Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 10504399

Roll No. 24079000007
Total Mark 54/75.00
Exam MASTER OF SCIENCE STATISTICS_ODD EXAM-DEC-2
Subject B060702T - MEASURE THEORY AND PROBABILITY

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	3/5						
1B	2/5						
1C	4/5						
1D	5/5						
1E	3/5						
1F	3/5						
1G	4/5						
1H	5/5						
1I	3/5						
2	NA/15						
3	12/15						
4	NA/15						
5	NA/15						
6	NA/15						
7	NA/15						
8	NA/15						
9	10/15						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

Date of Exam: 28/1/25 Shift: 1st Room No.: 9
Paper Code: B060702T Subject: Statistics Year/Sem: 1
Name of Candidate: Divya Dwivedi
Roll No.: 2 4 0 7 9 0 0 0 0 7

COE Facsimile

Signature of Invigilator

Divya
Signature of Candidate

PART-II

MARKS OBTAINED										
Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
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(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures — Max. Marks

Total Marks in Words

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Paper Code

Signature of Evaluator

Course (Statistics) MSC
Session 2024-25 Year/Semester 1
Subject Statistics

Paper Code
B 0 6 0 7 0 2 T

Exam Date
2 8 0 1 2 0 2 5

Name of Candidate
D I V Y A D W I V E D I

Father's Name
S A N T R A M D W I V E D I

महाविद्यालय का कोड
College Code
K N 0 3

परीक्षा केन्द्र का कोड
Exam Centre Code
K N 0 3

परीक्षा का प्रकार
Type of Exam

संस्थागत Regular ● — भूतपूर्व परीक्षार्थी Ex. Student
व्यक्तिगत Private — बैक पेपर परीक्षा Back paper Exam

ANSWER BOOKLET NO.
10504399

B 0 6 0 7 0 2 T
Paper Code

नामांकन संख्या
Enrollment Number: C S J M A 2 4 0 0 0 0 0 3 8 2 1

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number
2 4 0 7 9 0 0 0 0 0 7

पेपर कोड Paper Code
B 0 6 0 7 0 2 T

Divya
Signature of Candidate

Signature of Invigilator

C S Facsimile

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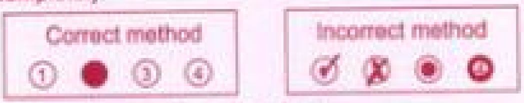
नोट : 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पन्ने के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. बॉक्स में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below blacken the circles completely.



4. Make no Stray marks on this sheet.
5. **DO NOT WRITE OR MARK ON THE BAR CODE.**

परीक्षार्थी के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र कोड सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1–24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्त पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर क निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा को कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पैंसिल का प्रयोग न करे।
10. B कोपी या अतिरिक्त ग्राफ नहीं दिया जायेगा।

IN ORDER TO AVOID UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/ electronic watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

INSTRUCTIONS TO THE CANDIDATE

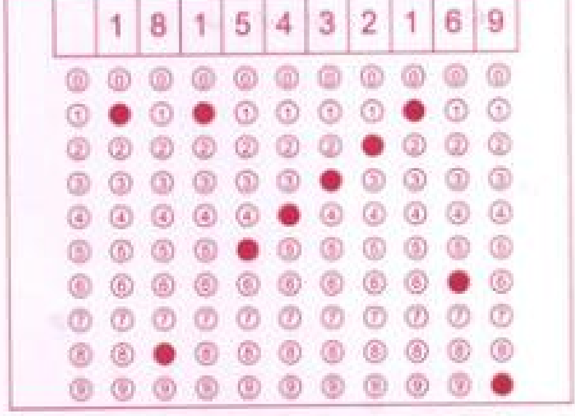
1. Read the instructions carefully given on the Question Pape Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-32) or any other kind of damag in your answer script, if found than change the answer scri immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g Subject Code, Subject Name and Question of the Questic Paper during first THIRTY MINUTES of the commencement the exam, so that it can be corrected in TIME. After that n corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over pape should fill in status as Carry Over. Those appearing as E Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

अनुचित साधन से बचने हेतु:

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कही और न लिखे तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुए साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, कोपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लैस साइंटिफिक केल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

INSTRUCTIONS TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.



Note - If your Roll No. is of 10 digits. Please leave first three column

Section - A

(a)

Borel field - It is the σ field which is generated by all open sets. and all open intervals. It is the smallest σ field for the open intervals. represented by $B(R)$

Borel sets - Borel sets is the collection of smallest sets in the open intervals.

