Chhatrapati Shahu Ji Maharaj University, Kanpur

Answer Script Details
Barcode 10597757

Roll No. 24079000007
Total Mark 50/75.00

Exam MASTER OF SCIENCE STATISTICS_ODD EXAM-DEC-2
Subject B060704T - REAL ANALYSIS

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5	6	10/15				
1B	3/5	7	NA/15				
1C	4/5	8	NA/15				
1D	3/5	9A	NA/5				
1E	3/5	9B	NA/5				
1F	4/5	9C	NA/5				
1G	3/5						
1H	3/5						
1I	3/5						
2A	NA/7						
2B	NA/7						
3A	NA/7						
3B	NA/7						
4A	NA/5						
4B	NA/5						
4C	NA/5						
5	10/15						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-I

Date of Exam: 30/1/25 Shift: 1st Room No. 9

Subject: Statistics Year/Sem: 1

Paper Code:

Name of Candidate: Divya Dwivedi

Roll No: 2407900000007

COE Facsimile

Signature of Invigilator

Signature of Candidate: Divya

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figure	Max. Marks
Total Marks in Words	

B060704T

Paper Code

Signature of Evaluator

PART-III

Course: MSC Statistics

Session: 2024-25 Year/Semester: 1

Subject: Statistics

Paper Code: B060704T

Exam Date: 30012025

Name of Candidate: DIVYA DWIVEDI

Father's Name: SANTRAM DWIVEDI

College Code: KN08

Exam Centre Code: KN03

Type of Exam: Regular ● Ex-Student ○ Private ○ Back paper Exam ○

ANSWER BOOKLET NO.

10597757

B060704T

Paper Code

PART-IV

Enrollment Number: CSJMA

Candidate's Roll Number: 2407900000007

Paper Code: B060704T

Divya

Signature of Candidate

Signature of Invigilator

C.S Facsimile

COE Facsimile

नोट : 1. परीक्षार्थी को निर्देशित किया जाता है कि समस्त पानों में पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. बॉक्स में भरी जाने वाली प्रविष्टियाँ काली स्याही से भरें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.

Correct method	Incorrect method
① ● ③ ④	✓ ✗ ● ⊖

4. Make no Stray marks on this sheet.
5. **DO NOT WRITE OR MARK ON THE BAR CODE.**

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1–24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जावेगी।
9. प्रश्नों के उत्तर लिखने के लिये पैंसिल का प्रयोग न करें।
10. B कोपी या अतिरिक्त ग्राफ नहीं दिया जावेगा।

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

INSTRUCTIONS TO THE CANDIDATE

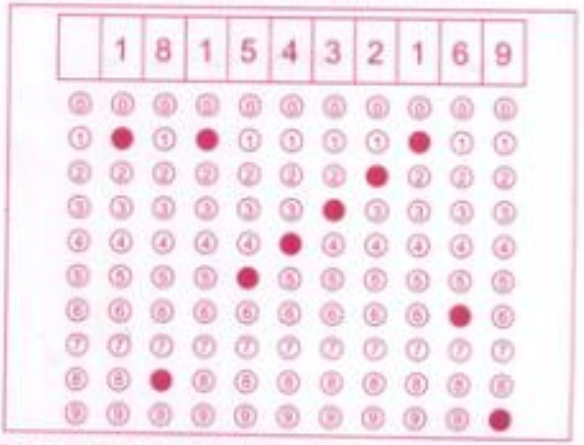
1. Read the instructions carefully given on the Question Paper Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code. Subject Name and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in status as Carry Over. Those appearing as Ex Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखे तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, कोपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लैस साइंटफिक केल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रूपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numberals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.



Note - If your Roll No. is of 10 digits. Please leave first three columns

Paper Code

B	0	6	0	7	0	4	T

01

Section-A

1(a) If a function $f(x)$ is said to be bounded if it converges to its integral. If a function $\int_0^\infty f(x)\,dx$ is a function than we take finite limit and obtain finite value to.

$$\int_0^\infty f(x)\,dx = \int_0^x f(x)\,dx .$$

(b) **Continuity-**

The function $f(x)$ is said to be continuous at a point $x=a$ if $\lim_{x\to\infty} f(x) = f(a)$

$$\boxed{\lim_{x\to\infty} f(x) = f(a)}$$

Left hand limit should be equal to Right hand limit

L.H.L = R.H.L

(C) **Uniform Continuity-** The function $f(x)$ is said to be Uniformly Continuous if there

if $\varepsilon > 0$ and such that $\delta > 0$ at variable x and function $f(x)$

~~$|f(x_1) - f(x_2)| > \varepsilon$~~ $|f(x) - f(y)| < \varepsilon$

then

~~$|x_1 - x_2| > \delta$~~ $|x - y| < \delta \; \forall$

If $f(x)$ is uniform continuous on ... point interval (a,b) then ... always continuous on (a,b)

(d) **Mean Value Theorem** :-

If $f(x)$ is continuous on closed interval $[a,b]$

$f(x)$ is differentiable at open interval (a,b)

where $f(x)$ is function on (a,b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Proof :-

Given that $f(x)$ is continous on closed interval $[a,b]$

then there exist a function

$$F(x) = f(x) + Ax$$

$f(a) = f(b)$ — (i)

$f(a) + Aa$ — (ii)

for b

$f(b) + Ab$ — (iii)

From Eq (ii) and (iii)

$f(a) + Aa = f(b) + Ab$

$Aa - Ab = f(b) - f(a)$

$A(a-b) = f(b) - f(a)$

$-A(b-a) = f(b) - f(a)$

$-A = \frac{f(b)-f(a)}{b-a}$

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

Proved

(e) Taylor's Theorem

if $f(a+h)$ is a function then there exist ascending power of h which is differentiable.

