश्नपत्र में ही सेनोरी लेस साइटफिक कैल्कुलेटर ले जाने की अनुमति होगी।
4. उत्तर पुस्तिकाओं में स्यादे न रब्लें न ही उत्तर पुस्तिका में शिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड साक्ष्यानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या फटे हुए हैं, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर उस निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा को कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेसिल का प्रयोग न करें।
10. B कोपी या अतिरिक्त चाक नहीं दिया जायेगा।

INSTRUCTIONS TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subject Name and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over paper should fill in status as Carry Over. Those appearing as Ex. Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9
0	0	0	0	0	0	0	0	0	0	0
1	●	1	●	1	1	1	●	1	1	1
2	2	2	2	2	2	2	●	2	2	2
3	3	3	3	3	3	●	3	3	3	3
4	4	4	4	4	●	4	4	4	4	4
5	5	5	5	●	5	5	5	5	5	5
6	6	6	6	6	6	6	6	●	6	6
7	7	7	7	7	7	7	7	7	7	7
8	8	●	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	●

Note - If your Roll No. is of 10 digits, Please leave first three column



Answer 1 (A)

find all the automorphisms of the multiplicative group $G = \langle 1, -1, i, -i \rangle$ of four fourth roots of unity.

Automorphism -

A function is said to be an automorphism if it is one-one and onto. Then we say a function is an automorphism of the group of four fourth roots of unity - $\{a, a^2, a^3, a^4 = e\}$

(i) $f(x)$ is one-one

(ii) $f(x)$ is onto

$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ a form of Homomorphism.

group $G = \langle 1, -1, i, -i \rangle$ is a multiplicative group G .

$\{a, a^2, a^3, a^4 = e\}$
 $\phi(a) = 4, \phi(a^2) = 2, \phi(a^3) = 4$
 order of two = 4

$x \backslash y$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	1	-1
-i	-i	i	-1	1

$$f(1) = 1 \quad | \quad f(-1, 1) = -1 \quad | \quad f(i, 1) = i$$

$$f(1, 1) = -1 \quad | \quad f(-1, -1) = 1 \quad | \quad f(i, -1) = -i$$

$$f(1, i) = i \quad | \quad f(-1, i) = -i \quad | \quad f(i, i) = -1$$

$$f(1, -i) = -i \quad | \quad f(-1, -i) = i \quad | \quad f(i, -i) = 1$$

this is called multiplicative group $G = \langle 1, -1, i, -i \rangle$ of four fourth roots of unity.

Answer 1 (B)

Show that no group of order 30 is simple.

$$\text{Group of order } 30 = \{0, 2, 4, 6, 8, \dots\} \\ \{0, 2, 3, 5\} \pmod{15}$$

No group of order 30 is simple because the group of order 30 is very other types. Then inside of any group of order 30 is simple.

Group and ring theory satisfies when these conditions are satisfied -

- (i) Closure Property
- (ii) Associative
- (iii) Existence of identity
- (iv) Existence of Inverse
- (v) Commutative law.

That's why group of order 30 is abelian.

So all group of order 30 is abelian that's we can say that

No group of order 30 is simple.

Do Not Write anything in this Portion

2/30
15
5
1



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Answer 1(C)

Show that $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over rational number \mathbb{Q} .

Irreducible -

That function $f(x)$ is irreducible if and only if

$$f(x) = a(x) \cdot b(x)$$

when this is satisfy then $f(x)$ is irreducible over rational number \mathbb{Q} .

