



Chhatrapati Shahu Ji Maharaj  
University, Kanpur

**Answer Script Details**  
**Barcode** 11923647

**Roll No.** 23081000409  
**Total Mark** 51/75.00

**Exam** BSC-V\_ODD\_EXAM\_NOV\_2025  
**Subject** B030501T - Group and Ring TheoryAndLinear Algebra

**Question wise Mark Summary**

**Q.No Mark Q.No Mark Q.No Mark Q.No Mark**

1A 2/5

1B 4/5

1C 2/5

1D 4/5

1E 5/5

1F 5/5

1G 0/5

1H 4/5

1I 4/5

2 9/15

3 0/15

4 0/15

5 0/15

6 12/15

7 0/15

8 0/15

9 0/15

# Chhatrapati Shahu Ji Maharaj University Kanpur, Uttar Pradesh

PART-I

Date of Exam: 25-11-2025 Shift: 1st Room No: 81  
 Paper Code: B030501T Subject: math I Year/Sem: 5th Sem  
 Name of Candidate: Shivani Prajapati

Roll No. 23081000409

Signature of Candidate: Shivani  
 Signature of Invigilator: Rajan  
 COE Facsimile: [Signature]

**PART-II**

MARKS OBTAINED										
Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										
Total Marks in Figures						Max. Marks				
Total Marks in Words										

Paper Code

Signature of Evaluator

PART-III

Course: B.Sc  
 Session: 2025-26 Year/Semester: 5th sem  
 Subject: Group and ring theory  
 Paper Code: B030501T  
 Exam Date: 25/11/2025  
 Name of Candidate: SHIVANI PRAJAPATI  
 Father's Name: SANJAY KUMAR

महाविद्यालय का कोड College Code: AU-03  
 परीक्षा केंद्र का कोड Exam Centre Code: AU-03

A	U	-	0	3
●	●	●	●	●
E	B	1	1	1
●	●	●	●	●
F	D	2	2	2
●	●	●	●	●
H	J	3	3	●
●	●	●	●	●
K	K	4	4	4
●	●	●	●	●
L	L	5	5	5
●	●	●	●	●
R	M	6	6	6
●	●	●	●	●
S	N	7	7	7
●	●	●	●	●
U	T	8	8	8
●	●	●	●	●
W	W	9	9	9
●	●	●	●	●

परीक्षा का प्रकार Type of Exam:  Regular  Ex. Student  
 Private  Back paper Exam

ANSWER BOOKLET NO. 11923647  
 Paper Code: B030501T

समावेशन संख्या Enrollment Number

C S J M A 2 3 0 0 0 0 3 8 5 2

परीक्षार्थी अनुक्रमिक संख्या Candidate's Roll Number

23081000409

0	0	●	0	0	●	●	●	0	●	0
1	1	1	1	●	1	1	1	1	1	1
●	2	2	2	2	2	2	2	2	2	2
3	●	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	●	4	4
5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7
8	8	8	●	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	●

B030501T

A	●	0	●	0	●	0	N
●	1	1	1	1	1	●	P
C	2	2	2	2	2	2	R
E	3	●	3	3	3	3	●
F	4	4	4	4	4	4	4
G	5	5	5	●	5	5	5
Z	6	6	6	6	6	6	6
Q	7	7	7	7	7	7	7
M	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

shivani

Signature of Candidate

Rajan

Signature of Invigilator

C S Facsimile

[Signature]

COE Facsimile

नोट : 1. परीक्षार्थी को निर्दिष्ट किया जाता है कि आवेदन पत्रों में पूरा ध्यान पर अधिक कभी निर्देशों को सावधानीपूर्वक पढ़ें।  
 2. बीसों में गरी जाने वाली प्रतिक्रियाएँ सभी तरफ से शुद्ध की जाएँ। 3. गीतों को काले या नीले बॉलपेन से भरा जाएँ।

### INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

### INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in  Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.



4. Make no Stray marks on this sheet.
5. DO NOT WRITE OR MARK ON THE BAR CODE.

### IN ORDER TO AVOID UFM (UNFAIR MEANS):

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

### अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्दिष्ट स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखें तथा कोई भी चिह्न न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाइल, डिजिटल डायरी, कोपी, पुस्तक वह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती हैं। केवल संबंधित प्रश्नपत्र में ही मेमोरी लैस साइटपिक कैल्कुलेटर ले जाने की अनुमति होगी।
4. उत्तर पुस्तिकाओं में रूपये न रखें न ही उत्तर पुस्तिका में विषयार्थ। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

### परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या फटे हुए हैं, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देखें, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेसिल का प्रयोग न करें।
10. B कोपी या अतिरिक्त ग्राफ नहीं दिया जायेगा।

### INSTRUCTIONS TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subject Name and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over paper should fill in status as Carry Over. Those appearing as E Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

### INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in  Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9
0	0	0	0	0	0	0	0	0	0	0
1	●	1	●	1	1	1	1	●	1	1
2	2	2	2	2	2	2	●	2	2	2
3	3	3	3	3	3	●	3	3	3	3
4	4	4	4	4	●	4	4	4	4	4
5	5	5	5	●	5	5	5	5	5	5
6	6	6	6	6	6	6	6	●	6	6
7	7	7	7	7	7	7	7	7	7	7
8	8	●	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	●

Note - If your Roll No. is of 10 digits. Please leave first three columns



## Section-A

### Answer-1.(B).

Let  $G$  be a group of order 30.  
It is written as  $5 \times 3 \times 2$ .

$n_5$  denotes 5-Psylow subgroup of number.

$n_3$  denotes number of 3-Psylow subgroup.

$n_2$  denotes number of 2-Psylow subgroup.

Since the definition of simple group -  
if  $G \neq \{e\}$  is a simple group if its only  
normal subgroup is identity and group  
itself.

By the theorem  $G$  is not simple.

### Answer-1 (C).

$$f(x) = 8x^3 + 6x^2 - 9x + 29.$$

irreducible domain = A domain is called irreducible  
if it has no proper subdomain.

$$f(x) = g(x) \cdot h(x)$$

it can not be written as  $g(x)h(x)$ .

So it is irreducible domain.

Answer-1 (D)Euclidean Domain

An integral domain is said to be a Euclidean domain if following properties are satisfied-

(i)  $d(a) \leq d(ab) \quad \forall a, b \in D.$

(ii)  $\forall a, b \in D$  and  $a \neq b \neq 0$

There exist unique polynomial such that

$$a = bq + r_1$$

if  $r_1 = 0$  then

$$d(a) \leq d(b)$$

Unique factorisation Domain:-

An integral domain is said to be a unique factorization domain if the following properties are satisfied-

(i) Every non-zero non-unity element of  $D$  is written as the product of irreducible elements in  $D$ .

(ii) Factorization is unique up to associates.

Answer-1 (E)

Let  $U_1$  and  $U_2$  be two subspaces of vector space  $V(F)$ .

We show that  $U_1 \cap U_2$  is also a subspace.

Let  $a \in U_1$  and  $b \in U_2$ . Then prove that  $a, b \in U_1 \cap U_2$ .

$$\begin{aligned} \therefore a \in U_1 & \quad a \notin U_2 \\ b \notin U_1 & \quad b \in U_2 \end{aligned}$$

$$\therefore U_1 \subseteq U_1 \cap U_2$$

$$\text{then, } a \in U_1 \cap U_2$$

Similarly,

$$U_2 \subseteq U_1 \cap U_2$$

$$\text{then } b \in U_1 \cap U_2$$

which is contradicts our assumption so we can say that

$$a, b \in U_1 \cap U_2$$

Thus  $U_1 \cap U_2$  is also a subspace.



