Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
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Exam MASTER OF SCIENCE _ODD EXAM-DEC-24
Subject B010702T - CLASSICAL MECHANICS

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5	8	NA/15				
1B	3/5	9	NA/15				
1C	3/5						
1D	4/5						
1E	3/5						
1F	4/5						
1G	4/5						
1H	4/5						
1I	3/5						
2	11/15						
3	NA/15						
4	NA/15						
5A	NA/5						
5B	NA/5						
5C	NA/5						
6	10/15						
7	NA/15						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

Date of Exam: 23/01/25 Shift: I Room No.: 25
Paper Code: B010702T Subject: CLASSICAL MECHANICS Year/Sem: I/I
Name of Candidate: FARHEEN RAHMAN
Roll No.: 2407700697

Signature of Candidate: Farheen Rahman
Signature of Invigilator
COE Facsimile

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
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Total										

Total Marks in Figures — Max. Marks
Total Marks in Words

B010702T
Paper Code

Signature of Evaluator

Course: M.Sc.
Session: 2024-25 Year/Semester: I/I
Subject Name: CLASSICAL MECHANICS
Medium: English ● Hindi ○

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Exam Date: 23012025
Name of Candidate: FARHEEN RAHMAN
Father's Name: HABIB UR RAHMAN

महाविद्यालय का कोड College Code: KN04

परीक्षा केंद्र का कोड Exam Centre Code: KN04

परीक्षा का प्रकार Type of Exam
संस्थागत Regular ● भूतपूर्व छात्र Ex-Student ○
व्यक्तिगत Private ○ बैक पेपर परीक्षा Back Paper Exam ○

ANSWER BOOKLET NO.
6433500

B010702T
Paper Code

नामांकन संख्या Enrollment Number: C S J M A 2 4 0 0 0 1 3 0 9 5 4

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number: 2407700697

पेपर कोड Paper Code: B010702T

Signature of Candidate: Farheen Rahman.
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नोट- 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पत्र के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. कॉपी में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below, blacken the circles completely.

Correct Method	Incorrect Method
① ● ③ ④	✓ ✗ ● ④

4. Make no Stray marks o n this sheet.

5. DO NOT WRITE OR MARK ON THE BAR CODE.

IN ORDER TO AVOD UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/electronic/digital/ watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु :

1. उत्तर पुस्तिका के निर्धारित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखें तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ छाड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, डिजिटल वॉच, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लेस साइंटिफिक कैल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

परीक्षार्थियों के लिए निर्देश

1. परीक्षा पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र ID सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसकी परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई सा नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेंसिल का प्रयोग न करें।
10. बी कॉपी या अतिरिक्त पत्रक नहीं दिया जायेगा।

INSTRUCTION TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper, Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-24) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Sub Name, and Question of the Question Paper during first THIRTY MINUTES of th commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in statu as Carry Over. Those appearing as Ex- Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9

Note- If your Roll No. is of 10 digits. Please leave first three columns .

Paper Code B010702T

1

Section-A

Short Answer Type Questions

Answer no. 1(A)

Principle of Least action → Principle of least action is defined as, " the time integral of twice of kinetic energy is called action integral"

$$\boxed{\int_{t_1}^{t_2} 2T\,dt = A} \quad -①$$

Definition → • The action integral A of twice of kinetic energy is equal to the zero"

$$\boxed{\int_{t_1}^{t_2} 2T\,dt = 0} \quad -②$$

we know that $\dot{q}\left(\frac{\partial T}{\partial \dot{q}}\right) = 2T$

$$\dot{q}\left(\frac{\partial L}{\partial \dot{q}}\right) = 2T$$

$$p\dot{q} = 2T$$

So, equation ② becomes as

$$\boxed{\int_{t_1}^{t_2} \Sigma p\dot{q}\,dt = 0}$$

Paper Code

B 0 1 0 7 0 2 T

since

$$\sum_i \int_{t_1}^{t_2} p\dot{q}\,dt = 0$$

and $\cancel{H = \sum p\dot{q}}$

by Hamilton principle

$$\delta \int_{t_1}^{t_2} L\,dt = 0$$

$$H + L = \sum p\dot{q}$$

$\therefore H + V = T$

$$H + L - \sum p\dot{q} = 0$$

$$H + L = \text{constant}$$

for the principle of least action.