$$f(x) = 8x^3 + 6x^2 - 9x + 24 \quad \checkmark$$

$$= 8x^3 + 12x^2 - 2x^2 - 9x + 24$$

$$\text{factor } 8x^3 + 6x^2 - 9x + 24 = 0$$

$$\uparrow 8x^3$$

$$8x^3 + 6x^2 - 9x + 24$$

$$x(8x^2 + 6x - 9) + 24$$

$$8x^3 + 12x^2 - 2x^2 - 9x + 24$$

$$x(8x^2 + 12x - 2x^2 - 9)$$

$$8x^2(x+1) - 2x$$

$$x(8x^2 + 12x - 2x^2 - 9) + 24$$

$$8x^3 + 6x$$

$$12x^6$$

$$x(4x(2x+3) - 3(2x+3) + 24)$$

$$12^2$$

$$x[(4x-3)(2x+3)] + 24$$

$$(x+24)(4x-3)(2x+3)$$

$$f(x) = (4x-3)(2x+3)(2x+3) \quad \checkmark$$

$$\begin{array}{r} 12 \overline{) 2} \\ 2 \overline{) 36} \\ 2 \overline{) 18} \\ 3 \overline{) 9} \\ 3 \overline{) 3} \\ 1 \end{array}$$

$$f(x) = a(x) \cdot b(x)$$

$$3 \times 8$$

$$12 \times 4$$

$$8 \times 24$$

then we said $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over rational number \mathbb{Q} .

$$f(x) = 8x^3 + 6x^2 - 9x + 24 \quad \checkmark \text{ is factor of}$$

$$f(x) = a(x) \cdot b(x) \text{ that's why}$$

$f(x)$ is irreducible over \mathbb{Q}



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Answer 1 (D)

Define factorisation Domains and Euclidean Domains

Factorisation Domains -

An integral domain D is called a factorisation Domain if the following conditions are satisfied -

factorisation Domains is field D over Domain.

factorisation Domains is Integral domain over field D .

$$d(a) \neq d(ab)$$

factorisation Domains is a other diff part of Euclidean Domains.

Euclidean Domains -

An integral domain D is called a Euclidean Domain if the following conditions are satisfied.

we can assign a non-negative integer $d(a)$ such that for $a, b \in R$ with $a \neq 0, b \neq 0$ we have

$$d(a) \leq d(ab)$$

(i) $a, b \in R$ with $b \neq 0$

$$a = bq + r, \text{ where either } r = 0$$

$$d(r) < d(b)$$

Euclidean domain by symbol $(D, +, \cdot, d)$



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Answer 1 (E)

Show that the intersection of two subspaces of a vector space is also a subspace.

P.T. $U_1 \cap U_2 \subseteq V(F)$

Let Vector space $V(F)$ is subspace when intersection of two subspaces is subspaces
let

$$\alpha \in V(F), \alpha \in U_1, \alpha, \beta \in F$$

$$\text{and } \gamma \in V(F), \gamma \in U_2, \alpha, \gamma \in U_1$$

$$\alpha, \gamma \in V(F), \text{ and } \alpha, \gamma \in U_1 \cap U_2$$

$$\text{subspace } U_1 \cap U_2 \subseteq V(F)$$

When

$$\alpha \in U_1 \& \gamma \in U_2 \mid \alpha, \beta \in F \Rightarrow \alpha + \beta \gamma \in U_1$$

$$\alpha, \gamma \in U_1 \cap U_2 \mid \alpha, \gamma \in U_2 \Rightarrow \alpha + \beta \gamma \in U_2$$

Ex -

$$\alpha + \beta \gamma \in U_1, \alpha + \beta \gamma \in U_2$$

then we said that

$$\alpha + \beta \gamma \in U_1 \cap U_2 \subseteq V(F)$$

'Intersection $U_1 \cap U_2$ of a $V(F)$ is also a subspace.

$U_1 \cap U_2$ is a subspace of V .

'Intersection of two subspaces of a vector space is also a subspace.'