### Answer-1(F)

Let  $B = (e_1, e_2, e_3)$  be the basis of the given set

$$e_1 = (2, -1, 0)$$

$$e_2 = (3, 5, 1)$$

$$e_3 = (1, 1, 2)$$

Let  $\alpha_1, \alpha_2, \alpha_3$  be three scalars in  $\mathbb{F}$ . then

$$\alpha_1 (2, -1, 0) + \alpha_2 (3, 5, 1) + \alpha_3 (1, 1, 2) = (0, 0, 0)$$

$$(2\alpha_1, -\alpha_1, 0) + (3\alpha_2, 5\alpha_2, \alpha_2) + (\alpha_3, \alpha_3, 2\alpha_3) = (0, 0, 0)$$

$$(2\alpha_1 + 3\alpha_2 + \alpha_3, -\alpha_1 + 5\alpha_2 + \alpha_3, \alpha_2 + 2\alpha_3) = (0, 0, 0)$$

Comparing the equations-

$$2\alpha_1 + 3\alpha_2 + \alpha_3 = 0 \quad \text{--- (1)}$$

$$-\alpha_1 + 5\alpha_2 + \alpha_3 = 0 \quad \text{--- (2)}$$

$$\alpha_2 + 2\alpha_3 = 0 \quad \text{--- (3)}$$

$$\text{equation (1)} + 2 \times \text{equ (2)}$$

$$(2\alpha_1 + 3\alpha_2 + \alpha_3) + (-2\alpha_1 + 10\alpha_2 + 2\alpha_3) = 0$$

$$13\alpha_2 + 3\alpha_3 = 0 \quad \text{--- (4)}$$

$$\text{equation (3)} \times 3 - \text{equ (4)} \times 2$$

$$(3\alpha_2 + 6\alpha_3) - (26\alpha_2 + 6\alpha_3) = 0$$

$$-23\alpha_2 = 0$$



$$\alpha_2 = 0$$

putting  $\alpha_2 = 0$  in eqn (3)

we get  $\alpha_3 = 0$

putting  $\alpha_2 = 0$  and  $\alpha_3 = 0$  in equation (1)

we get

$$2\alpha_1 = 0$$

$$\alpha_1 = 0$$

Hence  $\alpha_1 = \alpha_2 = \alpha_3 = 0$

Hence the set forms a basis of  $V_3(\mathbb{R})$ .

Answer-1 (H).

Given that  $T^2 - T + I = 0$

$$T^2 = (T - I)$$

$$T^2(\alpha_i) = (T - I)(\alpha_i)$$

$$T(T(\alpha_i)) = T(\alpha_i) - I(\alpha_i)$$

$$T(\beta_i) = \beta_i - \alpha_i$$

$$\beta_i = \beta_i + \alpha_i$$

(i)  $T$  is one-one.

$$T(\beta_1) = T(\beta_2)$$

$$\beta_1 - \alpha_1 = \beta_2 - \alpha_2$$

$$\text{since } \beta_1 = \beta_2$$

then

$$\alpha_1 = \alpha_2$$

Hence  $\tau_1 = \tau_2$

$T$  is one-one.



(ii)  $T$  is onto:-

let  $B_i - \alpha_i \in T$  then, we get a value which exist in  $T$ .

Hence  $T$  is onto.

Thus  $T$  is invertible.

Answer-1. (I)

let  $B^* = \{f_1, f_2, f_3\}$  be the dual basis

$$\text{Here } \alpha_1 = (1, -1, 3)$$

$$\alpha_2 = (0, 1, -1)$$

$$\alpha_3 = (0, 3, -2)$$

From Kronecker delta,

$$f_1(\alpha_1) = 1 \quad f_1(\alpha_2) = 0 \quad f_1(\alpha_3) = 0$$

$$f_1(\alpha_1) = a_1 a + a_2 b + a_3 c$$

$$f_2(\alpha_2) = b_1 a + b_2 b + b_3 c$$

$$f_3(\alpha_3) = c_1 a + c_2 b + c_3 c$$

$$f_1(\alpha_1) = 1 = f_1(1, -1, 3) = a_1 - a_2 + 3a_3 = 1 \quad \text{--- (1)}$$