(b) **Lebesque - Stieltjes Measure -**

Lebesque stieltjes Measure is the generlization ~~a~~ form of lebesque Measure. It is the Measure on the real line that generalise the concept of the length. In lebesque - stieltjes Measure the σ algebra is associated for all closed and open intervals. for all sets. which defined on the field.

(c) **Measurable function** - let (Ω, F) be a set in σ algebra or function σ in which f belongs to Ω such that $f : \Omega \rightarrow R$ where R belongs to the Borel set (B) which is the preimage of $B^{-1}(R)$

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02

$$f^{-1}(B) = \{f(\omega); (\omega) \in R\}$$

The measurable function is used to measure set which assigns value to non negative measurable sets.

It is also used to measure length, area and volume.

- Random Variable is a Measurable function(x) in probability space (Ω, F, P) where Ω is sample space.

If we toss a coin the probability of heads and tails which obtained [Result] is a random variable.

$$X(\omega) = P \begin{cases} H = 1 \\ T = 0 \end{cases}$$

Here, $P(H) = 1$ and $P(T) = 0$
the result we obtained is by random variable X.

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03

(e) Helly Bray theorem –

If the sequence of distribution function $\{Fn(x)\}$ has all the points of continuity of and $g(x)$ is bounded continuous function over the real lim $R(-\infty, \infty)$

$$\lim_{x \to \infty} \int_{-\infty}^{\infty} g(x)\, dF_n(x)$$

$$\int_{-\infty}^{\infty} g(x)\, d F_n(x)$$

$$\int_{-\infty}^{\infty} \cos t(x)\, d Fn(x)$$

$$\int_{-\infty}^{\infty}$$

(f) ~~Borel 0-1 law –~~

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(f) Borel 0-1 law-

It is fundamental law in statistics that gives result in stochastic process. ~~m~~ in tail t event. It this law is given by the mathematician Emile borel who has major contribution in the development of Statistics in Measure theory and Probability.

~~It is a binary~~ It gives result in binary form that the event happens almost surely $P(1)$ and the event never happens $P(0)$.

Formal statement.

let $(x_1, x_2 - - - - x_n)$ be a independent identically distributed random variable where T is σ tail then the condition of happening event is almost surely and the event of never happens.

$$\boxed{P(A) = P(0) \text{ or } P(1)}$$

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05

(d)

Inversion theorem

let $F(x)$ or $\phi(x)$ denote distribution function and characteristic function respectively. of random variable X. If $F(a+h, a-h)$ be the interval contain of the distribution function.

$$F(a+h) - F(a+h) = \lim_{T \to \infty} \frac{1}{\pi} \int_{-T}^{T} \frac{\sin h e^{-itx}}{t} d(x)\, dt.$$

(g) Linderberg feller theorem.

linderberg feller theorem is the generalisation of central limit theorem. In linderberg feller theorem. the Idependent ~~Identically distributed random variable~~ not necessary Identically distributed random variable. Converges the sum of sequence of iid into normal distribution

let $X_1, X_2 - - - X_n$ be the sequences of Independent not necessary Identically distributed random variable.

where

$E(X_i) = \mu_i$

$Var(X_i) = \sigma$

then the linderberg feller satisfies the sum of given condition.

$$\cancel{Sn} \; z = \frac{Sn}{\sqrt{\sigma^2 n}}$$

(h) WLLN

Weak law of large Number.

The law of large Number states the repeatation of event

Weak law of large Number states

that the sum of independent identically distributed random variable converges its probability to the population Mean.

$X_1, X_2 - - - - - X_n$ be a random variable of independent indentically distributed random variable

where mean
$E(Xi) = \mu$

for
$Var(Xi) = \sigma$

then

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$|a_n - \mu| > \varepsilon$

this satisfies the ~~cond~~ equation.

Conditions for WLLN.

Random variable should be Independent.

Random Variable should be Identical. and finite.

Weak law of large Number is applicable for gives the result slow and not acurate so the sample should not be very large.

(1) Caratheodary Extension theorem -

Let A be any subset of given set E & E∈A and μ be a measurable set than there exist a unique measurable set $\bar{\mu}$ such that

$$\boxed{E(\mu) = \bar{\mu}}$$

where

μ = measurable set

$\bar{\mu}$ = unique Measurable set

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08

9

Section C

Central limit theorem.