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Answer ✓

Show that the set $\{(2, -1, 0), (3, 5, 1), (1, 1, 2)\}$ forms a basis of $V_3(\mathbb{R})$

Basis forms a basis of $V_3(\mathbb{R})$ satisfied when conditions are applied

- (i) S is linearly independent. $|A| \neq 0$
 (ii) set is linearly functional &

$$LCS = V_3(\mathbb{R})$$

linearly transform.

$$f(x) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0 \dots \alpha_n x_n = \{0, 0, 0\}$$

$$2\alpha_1 - \alpha_2 + 0\alpha_3 = 0$$

$$3\alpha_1 + 5\alpha_2 + \alpha_3 = 0$$

$$\alpha_1 + \alpha_2 + 2\alpha_3 = 0$$

$$\begin{vmatrix} 2 & -1 & 0 \\ 3 & 5 & 1 \\ 1 & 1 & 2 \end{vmatrix} \neq 0$$

$$= 2(10-1) + 1(6-1) \neq 0$$

$$18 + 5 \neq 0$$

$$23 \neq 0$$

$$|A| \neq 0$$

function is linearly independent & linearly transform

$$F(x, y, z) = \alpha_1(2, -1, 0) + \alpha_2(3, 5, 1) + \alpha_3(1, 1, 2)$$

set $\{(2, -1, 0), (3, 5, 1), (1, 1, 2)\}$ forms a basis of $V_3(\mathbb{R})$.



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Answer 1 (G1)

State and prove the Bessel's inequality.

Bessel's Inequality -

Bessel's inequality is theorem of Laplace transform. This topic is taken by IVth sem. ✓

Laplace & Fourier transform is the topic of IVth sem.

Basis of $V_3(\mathbb{R})$ -

Basis is said that if set of functions is linearly independent and form of linearly transformation is called Basis -

Natural Basis - $\langle (1, 0, 0), (0, 1, 0), (0, 0, 1) \rangle$

This is the natural Basis of Bessel's inequality

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

$$|\alpha| \neq 0, \quad (\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0)$$

In this transform we can say that Bessel's inequality is not a topic of Vth sem. That's why I wrote Basis of $V_3(\mathbb{R})$ ✓



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Answer 1 (H)

If T is a linear operator on a vector space V such that $T^2 - T + I = \hat{O}$, then show that T is invertible.

$$T^2 - T + I = \hat{O}$$

$$T^2 = T - I$$

$$T^2(\alpha_i) = (T - I)\alpha_i$$

$$T(T(\alpha_i)) = T(\alpha_i) - \alpha_i$$

$$\gamma_i = \beta_i - \alpha_i$$

$$T(\beta_i) = \gamma_i \in V$$

Show that T is one-one

$$T(\alpha_1) = T(\alpha_2)$$

$$\beta_1 = \beta_2$$

$$T(\beta_1) = T(\beta_2) \Rightarrow \gamma_1 = \gamma_2$$

$$\beta_1 - \alpha_1 = \beta_2 - \alpha_2$$

$$\beta_1 - \beta_2 = \alpha_2 - \alpha_1$$

$$0 = \alpha_2 - \alpha_1$$

$$\text{Since } \beta_1 = \beta_2$$

$$\alpha_1 = \alpha_2$$

T is one-one

For each $\beta_i \in V$ there exist $\beta_i - \alpha_i \in V$
 as T is one-one so for $\beta \in V$
 $\gamma \in V$

Thus T being one-one and onto is invertible.



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Answer 1(I)

find the dual basis of the basis set

$$B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\} \text{ for } \mathbb{R}^3$$