$$f_1(\alpha_2) = 0 = f_2(0, 1, -1) = a_2 - a_3 = 0 \quad \text{--- (2)}$$

$$f_1(\alpha_3) = 0 = f_3(0, 3, -2) = 3a_2 - 2a_3 = 0 \quad \text{--- (3)}$$

$$\text{eqn (2)} \times 2 + \text{eqn (3)}$$

$$2a_2 - 2a_3 - 2a_2 + 2a_3 = 0$$



$$a_2 = 0$$

putting  $a_2 = 0$  then we get

$$a_3 = 0$$

$$a_1 = 1$$

$$\boxed{f_1(a, b, c) = a}$$

$$f_2(a_1) = 0 = f_2(1, -1, 3) = b_1 - b_2 + 3b_3 = 0 \quad \text{--- (4)}$$

$$f_2(a_2) = 1 = f_2(0, 1, -1) = b_2 - b_3 = 1 \quad \text{--- (5)}$$

$$f_2(a_3) = 0 = f_2(0, 3, -2) = 3b_2 - 2b_3 = 0 \quad \text{--- (6)}$$

$$\text{equ (5)} \times 2 - \text{equ (6)}$$

$$2b_2 - 2b_3 - 3b_2 + 2b_3 = 2$$

$$-b_2 = 2$$

$$\boxed{b_2 = -2}$$

$$-2 - b_3 = 1$$

$$-2 - 1 = b_3$$

$$b_3 = -3$$

$$b_1 + 2 - 9 = 0$$

$$\boxed{b_1 = 7}$$

$$\boxed{f_2(a, b, c) = 7a - 2b - 3c}$$

$$f_3(a_1) = 0 = f_3(1, -1, 3) = c_1 - c_2 + 3c_3 = 0 \quad \text{--- (7)}$$

$$f_3(a_2) = 0 = f_3(0, 1, -1) = c_2 - c_3 = 0 \quad \text{--- (8)}$$

$$f_3(a_3) = 1 = f_3(0, 3, -2) = 3c_2 - 2c_3 = 1 \quad \text{--- (9)}$$



$$\text{equ (8)} \times 2 - \text{equ (9)}$$

$$2C_2 - 2C_3 - 3C_2 + 2C_3 = -1$$

$$-C_2 = -1$$

$$C_2 = 1$$

$$C_2 - C_3 = 0$$

$$C_3 = 1$$

$$C_1 - 1 + 3 = 0$$

$$C_1 = -2$$

$$f_3(a, b, c) = -2a + b + c$$

Answer-1. (A).

$$G_1 = \{1, -1, i, -i\}$$

Let the group is generated by  $i$ .

$$o(i)^1 = i$$

$$o(i)^2 = -1$$

$$o(i)^3 = 1$$

$$o(i)^4 = 1$$

$$G_1 = \{i, i^2, i^3, i^4\}$$

Hence the auto. group of the multiplicative group =  $\{i, i^2, i^3, i^4\}$



Answer - 1 (b)

Let  $x$  and  $y$  be two polynomials such that

$$\|x\| \|y\| \geq |x, y|$$

Proof:- if  $\|x\| = 0$  then  $(x, x) = (0, 0)$

$$(x, y) = (0, y) \text{ then}$$

$$\|x, y\| = 0$$

if  $\|x\| \neq 0$   $\frac{1}{\|x\|^2}$  be a positive.

$$z = y - \frac{(y, x)}{\|x\|^2} x$$

$$(z, z) = \left( y - \frac{(y, x)x}{\|x\|^2} \right) \left( y - \frac{(y, x)x}{\|x\|^2} \right)$$

$$(z, z) = (y, y) - \frac{(y, x)(x, x)}{\|x\|^2} - \frac{(y, x)(x, y)}{\|x\|^2}$$

$$+ \frac{(y, x)(y, x)}{\|x\|^2}$$

$$(z, z) = (y, y) - \frac{(y, x)(y, x)}{\|x\|^2}$$

$$\|z\| \geq 0$$

$$\|y\|^2 - \frac{(y, x)^2}{\|x\|^2} \geq \|z\|^2$$

$$\|y\| \|x\| \geq |x, y|$$

Hence proved.