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a)$$

Proof —

$f(a+h) = A_0 + hA_1 + h^2A_2 + h^3A_3 + h^4A_4 \ldots$ — (i)

$f(a+h) = hA_1 + h^2A_2 + h^3$

$f'(a+h) = hA_1 + 2hA_2 + 3h^2A_3 + 4h^3A_4$ —(ii)

$f''(a+h) = hA_2 + 6hA_3 + 12h^2A_4$ —(iii)

$f'''(a+h) = 6hA_3 + 24hA_4$ —(iv)

on taking $h = 0$

$f(a) = A_1$

$f'(a) =$

$f(a+h) = f(a) + h'f(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a)$ = Proved

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(f) **Multiple Integrals—**

Multiple Integrals is used to Integral the volume of a given region or surface.

Multiple Integral is of two types

1. **Double Integral**

$$\iint_R xy\,dx\,dy$$

double Integral is used to the find the Area/ volume of a given surface where R is the region and used to Integrate the different Variable.

Triple Integral -

$$\iiint_V xyz\,dx\,dy\,dz$$

Here V is the volume, and is used to Integrate the Variables.

(g) Leibniz's Rule -

If (x, α) and $\frac{df(x,\alpha)}{d\alpha}$ are Variables and function which is differentiable then in the function $\frac{d}{d\alpha}\int_0^\infty f(x,\alpha)$ than it is convergent in $\int_0^\infty \frac{df(x,\alpha)}{d\alpha}$

h ~~If f~~

The integral (a,b) which is finite and bounded

X

(h) Improper integrals are the integral which is infinite and ~~be~~ unbounded. If f(x) is function in (a, ∞) than it converges whe there exist a $\int_a^\infty f(x)dx$ which converges into $\int_a^x F(x)dx$.

for uniform convergence the integral should be bounded and integrant and finite. $\int_a^\infty f(x)dx = \int_a^x f(x)dx$

Ex- $\int_0^\infty \frac{1}{1+x^2} dx$

✓

$$\int_0^x \frac{1}{1+x^2}\,dx$$

$$\lim_{x\to\infty} \int_0^a \frac{1}{1+x^2}\,dx$$

$\lim_{x\to\infty} \tan^{-1}x$ on putting limit.

$$\tan^{-1}x - \tan^{-1}0 \Big|_0^\infty$$

$$\tan^{-1}\infty - \tan^{-1}0$$

$$\tan^{-1}\infty = \pi/2$$

Here we convert the infinite integral into finite.

(F) Dirichlet Multiple Integral —

For Uniform Convergence we use different test —

• P test — If the integral is Unbounded then there exist $\int_0^\infty \frac{dx}{x^p}$ is convergent

If $P > 1$ and

- **Comparison Test –**

function $f(x)$ and $g(x)$ are two function in which $\int_0^\infty g(x) \geq f(x)$ then there exist $x \geq a$ than $\int_0^\infty g(x)\,dx$ converges if $\int_0^x f(x)\,dx$.

- **Absolute Convergent –**

if $\int_0^\infty |f(x)|\,dx$ is a function than it is absolutely convergent in $\int_0^x f(x)\,dx$.

(i) **Dirichlet's Multiple Integral –**

The classic Dirichlet is $\int_0^\infty \frac{\sin x}{x}$ which is Oscillatory and cannot be convergent

$$\int_0^\infty F(x)\, g(x)\, dx$$

Dirichlet Multiple –

$$\iiint x_1^{m_1-1} x_2^{m_2-1} \cdots \{x_n^{m_n-1} dx_n\, dx_1 \cdots dx_n$$

$$\frac{\Gamma m_1\, \Gamma m_2 \cdots \Gamma m_n}{\Gamma(m_1 + m_2 + m_3 \cdots 1)}$$

Where Integral is Extended to all the positive values of variable $x_1, x_2 \cdots x_n$ subjecting to condition $x_1 + x_2 \cdots x_n \leq 1$

Section- B

5. State and Prove Mean Value Theorem for Integral.

If F is bounded and integrable on $[a, b]$ m, M is infimum and supremum then

$$\int_a^b F(x)\, dx = \mu(b-a)$$

$$m \leq \mu \leq M$$

c count on (a,b) there exist $c \in [a,b]$ such that

$$\int_a^b F(x)dx = (b-a)F(c)$$

$$m \leq f(x) \leq M$$

$$\int_a^b m \leq n\,dx\,dy$$ (u exist between m and M)

$$\int_a^b f(x)\,dx$$

$$m \leq \int_a^b F(x)dx \leq 1$$ ✓

$$m(b-a) \leq \int_a^b F(x)dx \leq M(b-a)$$

$$u(b-a) \leq F(b-a)$$

$$m(b-a) \leq u(b-a) \leq M(b-a)$$

$$\boxed{\int_a^b F(x)dx = u(b-a)}$$

~~Section-C~~

6.