$$\alpha_1 = (1, -1, 3)$$

$$\alpha_2 = (0, 1, -1)$$

$$\alpha_3 = (0, 3, -2)$$

$$f_1(a, b, c) = a_1 a + a_2 b + a_3 c$$

$$f_2(a, b, c) = b_1 a + b_2 b + b_3 c$$

$$f_3(a, b, c) = c_1 a + c_2 b + c_3 c$$

by theorem $f_i \alpha_j = \delta_{ij}$, $i \neq j$
 $f_i \alpha_j = 1$, $i = j$

$$f_1 \alpha_1 = 1, \quad f_1 \alpha_2 = 0, \quad f_1 \alpha_3 = 0$$

$$f_2 \alpha_1 = 0, \quad f_2 \alpha_2 = 1, \quad f_2 \alpha_3 = 0$$

$$f_3 \alpha_1 = 0, \quad f_3 \alpha_2 = 0, \quad f_3 \alpha_3 = 1$$

$f_1(a, b, c) \rightarrow$

$$f_1(1, -1, 3) = 1$$

$$f_1(0, 1, -1) = 0$$

$$f_1(0, 3, -2) = 0$$

$$a_1 - a_2 + 3a_3 = 1$$

$$0 + a_2 - a_3 = 0$$

$$0 + 3a_2 - 2a_3 = 0$$

$$3a_2 - 2a_3 = 0$$

$$2(a_2 - a_3) = 0$$

$$3a_2 - 2a_3 = 0$$

$$2a_2 - 2a_3 = 0$$

$$= a_2 = 0, a_3 = 0$$



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$$a_1 = 1$$

$$f(1, 0, 0) = \dots = a$$

$$f_2(a, b, c) -$$

$$f_2 a_1 = a_1$$

$$b_1 - b_2 + 3b_3 = 1$$

$$0 + b_2 - b_3 = 1$$

$$0 + 3b_2 - 2b_3 = 0$$

$$2(b_2 - b_3 = 1)$$

$$3b_2 - 2b_3 = 0$$

$$2b_2 - 2b_3 = 2$$

$$3b_2 - 2b_3 = 0$$

$$= -b_2 = 2$$

$$b_2 = -2$$

$$\downarrow -2 - b_3 = 1$$

$$-2 - 1 = b_3$$

$$b_3 = -3$$

$$b_1 + 2 - 9 = 0$$

$$b_1 = -7$$

$$f_2(a, b, c) = -7, -2, -3$$

$$f_3(a, b, c) -$$

$$c_1 - c_2 + 3c_3 = 0$$

$$0 + c_2 - c_3 = 0$$

$$0 + 3c_2 - c_3 = 0$$

$$= c_2 = c_3$$

$$3c_2 - 2c_2 = 1$$

$$c_2 = 1, c_3 = 1$$

$$c_1 - 1 + 3 = 0$$

$$c_1 = -2$$

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$$f_3(a, b, c) = 2, 1, 1$$

$$f_1(a, b, c) = a$$

$$f_2(a, b, c) = -7a - 2b - 3c$$

$$f_3(a, b, c) = -2a + b + c$$

$B^* = (f_1, f_2, f_3)$ is dual basis of $(\mathbb{N}_3 \mathbb{R})$

$f_1(a, b, c), f_2(a, b, c), f_3(a, b, c)$
dual basis





Section B

- ④ $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$
if W_1 and W_2 are two subspace of
a finite-dimensional vector space V .

W_1 and W_2 are two subspace of
a finite dimensional vector space V
then

$$f(x) \in V(\mathbb{R}),$$

$$x \in W_1 \quad \& \quad y_2 \in W_2$$

$$x \in \dim W_1, \quad y \in \dim W_2$$

When $\dim W_1 \cap W_2$ then $x, y \in V(\mathbb{R})$
and $x, y \in \dim W_1 \cap W_2$

$$V = W_1 + W_2$$

domain addition

$$V \oplus V = W_1 \oplus W_2$$

$$\dim V = \dim(W_1 + W_2)$$

$$\dim W_1 \in V(\mathbb{R}), \quad \dim W_2 \in V(\mathbb{R})$$

$$0 \in W_1 \quad \& \quad 0 \in W_2$$

$$0 \in V(\mathbb{R})$$

$$0 \in W_1 \cap W_2$$

$$W_1 \text{ and } W_2 \subseteq V(\mathbb{R})$$

$$\alpha x + \beta y \in W_1$$

$$\alpha x + \beta y \in W_2$$

$$x, y \in W_1 + W_2$$

$$\alpha(\alpha x + \beta y) \in W_1 + W_2$$



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Vector space V ($V(R)$) intersection of two subspaces.