### Section-C

Answer-7.

Principal ideal domain :-

A integral domain for which every ideal is principal is called principal ideal domain.

Proof:- Now we show that euclidean domain is a principal ideal domain.

Let  $I$  be a principal ideal domain.

Case-I  $I = \{0\}$ .

the  $I$  is principal ideal domain.

Case-II  $I \neq \{0\}$

Now suppose  $a \in I$  then  $d(a)$  is minimally along with  $I$ .

$a \in I$  and  $b \in I$  be a arbitrary element. there exist unique polynomial  $q$  and  $r$  such that

$$a = bq + r$$

here  $d(a) \leq d(b)$   
and if  $r = 0$  then

$$d(a) \leq d(b)$$

$$d(b) \leq d(b)$$



Hence proved: that euclidean domain is principal ideal domain.

Answer - 6:

It is given that the set  $K[x]$  of all polynomials over an arbitrary ring  $R$  forms a ring with respect to addition and multiplication of polynomials.

Let  $f(x)$  and  $g(x)$  be two polynomials of  $K[x]$ .

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad a_n \neq 0$$

$$g(x) = b_0 + b_1x + b_2x^2 + \dots + b_mx^m \quad b_m \neq 0.$$

$$f(x) = \sum_{k=0}^n a_k x^k \quad g(x) = \sum_{k=0}^m b_k x^k$$

(i) Associative property :-

Let  $f(x), g(x), h(x) \in F(x)$ .

$$f(x) = \sum_{k=0}^n a_k x^k \quad g(x) = \sum_{k=0}^m b_k x^k$$

$$h(x) = \sum_{k=0}^p c_k x^k$$

$$\{f(x) + g(x)\} + h(x) = \sum_{k=0}^n (a_k + b_k) x^k + \sum_{k=0}^p c_k x^k$$



$$= \sum_{k=0}^{\infty} a_k x^k + \sum_{k=0}^{\infty} b_k x^k + \sum_{k=0}^{\infty} c_k x^k$$

$$= \sum_{k=0}^{\infty} a_k x^k + \sum_{k=0}^{\infty} (b_k + c_k) x^k$$

$$= f(x) + \{g(x) + h(x)\}$$

(ii) Existence of identity :-

let  $0 \in F(x)$

$$0(x) = a_0 + a_1 x + a_2 x^2 + \dots$$

$$f(x) + 0 \cdot x = (a_0 + a_1 x + a_2 x^2 + \dots) + (0 + 0x + 0x^2 + \dots)$$

$$= (a_0 + 0) + (a_1 + 0)x + (a_2 + 0)x^2 + \dots$$

$$= a_0 + a_1 x + a_2 x^2 + \dots$$

$$= f(x)$$

$e(x)$  is the identity element

(iii) Existence of inverse.

let  $-f(x) \in R(x)$

$$f(x) + (-f(x)) = (a_0 + a_1 x + a_2 x^2 + \dots) + (-a_0 + a_1 x - a_2 x^2 + \dots)$$

$$= (a_0 - a_0) + (a_1 - a_1)x + (a_2 - a_2)x^2 + \dots$$

$$= 0 + 0x + 0x^2 + \dots$$

$$= 0$$



-  $f(x)$  is the inverse element.

(iv) Commutative property:-

Let  $f(x), g(x) \in R(x)$ .

$$f(x) + g(x) = \sum_{k=0}^{\infty} a_k x^k + \sum_{k=0}^{\infty} b_k x^k$$

$$= \sum_{k=0}^{\infty} (a_k + b_k) \cdot x^k$$

$$= \sum_{k=0}^{\infty} (b_k + a_k) \cdot x^k$$

$$= \sum_{k=0}^{\infty} b_k \cdot x^k + \sum_{k=0}^{\infty} a_k \cdot x^k$$

$$f(x) + g(x) = g(x) + f(x)$$

Hence commutative property satisfied.