The above equation satisfies the Mean value Theorem for the Integral

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Section-C

6.

Improper Integrals are the Integrals which is unbounded and ifinite.

The conditions under which the Improper Integral

$$I = \int_a^{\infty} F(x)\,dx$$ Converges

are-

Comparison Test - Comparison test is used to compare between two Integrals $F(x)$ and $g(x)$ in which if $g(x) \geq F(x)$ and $x \geq a$ then $\int_a^{\infty} g(x)\,dx$ Converges into $\int_a^{\infty} F(x)\,dx$.

for $I = \int_{\alpha}^{\infty} f(x)\,dx$

$$\int_{\alpha}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$$

then it converges into

$$\int_{\alpha}^{a} f(x)\,dx$$

Convergence and divergence for Improper Integral—

The Improper Integral ~~of f(x)~~ $f(x)$ is in (a, ∞) converges if $\int_{0}^{\infty} f(x)\,dx = \int_{0}^{x} f(x)\,dx$ which is bounded and finite.

- for the divergence the Improper Integral should ~~not~~ be

unbounded and finite on the Given Integral.

Test for Integral for convergence and divergence -

P test - If the Integral $\int_0^\infty \frac{dx}{x^p}$ converges

If $P>1$ and diverges IF $P \leq 1$.

u test - If $f(x)$ is a function in (a, ∞) in $\int_0^\infty f(x)dx$ then there exist a num value u such that $\int_0^\infty x^u dx$ and $u>1$ than it converges into $\lim_{x \to a} \int_a^x f(x)\,dx$ and if $u \leq 1$ than it diverges and same interval $\lim_{x \to \infty} x^u f(x)$ $\infty, -\infty$

Limit comparison Test -

if $f(x)$ and $g(x)$ are two function such that $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to b} \frac{f(x)}{g(x)}$

than $\int_a^b f(x)\,dx\, g(x)\,dx$.

Able test – If $\int_a^\infty f(x)$ is convergent

and $\int_a^\infty g(x)$ is bounded on $x > a$

$\int_a^\infty f(x)g(x)\,dx$ is convergent

$$I = \int_1^\infty \frac{1}{1+x^2}\,dx$$

$$\lim_{x\to\infty} \int_1^\infty \frac{1}{1+x^2}\,dx = \tan^{-1}|x|\Big|_1^\infty$$

$$\tan^{-1}|\infty| - \tan^{-1}|a|\Big|_1^0$$

On putting value

$$\tan^{-1}|1| - \tan^{-1}|0|$$

$$\tan^{-1}\frac{\pi}{4} - \frac{\pi}{2}$$

$$\tan^{-1}|\infty| - \tan^{-1}|1|$$

$$\frac{\pi}{4} - \frac{\pi}{2}$$

$$\frac{\pi}{2}//$$

~~$\tan^{-1}|\infty| = \tan^{-1}|1|$~~

$\tan^{-1}|\infty| - \tan^{-1}|1|$

$\frac{\pi}{2} - \frac{\pi}{4}$ ✗

$\frac{\pi}{2}$ Proved.

$$I = \int_1^{\infty} \frac{1}{1+x^2}$$

$$I = \int_1^{\infty} \frac{1}{x^2+1}$$

$$\lim_{x\to\infty} \int_1^{\infty} \frac{1}{x^2+1}\,dx$$ ✓

$$\left|\tan^{-1}|x|\right|_1^{\infty}$$

$\tan^{-1}|\infty| - \tan^{-1}|1|$

$\frac{\pi}{2} - \frac{\pi}{4}$

$\frac{\pi}{2} - \frac{\pi}{4}$ which is finite value.

$\frac{\pi}{4}$ //

Section-A

1(a) If a function $f(x)$ is lies between closed interval $[a,b]$ and Open Interval (a,b) than it is said to be bounded.

If a function $f(x)$ s.t

$\int_0^{\infty} f(x)\,dx$ converges into

$\int_0^{x} f(x)\,dx$ from where can

obtain the finite values between $(0,x)$ than s.t. function is said to be bounded.

Paper Code

17

X

Do Not Write anything in this Portion

X

X

Paper Code

20

Do Not Write anything in this Portion

Paper Code

21

Do Not Write anything in this Portion

Paper Code

22

Do Not Write anything in this Portion

$|x_1 - x_2| > \delta$

$|f x_1 - f x_2| > \varepsilon$

$|f x_1 - f a| < \delta$

$|f x_1 - f x_2| > \varepsilon$

$\frac{\pi}{4} - \frac{\pi}{2}$

$\frac{\pi - 2}{4}$

$\frac{\pi}{4} - \frac{\pi}{2}$

$\frac{2\pi - \pi}{4}$

$\frac{\pi - \pi/2}{4}$