$$\dim W_1 \cap W_2 \in \mathbb{N}$$

$$\dim W_1 \in \mathbb{N}$$

$$\dim W_2 \in \mathbb{N}$$

by theorem

$$\dim(W_1 + W_2) \in \mathbb{N} \in \mathbb{N}$$

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

because by the theorem of intersection $W_1 \cap W_2$ is also a subspace of finite dimensional vector space V , then $\dim W_1 \cap W_2 \in \mathbb{N}$ we can say that -

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

$$\dim W_1 = 0, \dim W_2 = 0, \dim(W_1 \cap W_2) = 0$$

$$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

Hence prove -

We can say that if W_1 and W_2 are two subspaces of a finite-dimensional vector space V then -

$$\boxed{\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)}$$



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Section - c

Apply Gram-Schmidt process to the vectors $y_1 = (1, 0, 1)$, $y_2 = (1, 0, -1)$ and $y_3 = (0, 3, 4)$ to obtain an orthonormal basis for $V_3(\mathbb{R})$ with the standard inner product.

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

~~$T(1, 0, 1)$~~

Inner product

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Section (C)

Answer - 6

The set $R[x]$ of all polynomials over an arbitrary ring R forms a ring with respect to addition and multiplication of polynomials -

Polynomial ring - the polynomial ring is said that set $R[x]$ is associative, existence of identity, existence of inverse, commutative ring, scalar multiplication, distributive law, extra.

An arbitrary ring R forms a ring with respect to addition and multiplication of polynomial.

An arbitrary ring R forms a ring with respect to multiplication of polynomial.

P.T. $f(x) + g(x)$ is polynomial of addition.

$$f(x) = ax^2 + bx + c$$

$$g(x) = a_2x^2 + b_2x + c_2$$

$$f(x) + g(x) =$$

$$a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2$$

$$f(x) + g(x) = \text{polynomial ring.}$$



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The set $R[x]$ of all polynomials over an arbitrary ring R forms a ring with respect to multiplication of polynomials

$$f(x) \cdot g(x) = \text{polynomial ring } v$$

$$f(x) = a_1 x^2 + b_1 x + c_1$$

$$g(x) = a_2 x^2 + b_2 x + c_2$$

with respect to multiplication of polynomials.

$f(x) \cdot g(x)$ is polynomials over an arbitrary ring R forms a ring with respect to multiplication of polynomial.

Polynomial ring allows the set $R[x]$ of all polynomial -

- (i) Associative
- (ii) Existence of identity
- (iii) Existence of inverse
- (iv) Commutative Ring
- (v) Scalar multiplication.
- (vi) Associative with respect to multiplication.
- (vii) Distributive Law
- (viii) Existence of inverse with respect to multiplication.

Set $R[x]$ of all polynomial over an arbitrary ring R forms a ring with respect to addition
 $f(x)$ and $g(x)$ are two set $R[x]$
 $f(x) + g(x)$ is polynomial

P.T.O.



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The set $R[x]$ of all polynomials over an arbitrary ring R forms a ring with respect to multiplication of polynomials.

Here $f(x)$ and $g(x)$ are two set $R[x]$ is polynomial when $f(x) \cdot g(x)$ is satisfy-

~~$$f(x) = x^2 + ax - a$$~~

$f(x)$ and $g(x)$ addition are totally satisfy in over set of $R[x]$ when we let $f(x)$ is follows all condition of polynomial ring & $g(x)$ is also satisfy polynomial ring.

