Chhatrapati Shahu Ji Maharaj
University, Kanpur

Answer Script Details
Barcode 6428374

Roll No. 24077000697
Total Mark 53/75.00

Exam MASTER OF SCIENCE _ODD EXAM-DEC-24
Subject B010701T - MATHMATICAL PHYSICS - I

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5	8B	NA/7				
1B	4/5	9A	NA/5				
1C	4/5	9B	NA/5				
1D	4/5	9C	NA/5				
1E	4/5						
1F	3/5						
1G	4/5						
1H	4/5						
1I	3/5						
2	NA/15						
3	10/15						
4	NA/15						
5	NA/15						
6A	NA/7						
6B	NA/7						
7	9/15						
8A	NA/7						

PART-I

Date of Exam: 22/01/25 Shift: I Room No.: 12

Paper Code: B010701T Subject: MATHEMATICAL PHYSICS Year/Sem: I/I

Name of Candidate: FARHEEN RAHMAN

Roll No. 2407700697

Signature of Candidate: Farheen Rahman

Signature of Invigilator

COE Facsimile

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures — Max. Marks

Total Marks in Words

B010701T
Paper Code

Signature of Evaluator

PART-III

Course: M.Sc.

Session: 2024-25 Year/Semester: I/I

Subject Name: MATHEMATICAL PHYSICS

Medium: English ● Hindi ○

Paper Code: B010701T

Exam Date: 22012025

Name of Candidate: FARHEEN RAHMAN

Father's Name: HABIB URRAHMAN

महाविद्यालय का कोड College Code: KN04

परीक्षा केन्द्र का कोड Exam Centre Code: KN04

परीक्षा का प्रकार Type of Exam: संस्थागत Regular ● / भूतपूर्व छात्र Ex-Student ○ / व्यक्तिगत Private ○ / बैक पेपर परीक्षा Back Paper Exam ○

ANSWER BOOKLET NO. 6428374

B010701T
Paper Code

PART-IV

नामांकन संख्या Enrollment Number: CSJMA24000130954

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number: 2407700697

पेपर कोड Paper Code: B010701T

Signature of Candidate: Farheen Rahman

Signature of Invigilator

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नोट- 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पन्ने के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें। 2. ओएमआर में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below, blacken thecircles completely.

Correct Method	Incorrect Method
① ● ③ ④	✓ ✗ ◉ ❹

4. Make no Stray marks o n this sheet.

5. DO NOT WRITE OR MARK ON THE BAR CODE.

IN ORDER TO AVOD UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/electronic/digital/ watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु :

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखें तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ छाड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, डिजिटल वॉच, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लेस साइंटिफिक कैल्कुलेटर ले जाने की अनुमति होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

परीक्षार्थियों के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र ID सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या कटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसको परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्य नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेंसिल का प्रयोग न करें।
10. बी कॉपी या अतिरिक्त शाक नहीं दिया जायेगा।

INSTRUCTION TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper, Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-24) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subje Name, and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in status as Carry Over. Those appearing as Ex- Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9

Note- If your Roll No. is of 10 digits. Please leave first three columns .

Paper Code B010701T

1

Section - A

Short Answer Type Questions -

Answer no. 1 (A)

given: $f(z) = |z|$

we know z is a complex number
we can write it as

$$z = x + iy \quad \text{or} \quad z = u + iv$$

$$f(z) = |z| = |u + iv|$$

Condition for analytic function must satisfies -

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial [u(x,y)]}{\partial x} = \frac{\partial y}{\partial x} \quad (1)$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}[v(x,y)] = \frac{\partial x}{\partial y} \quad (2)$$

And

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[u(x,y)]$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}$$ —③

$$-\frac{\partial v}{\partial x} = -\frac{\partial}{\partial x}[v(x,y)]$$

$$= -\frac{\partial y}{\partial x}$$ —④

equating the equation ① and ② and ③ and ④, we get

$\frac{\partial y}{\partial x} \neq \frac{\partial v}{\partial y}$ and

$\frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$ are equall

So, the given function $f(z) = |z|$ is doesnt satisfies the condition

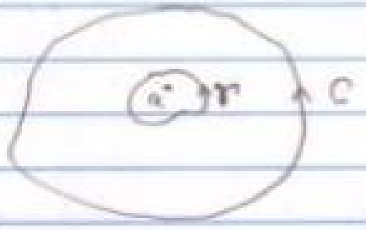
Hence, $f(z) = |z|$ is nowhere analytic.