Hence, principle of least action deduce is

$$\int_{t_1}^{t_2} (2 \times \text{kinetic energy})\,dt = 0$$

$$\boxed{\int_{t_1}^{t_2} 2T\,dt = 0}$$

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B 0 1 0 7 0 2 T

Answer : 1(b)

given:

$$L = a\dot{x}^2 + \dot{y}^2 - kxy$$

we know $\frac{\partial L}{\partial \dot{x}} = p_x$ and $\frac{\partial L}{\partial x} = \dot{p}_x$

$$\frac{\partial L}{\partial \dot{x}} = 2a\dot{x} + 0 - 0$$

$$p_x = 2a\dot{x} \Rightarrow \dot{x} = \frac{p_x}{2a}$$

Hamilton is $H = \sum_i p_i \dot{q}_i - L$

or

$$H = \sum p\dot{x} - L$$

$$H = p_x\left[\frac{p_x}{2a}\right] - [a\dot{x}^2 + \dot{y}^2 - kxy]$$

$$H = \frac{p_x^2}{2a} - a\left[\frac{p_x^2}{2^2a^2}\right] + \left[\frac{p_y^2}{2^2}\right] - [kxy]$$

$$H = \frac{p_x^2}{2a} - \frac{p_x^2}{4a} + \frac{p_y^2}{4} - kxy$$

$$H = \frac{p_x^2}{2a} - \frac{p_x^2}{4a} + \frac{p_y^2}{4} - kxy$$

or

Paper Code: B O 1 0 7 0 2 T

$$\boxed{H = \frac{1}{4}p_x^2 + \frac{1}{4}p_y^2 - bxy}$$

This is the required Hamiltonian.

Answer : 1 (C)

To show: generalised momentum is conserved when corresponding coordinate is cyclic

If Lagrangian is independent of the coordinate q_k's then

Lagrange: $\frac{\partial L}{\partial q_k} = 0$ —①

by Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0 \quad —②$$

by eq.①

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = 0 \quad —③$$

$$\frac{\partial L}{\partial \dot{q}_k} = \text{constant} \quad —④$$

we know that $p_k = \frac{\partial L}{\partial \dot{q}_k}$

Paper Code B010702-T

so equation ④ becomes,

$$\frac{\partial L}{\partial \dot{q}_k} = p_k = \text{constant}$$

$$\boxed{p_k = \text{Constant}}$$ proved

Here, q_k's is the cyclic coordinate because due to q_k Lagrange's corresponding momentum is conserved.

Answer : 1(D)

given; transfomation equations

$$Q = q^{\alpha} \cos\beta p \quad -①$$

$$P = q^{\alpha} \sin\beta p \quad -②$$

To proof; these equation are cannonical

Condition for cannonical : equation must satisfied -

$$pdq - PdQ = \text{Exact differential eqn} \quad -③$$

$$dQ = \dot{Q} = d\left[q^{\alpha} \cos\beta p \right]$$

$$= q^{\alpha}\left[-\sin\beta p \, dp \right] + \cos\beta p \left[\alpha q^{\alpha-1} dq \right]$$

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Paper Code 8 0 1 0 7 0 2 T

eq^n (3) becomes.

$$pdq - PdQ = pdq - q^{\alpha}\sin\beta p\left[-q^{\alpha}\sin\beta p\,dp + (\alpha q^{\alpha-1}dq)\right]$$

$$pdq - PdQ = pdq + q^{2\alpha}\sin^2\beta p\,dp - \alpha q^{2\alpha-1}\sin\beta p\,dq$$

$$pdq - PdQ = (p - \alpha q^{2\alpha-1}\sin\beta p)dq + q^{2\alpha}\sin^2\beta p\,dp \quad -(4)$$

eq^n (4) becomes as

$$Mdx - Ndy = 0$$

so $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ must satisfied

on compareing

$$\frac{\partial}{\partial p}(p - \alpha q^{2\alpha-1}\sin\beta p) = \frac{\partial}{\partial q}(q^{2\alpha}\sin^2\beta p)$$

$$1 - \alpha q^{2\alpha-1}\cos\beta p = 2\alpha q^{2\alpha-1}\sin^2\beta p$$

equation becomes equal to zero

if $2\alpha - 1 = 0$ $\quad q^0 = 1$

Paper Code B010702T

then $\alpha = \frac{1}{2}$ put in last expression

we get

$\beta = 2$

And given equation is an exact differential equation because it satisfied exact differential equation condition

$\boxed{\alpha = \frac{1}{2}}$ and $\boxed{\beta = 2}$ Ans.