Remainder theorem-

If the $f(x), g(x)$ and $r(x) \in$ polynomial ring then $(x-a)$ is remainder.

$$f(x) = q(x) + r(x) \cdot (x-a)$$

$$x = a$$

$$f(a) = q(a) + r(a) \cdot 0$$

$$\underline{f(a) = 0}$$



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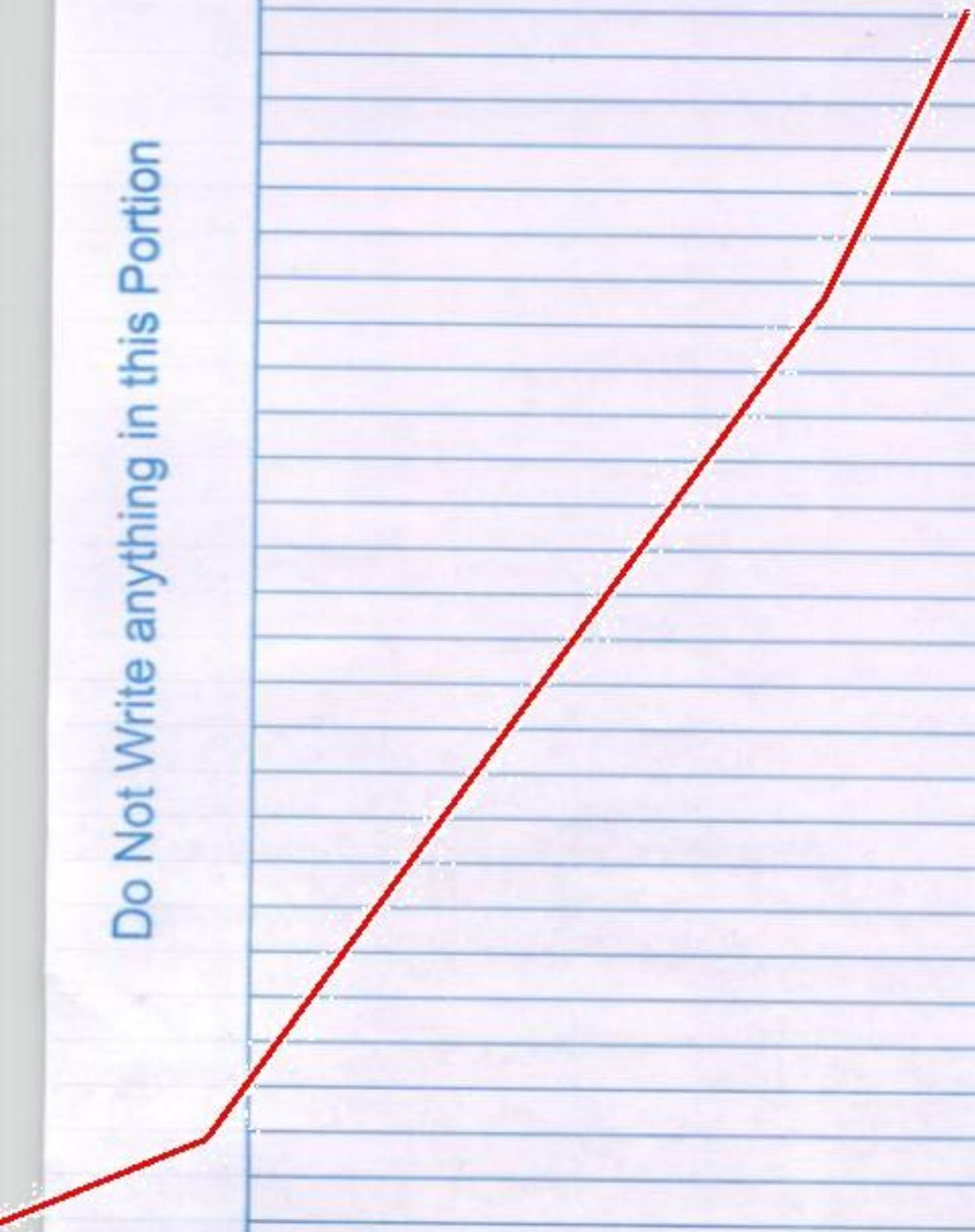
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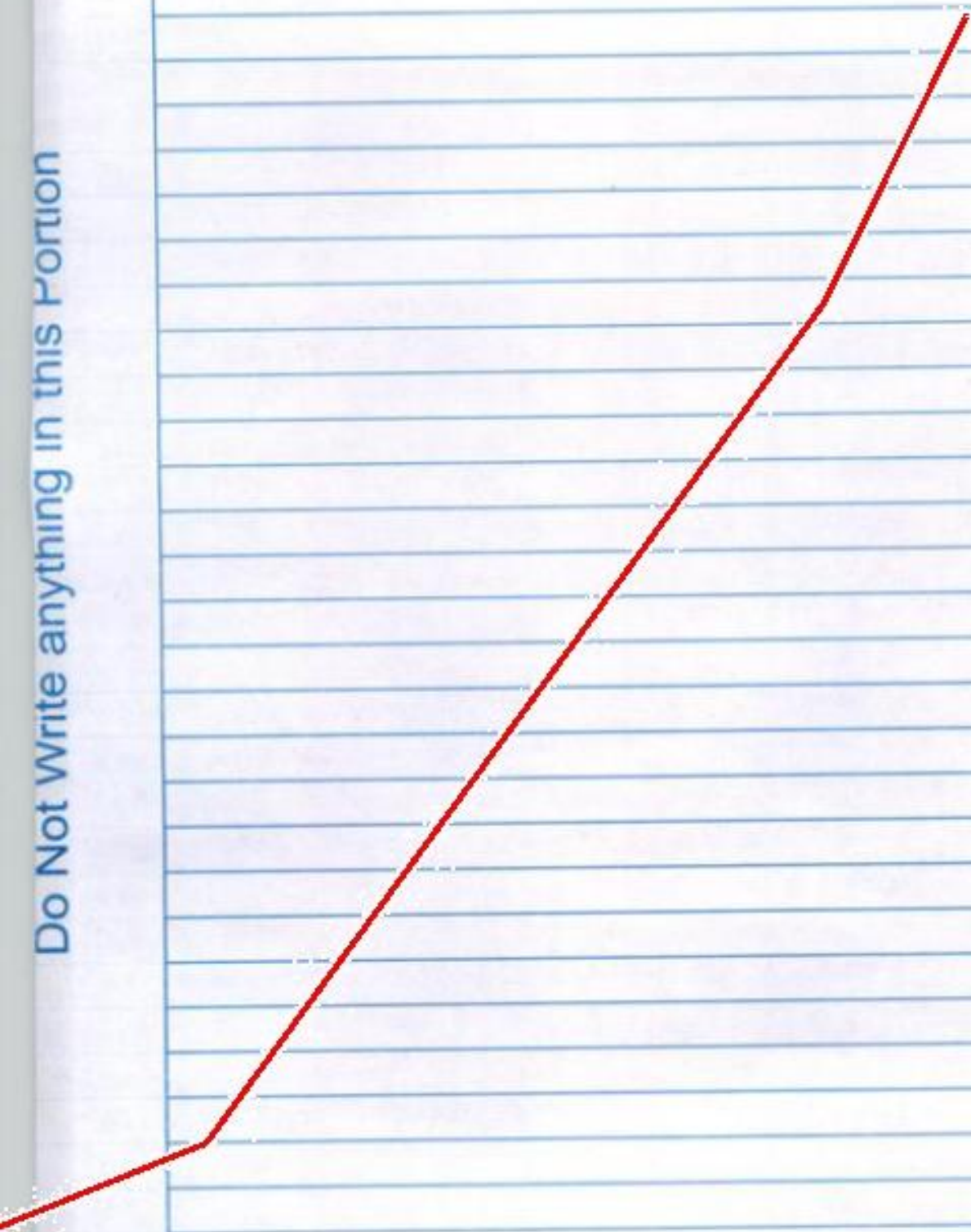