(v) Associative property in multiplication:-

Let  $f(x), g(x), h(x) \in R(x)$

$$\{f(x) \cdot g(x)\} \cdot h(x) = \sum_{k=0}^{\infty} (a_k \cdot b_k) \cdot x^k \cdot \sum_{k=0}^{\infty} c_k x^k$$

$$= \sum_{k=0}^{\infty} a_k x^k \cdot \sum_{k=0}^{\infty} b_k x^k \cdot \sum_{k=0}^{\infty} c_k x^k$$



$$= \sum_{k=0}^{\infty} a_k x^k \sum_{k=0}^{\infty} (b_k c_k) x^k$$

$$= f(x) \{g(x) \cdot h(x)\}$$

(vi) Existence of identity :-

let  $1(x) \in R(x)$

$$1(x) = a_0 + a_1 x + a_2 x^2 + \dots$$

$$f(x) \cdot 1(x) = (a_0 + a_1 x + a_2 x^2 + \dots) \cdot (1 + 0 \cdot x + \dots)$$

$$= (a_0 + a_1 x + a_2 x^2 + \dots)$$

$$= f(x)$$

Hence  $1(x)$  is the identity element.  $\square$



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### Section- B.

#### Answer-2.

Sylow's First theorem:-

Let  $G$  be a finite group and  $p$  is a prime number. if  $p^m \mid o(G)$  and  $p^{m+1}$  is not divides  $o(G)$ , then sub group of every element  $p^m$  order.

Proof:- let  $G$  be a finite group and  $o(G) \neq 1$ .  
then the theorem is obvious.

$K$  is a subgroup of  $G$ .  $K \subseteq G$  then  $p^m \mid o(K)$ .  
 $H$  be a subgroup of  $K$ .  $o(H) = p^m$   
 $K \subseteq G \Rightarrow H \subseteq K \subseteq G$

By class equation -

$$o(G) = o(Z(G)) + \sum_{a \notin Z(G)} \frac{o(G)}{o(N(a))}$$

$$a \notin Z(G) \Rightarrow N(a) \neq G.$$

$$\frac{p^m}{o(G)} = \frac{p^m}{o(G)} \left( \frac{o(G)}{o(N(a))} + \dots \right)$$

$$= \frac{p^m}{o(G)} \left( \frac{o(G)}{o(N(a))} + \dots \right)$$

$$= \sum_{a \notin Z(G)} \frac{o(G)}{o(N(a))}$$



$$= o(H) - \sum_{a \notin Z(H)} \frac{o(H)}{o(N(a))}$$

Hence the order of  $H$  is  $pm$ .

$$o(H) = pm$$

Hence proved.

Sylow's second theorem:-

Let  $G$  be a finite group and  $p^m | o(G)$ .  
Then two  $p$ -Sylow subgroups are conjugate to one-another. ✓

Proof:- Let  $P$  and  $Q$  be two  $p$ -Sylow subgroups.  
 $o(P) = p^m = o(Q)$

$$p^m | o(G)$$

If possible we assume that  $P \neq g^{-1}Qg$ .

$$\frac{o(P \times Q)}{o(P \cap Q \cap g^{-1}Qg)}$$

if  $m \leq n$ .

$$o(P \cap g^{-1}Qg) = p^m \quad m \leq n.$$

if  $m = n$ .

$$o(P \cap g^{-1}Qg) = p^m$$

$$= o(Q)$$

✓



which is contradiction,

hence, two  $p$ -Sylow subgroups are conjugate to one-another. ✓

Sylow's third theorem:-

$G$  be a finite group and  $p \mid |G|$ . then all  $p$ -Sylow subgroups are  $1 + kP$ , where  $k$  is integer.

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