Answer : 1 (e)

Inertia Tensor → consider the system of n particles having Tax matrices and ω is the angular frequency.

It is defined as for normal mode particles oscillate will same frequency and phase and amplitude are not same in normal mode.

It is defined by matrix $T = I\omega$

having 3×3 row matrix and contains 9 element

It can be write it as

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

which represent the inertia tensor.

Properties of Inertia tensor →

i) Inertia tensor has the angular frequency of particles of system oscillation.

ii) Inertia Tensor represent by matrix (In above expression) withe relates the angular frequency and tensor quantity.

Answer : 1(f)

Hamilton is the advance part of Lagrange formalism.
In lagrange has contain generalised coordinate q_i

$q, \dot{q}$ and time t which are dependent to each other but Hamilton has generatised coordinate are generalised momentum, generalised position and time which are independent to each other.

$$L = L(q, \dot{q}, t)$$

and

$$H = H(q, p, t)$$

Hamilton coordinates are independent to each other so it is an explicit function of time

we know $\frac{-\partial L}{\partial t} = \frac{dH}{dt}$

If Lagrange's is independent of time then

$$0 = \frac{dH}{dt}$$

$$\boxed{H = constant} \text{ proved}$$

Hence, H is not an explicit function of time and
H is constant

Answer : 1(g)

given the system 
is spring
mass system so

Kinetic energy $T = \frac{1}{2}(m_1\dot{x}^2 + m_2\dot{x}^2)$

and $V = \frac{1}{2} k x^2$

Lagrange $L = T - V$

$$L = \frac{1}{2} m_1(\dot{x}^2 + m_2\dot{x}^2) - \frac{1}{2} k x^2 \quad —①$$

by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \quad —②$

$$\therefore \frac{\partial L}{\partial x} = 0 - kx.$$

$$\frac{\partial L}{\partial \dot{x}} = m_1\dot{x} + m_2\dot{x} \quad \text{put in ②}$$

$$\frac{d}{dt}\left[(m_1+m_2)\dot{x}\right] - (-kx) = 0$$

$$(m_1+m_2)\ddot{x} + kx = 0$$

$$\boxed{\ddot{x} = \frac{-kx}{m_1+m_2}}$$

Paper Code 8 0 1 0 7 0 2 T

frequency $\omega = \sqrt{\frac{K}{V}}$ — ③

$$\ddot{x} = \frac{-Kx}{m_1+m_2}$$

$$K = \frac{(m_1+m_2)\ddot{x}}{-x}$$ — ④

solve them we get

frequency $\omega = \sqrt{\frac{K(m_1+m_2)}{m_1 m_2}}$ proved

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Answer : 1(h)

Principle of virtual work → Let a motion imposed on the body in static equilibrium. which represend virtual situation. the Individual force is zero so the complete work done by the body is also zero. This is done. Work done by the force of constrain as way that virtual displacement is zero.

$$\delta x_i = \delta x_i (q_1, q_2 \ldots q_n \ldots q_f, t)$$ ①

$$W = F \times Disp.$$

$$\delta W_i = \sum_i F_i \times \partial x_i$$ — ②

for total force

$$F_i = F_i^a + f_i \quad -③$$

in equilibrium state, work done becomes

$$\delta W_i = \sum_i (F_i^a + f_i) \cdot \delta r_i = 0 \quad -④$$

here force of constraints $f_i = 0$

becomes as

$$\delta W_i = \sum_i (F_i^a + 0) \cdot \delta r_i = 0$$

or. ← Principle of virtual work →

$$\boxed{\delta W_i = \sum_i F_i \cdot \delta r_i = 0}$$

This is the required principle of virtual work.