Central limit theorem is fundamental theorem in statistics. In central limit theorem the sample mean convergesm into Normal distribution.

let,

$x_1, x_2 - - - - x_n$ be a sequence of Independent Identically distributed random variable.

where

$S = (x_1, x_2 - - - - x_n)$

$E(x) = E(x_1 + x_2 - - - - x_n)$

$E(x_1) + E(x_2) - - - - E(x_n)$

$\mu + \mu + - - - n\mu$

$E(x) = n\mu.$

$V(x_i) = V(x_1 + x_2 - - - x_n)$

$V(x_1) + V(x_2) - - - - V(x_n)$

$cov(x_i, x_j)$

$\sigma + \sigma - - - - - \sigma$

$\sigma 2n$

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09

$$\boxed{S_n = N(\mu n, n\sigma^2)}$$

$\bar{x} =$

$$E(\bar{x}) = E(\bar{x}_1 + \bar{x}_2 - - - \bar{x}_n)$$
$$E(\bar{x}_1) + E(\bar{x}_2) - - - E(\bar{x}_n)$$
$$\frac{\mu}{n} + \frac{\mu}{n} - - - \frac{\mu}{n}$$

$$E(\bar{x}) = \frac{\mu}{n}$$

$$V(\bar{x}) = V(\bar{x}_1 + \bar{x}_2 - - - \bar{x}_n)$$
$$V(\bar{x}) + V(\bar{x}_2) - - - Var(\bar{x})$$
$$\frac{\sigma^2}{n} + \frac{\sigma^2}{n} - - - \frac{\sigma^2}{n}$$

$$\frac{\sigma^2}{n}$$

$$\boxed{S_n = N\left(\mu, \frac{\sigma^2}{n}\right)}$$

In central limit theorem it converges the Independent Identically distributed random variable sequence ~~to its~~ ~~ts~~ of sample mean to its normal distribution where law of large Number is the sum of average of Idenpendent Identically ~~norm~~ distributed random Variable ~~to~~ into almost surely ~~to~~ its Expected Value.

Where
$(X_1, X_2 - - - X_n)$

$E(X_i) = \mu$
$V(X) = \sigma$

~~Sn (sum~~
→
~~$S_n = n$~~

Sn is the sum in which $n \to \infty$
asympotically to its ~~A~~ expected value

If we toss a coin than the repeatation
of event of Head comes is
0.5 as expected.

Hence, in law of law of large Number
we repeat the event until the
result becomes expected. as in
central limit theorem.

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3. ## Section-B

Set function —

let A be the Value of the function.

A set is a Well defined Collection of Objects called and elements [numbers, symbols]

let S be the set and {a, b, c - - } be the element of the set.

4 the elements of set is represented by curly brackets.

the function defined on set is called set function. (A)

$A \in S = \{a, b, c\}$

Measure and Probability Measure.

Measure function is the set ~~of~~ function which defined on the Measurable sets which is non empty. It is used to Measure the set. ~~which~~

from Measure function we measure the length, Area and volume of a given set. It has a huge role in statistics.

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Paper Code B 0 6 6 7 0 2 7

Probability Measure is the measure in which all the additional masses of sum . is 1

$$P(A) = 1$$

probability Measure is defined on probability space (Ω, f, p) where ~~it~~ Ω is a set . f is a function and p be

If we a toss a coin in which

$\Omega = \{H, T\}$

$$P(A) = \begin{cases} 1 \\ 0 \end{cases}$$

the probability here we let the $p(H) = 1$ and the probability of tails is 0 #

(2) Monotone sequence of set -

Monotone sequence of set is an increasing Order which converges its sequence to its least upper bound.

To prove -

$$|a_n - u| < \varepsilon$$

Proof -

let $A_1, A_2 --- A_n$ be a sequence of set

and $\langle a_n \rangle \longrightarrow 0$

and u be its least upper bound then

$$a_n \leq u \quad ; \quad \varepsilon > 0$$

and
but, $u - \varepsilon$ is not least upper bound.

such that $\quad u - \varepsilon < a_n \quad -(i)$
there exist a_m
$$a_m \leq a_n$$

then
$$\cancel{a_n} \; u + \varepsilon \geq a_n \quad -(i)$$

from Eq (i) and (ii)

Paper Code

$\mu - \varepsilon \leq a_n \leq \mu + \varepsilon$

$\cancel{\mu} - \varepsilon$

$|a_n - \mu| < \varepsilon$

Hence proved

In limit of sequence of set

There are two limits.

limit Inferior

lim inf –

Represents all the set which are finite. in nature

limit superior

lim sup –

Represent all the set which are infinite in nature.

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X

Paper Code

Do Not Write anything in this Portion

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19

Do Not Write anything in this Portion

Paper Code

Do Not Write anything in this Portion

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Do Not Write anything in this Portion

Paper Code

23

Do Not Write anything in this Portion

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24

X