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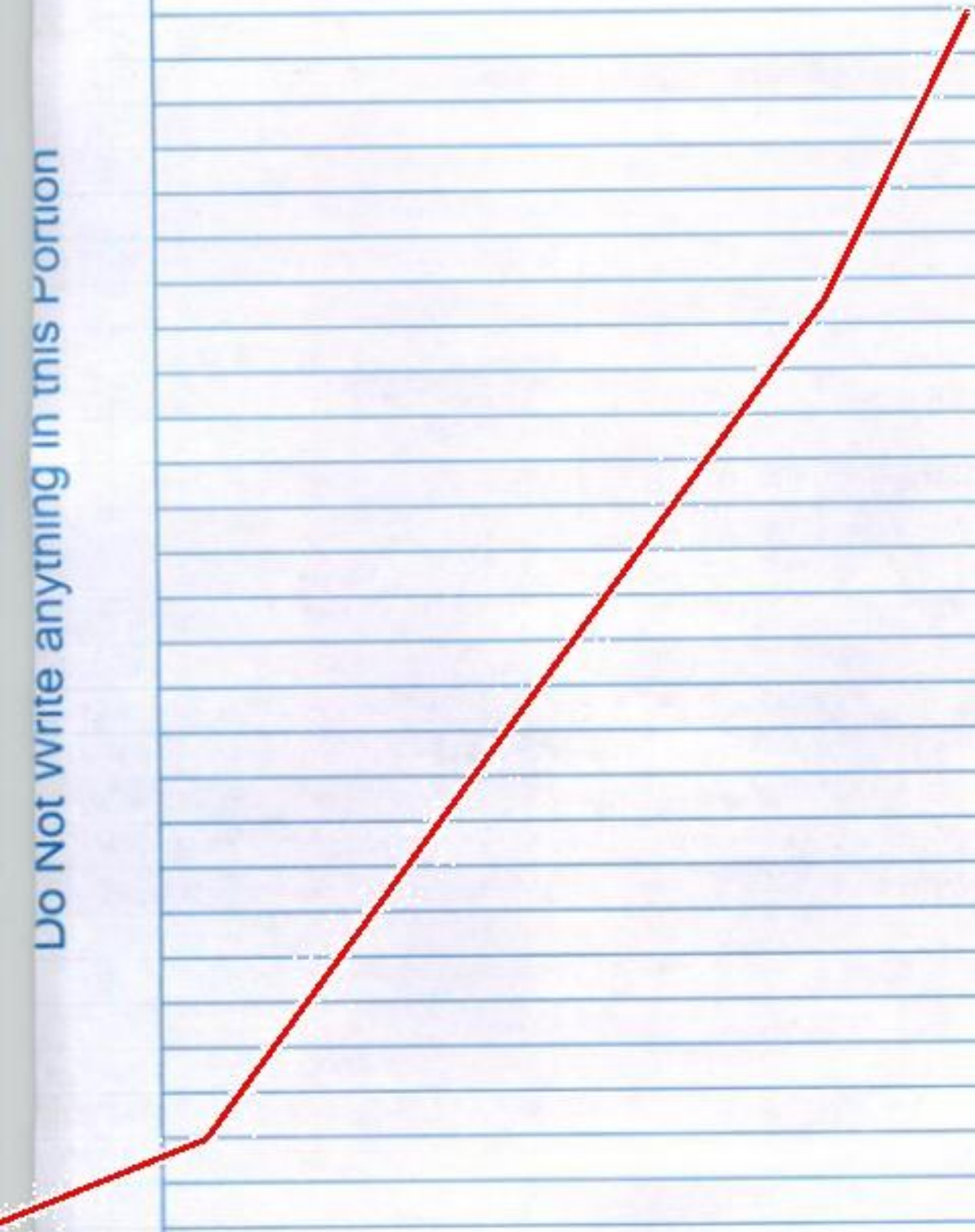


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