Answer : 1(i)

Theory of small oscillation → Theory of small oscillation is for large particle system having i^{th} particle of the given system of oscillation.

Paper Code B 0 1 0 7 0 2 T

Normal modes → If system of particle having same frequency of oscillation then called it as normal mode.

Normal mode does exit in same phase and same amplitude of system.

Oscillation of i^{th} particle of system are represented by

$$\boxed{|V - \omega T| = 0}$$

where V = function of Potential energy magnitude
T = Magnitude of kinetic energy
ω = angular frequency of i^{th} Particle

$\dot{q}_1 \dot{q}_{21}$ = function of V_{12} and V_{21}

$\dot{q}_1^2$ = function of V_{11}

$\dot{q}_2^2$ = function of V_{22}

This theory explain the system of particles having same frequency of oscillation for system.

Paper Code B0107027

14

Section - B

Answer : 2

Generalised coordinate → The minimum numbers of independent coordinate that describe the motion of particle of system compatible with constraints known as 'generalised coordinate'.

transformation equation for generalised coordinate are

$$\left.\begin{aligned} x_i &= x_i(q_1, q_2 \cdots q_f \cdots q_n, t) \\ y_i &= y_i(q_1, q_2 \cdots q_f \cdots q_n, t) \\ z_i &= z_i(q_1, q_2 \cdots q_f \cdots q_n, t) \end{aligned}\right\} - 1(a)$$

for position vector

$$\vec{r}_i = \vec{r}_i(q_1, q_2 \cdots q_f \cdots q_n, t) \quad -1(b)$$

1(a) and 1(b) are transformation eqn

D' Alembert principle for generalised coordinate →

we can derive the lagrange equation

Paper Code B 0 1 0 7 0 2 T

of motion by D'Alember principle.
which defined as " The resultant force together with the reverse effective force are in equilibrium".

by Newton's 2nd law of motion

$$F = \frac{dp}{dt} = \dot{p} \quad -①$$

virtual displacement $\frac{\partial r_i}{\partial q_k} \delta q_k = \delta r_i$

and work done

$$\delta W = F \times \delta r_i$$

$$\sum_i (F - \dot{p}) \delta r_i = 0 \quad -②$$

for total force. $F_i = F_i^a + f_i \quad -③$

$$\sum_i (F_i^a + f_i) \delta r_i - \sum_i \dot{p}_i \cdot \delta r_i = 0$$

here force of castraint are zero

$$\sum_i (F_i^a + 0) \delta r_i - \sum_i \dot{p}_i \cdot \delta r_i = 0$$

write it as

$$\boxed{\sum_i (F_i - \dot{p}_i) \cdot \delta r_i = 0}$$

This is the D'Alempert principle.

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for generalised coordinate q_k

$$(F_k - \dot{p}_k) \cdot \delta q_k = 0$$

<u>Lagrange equation of motion</u> →

we can derive Lagrange's equation of motion from D'Alembert principle for taking system of i^{th} particle having position vector

$$r_i = r_i(q_1, q_2 \ldots q_n, t) \quad \text{--①}$$

and

virtual displacement is $\sum_i F_i \cdot \delta r_i = 0$ --②

$$\delta r_i = \frac{\partial r_i}{\partial q_k} \delta q_k + \frac{\partial r_i}{\partial q_k} \frac{\partial t}{\partial t}$$

By definition of D'Alembert principle.

$$(F_i - \dot{p}_i) \cdot \delta r_i = 0$$

$$F_i \cdot \delta r_i = \dot{p}_i \cdot \delta r_i \quad \text{--③}$$

Take LHS

$$F_i \cdot \delta r_i = \sum_i F_i \sum_k \frac{\partial r_i}{\partial q_k} \delta q_k$$

Let $G_k = F_i \cdot \frac{\partial r_i}{\partial q_k}$

$$F_i \cdot \delta r_i = \sum_k G_k \delta q_k \quad \text{--④}$$

Paper Code B 0 1 0 7 0 2 T

Take RHS

$$p_i \cdot \delta r_i = \sum m \dot{r}_i \delta r_i$$

$$= \sum \frac{d}{dt}(m\dot{r}) \delta r_i \quad -⑤$$

$$\therefore \left[\frac{d}{dt}(m\dot{r}_i \delta r_i) = m\dot{r} \frac{d}{dt}(\delta r_i) + \delta r_i \frac{d}{dt}(m\dot{r}) \right]$$

put in above eqn ⑤

$$= \sum \frac{d}{dt}(m\dot{r}_i \delta r_i) - m\dot{r}_i \frac{d}{dt}(\delta r_i)$$

$$= \sum \frac{d}{dt}\left(m\dot{r}_i \frac{\delta v}{\partial q} \right) - m v_i \delta \frac{d}{dt} \dot{r}_i$$

$$= \sum \frac{d}{dt}\left(\frac{m v_i^2}{\partial \dot{q}} - \delta m v_i^2 \right)$$

$$\therefore T = \frac{1}{2} m v^2$$

$$= \left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} \right] \quad -⑥$$

now L.H.S = R.H.S

$$\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = G_k} \quad -⑦$$

This is the general form of Lagranges equation of motion.

Paper Code B010702T

For Conservative System → force is derive by some potential

$$G_K = \frac{-\partial V}{\partial q} \quad — (a)$$

we know

$$L = T - V \quad — (b)$$

(a) and (b) put in eq. (7)

$$\frac{d}{dt}\left[\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial V}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} + \frac{\partial V}{\partial q} = G_K \quad — (8)$$

use $G_K = \frac{-\partial V}{\partial q}$

and Potential energy is not a function of velocity for conservative system.

for conservative system, $\frac{\partial V}{\partial \dot{q}} = 0$

then eq. (8) becomes

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0}$$

This is the Lagrange equation of motion for conservative system. where L is Lagrangian

Paper Code B 0 1 0 7 0 2 T

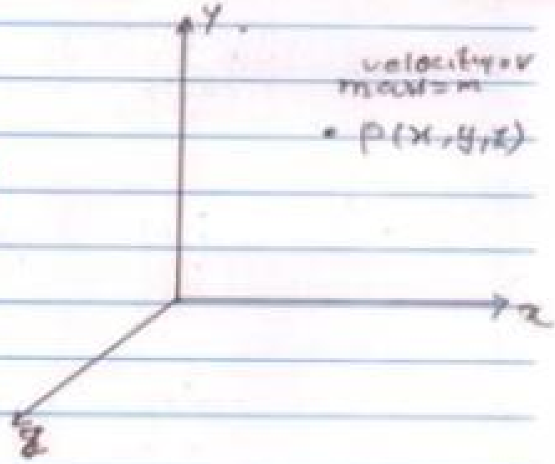
Section - C

Hamilton equation of motion

Let a particle of mass m is moving with velocity v about force f with mass m directed toward centre.

then particle contain energies like potential and kinetic energy.

Hamilton eqⁿ in Cartesian Coordinate



from Cartesian system particle has kinetic energy

$$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

and

Potential energy

$$V = V(x, y, z)$$

so

Lagrange is $L = T - V$

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y, z)$$

generalised momenta -

$$p_x = \frac{\partial L}{\partial \dot{x}} = \frac{1}{2} 2m\dot{x}^2 = m\dot{x} \quad \text{or} \quad \dot{x} = \frac{p_x}{m}$$

$$p_y = \frac{\partial L}{\partial \dot{y}} = m\dot{y} \Rightarrow \dot{y} = \frac{p_y}{m}$$

$$p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z} \Rightarrow \boxed{\dot{z} = \frac{p_z}{m}}$$

Hamilton's : $H = T + V$

$$H = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + V(x,y,z)$$

$$\boxed{H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(x,y,z)}$$

Hamilton equations are

$$\frac{\partial H}{\partial p_x} = \dot{x} \Rightarrow \boxed{\frac{p_x}{m} = \dot{x}}$$

$$\frac{\partial H}{\partial p_y} = \dot{y} \Rightarrow \boxed{\frac{p_y}{m} = \dot{y}}$$

$$\frac{\partial H}{\partial p_z} = \dot{z} \Rightarrow \boxed{\frac{p_z}{m} = \dot{z}}$$

and

$$\frac{\partial H}{\partial x} = -\dot{p}_x \Rightarrow \boxed{-\frac{\partial V}{\partial x} = \frac{\partial H}{\partial x}}$$

$$\frac{\partial H}{\partial y} = -\dot{p}_y \Rightarrow \boxed{-\frac{\partial V}{\partial y} = \frac{\partial H}{\partial y}}$$

$$\frac{\partial H}{\partial z} = -\dot{p}_z \Rightarrow \boxed{-\frac{\partial V}{\partial z} = \frac{\partial H}{\partial z}}$$

these are required Hamilton eq^n for Cartesian coordinate

Hamilton eq^n in cylindrical coordinate

Kinetic energy $T = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2)$

for cylindrical (r, θ, z)

$$V = V(r, \theta, z)$$

$$L = T - V$$

Lagrange $L = \frac{1}{2} m(\dot{r}^2 +$ [illegible] $) + V(r, \theta, z)$

generalised momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} \Rightarrow m\dot{r}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} \Rightarrow mr^2\dot{\theta}$$

$$H = T + V$$

$$H = \frac{1}{2} m(r^2\dot{\theta}^2 + \dot{r}^2) + V(r, \theta, z)$$

or

Hamilton $H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{mr^2} + V(r, \theta, z)$

Paper Code
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Cannonical equation are

$$\frac{\partial H}{\partial p_r} = \dot{r}$$

$$\frac{\partial H}{\partial p_r} = \dot{r} \Rightarrow \boxed{\frac{p_r}{m} = \dot{r}} \quad -(1)$$

$$\frac{\partial H}{\partial p_\theta} = \dot{\theta} \Rightarrow \boxed{\frac{p_\theta}{mr^2} = \dot{\theta}} \quad -(2)$$

$\frac{\partial H}{\partial}$ and

$$\frac{\partial H}{\partial r} = -\dot{p}_r \Rightarrow \theta \; mr\dot{\theta}^2 + \boxed{\frac{\partial V}{\partial r} = -\dot{p}_r \quad -(3)}$$

$$\boxed{\frac{\partial H}{\partial \theta} = -\dot{p}_\theta = \frac{\partial V}{\partial \theta}} \quad -(4)$$

These are Hamilton equations in cyclindrical.

Paper Code B 0 1 0 7 0 2 7

Hamilton eq^n in spherical coordinate

$$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2\theta\dot{\phi}^2) - V(r,\theta,\phi)$$

momenta

$$p_r = \frac{\partial L}{\partial \dot{r}} \Rightarrow m\dot{r}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} \Rightarrow m\dot{\theta}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} \Rightarrow mr^2\sin^2\phi\dot{\phi}$$

$$H = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) + V(r,\theta,\phi)$$

or

$$H = \left(\frac{p_r^2}{m} + \frac{}{mr^2} + \frac{r^2\sin^2\theta\, p_\phi^2}{m^2r^4\sin^4\theta}\right) + V(r,\theta,\phi)$$

or

$$H = \frac{p_r^2}{m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\phi^2}{m^2r^2\sin^2\phi}$$

Hamilton eq^n are

$$\boxed{\frac{\partial H}{\partial p_r} = \frac{p_r}{m}} \quad -①$$

$$\boxed{\frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{mr^2}} \quad -②$$

$$\boxed{\frac{\partial H}{\partial p_\phi} = \frac{p_\phi}{m^2r^2\sin^2\phi}} \quad -③$$

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$$\boxed{\frac{\partial H}{\partial r} = -\dot{p}_r \Rightarrow \frac{\partial V}{\partial r}} \quad -④$$

$$\boxed{\frac{\partial H}{\partial \theta} = -\dot{p}_\theta \Rightarrow \frac{\partial V}{\partial \theta}} \quad -⑤$$

$$\boxed{\frac{\partial H}{\partial \phi} = -\dot{p}_\phi \Rightarrow \frac{\partial V}{\partial \phi}} \quad -⑥$$

these are Hamilton eqn for spherical system