Answer no. 2

Cauchy's Integral Theorem

Let a closed contour C where the funtion $f(x)$ is defined and it is analytic but a point 'a' with curve where the function is not analytic then by Cauchy's integral Theorem

$\int_C f(z)dz = 0$



Let the curve is circle then eq. of circle

$$z - a = r$$
$$z - a = re^{i\theta}$$
$$\text{or} \quad z = a + re^{i\theta}$$

$$dz = 0 + re^{i\theta}(i)d\theta$$

By Cauchy Theorem,

$$\int f(z)dz = 2\pi i f(a)$$

we can write it

$$\int \frac{f(z) - f(a)}{z-a} dz + \int \frac{f(a)\,dz}{z-a} = 2\pi i f(a)$$

$$\because \ |f(z) - f(a)| < \varepsilon$$

so

$$= \int f \frac{(a).\ re^{i\theta}}{re^{i\theta}} i d\theta \qquad \because z - a = re^{i\theta}$$

$$= i\int_0^{2\pi} 1.d\theta\, f(a)$$

$$= i\,[\theta]_0^{2\pi}\, f(a)$$

Paper Code B D 1 0 7 0 1 T

Do Not Write anything in this Portion

$= (2\pi i - 0 i] f(a)$

$$\boxed{\oint_c \frac{f(z)dz}{z-a} = 2\pi i f(a)}$$

This is the required cauchy integral formula

Answer no. C

given, $f(z) = \log(1+z)$

at $z=-1$ $\quad \therefore e^x = 1+x+x^2+\cdots$

$\therefore f(z) = e^{(1+z)}$

$f(z) = 1 + (1+z) + (1+z)^2 + \cdots$

$f(z) = \lim_{z \to -1} \left[1 + (1+z) + (1+z)^2 + \cdots\right]$

$f(z) = 1$ finite

given $f(z)$ has finite value so there is no isolated singularity.

for isolated singularity $f(z)$ must be zero.

Hence, at $z=-1$ function $f(z) = \log(1+z)$ doesn't exit isolated singularity

Answer no. D

To show $P_n^m(-x) = (-1)^{n+m} P_n^m(x)$

we know the Rodrigue formula

$$\boxed{P_n(x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2-1)^n} \quad -①$$

for $P_n(-x)$

replace x by $-x$ in the formula

$$P_n(-x) = \frac{1}{2^n n!}\left[\frac{d}{dx}\right]^n [(-x)^2-1]^n$$

$$P_n(-x) = \frac{1}{2^n n!}\left[\frac{d}{dx}\right]^n (-1)^n[(x)^2-1]^n$$

$$P_n(-x) = (-1)^n \frac{1}{2^n n!}\left[\frac{d}{dx}\right]^n [x^2-1]^n$$

$$P_n(-x) = (-1)^n P_n(x) \quad -②$$

for $P_m(-x)$

replace x by $-x$ and
replace n by m in the
formula

$$P^m(-x) = \frac{1}{2^m m!}\left[\frac{d}{dx}\right]^m [(-x)^2 - 1]^m$$

$$P^m(-x) = \frac{1}{2^m m!}\left[\frac{d}{dx}\right]^m (-1)^m [x^2 - 1]^m$$

$$P^m(-x) = (-1)^m \frac{1}{2^m m!}\left[\frac{d}{dx}\right]^m [x^2-1]^m$$

$$P^m(-x) = (-1)^m P_m(x) \quad -③$$

from equation ② and ③, we get

$$P_n(-x) \times P^m(-x) = (-1)^n P_n(x) \times (-1)^m P_m(x)$$

$$\boxed{P_n^m(-x) = (-1)^{n+m} P_n^m(x)}$$

Hence proved.

$\because a^n \times a^m = a^{n+m}$

Answer no. 1 (E)

we know $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!(n+r)!}\left(\frac{x}{2}\right)^{n+2r}$

$$J_n(x) = \frac{(-1)^0}{0(n+0)!}\left(\frac{x}{2}\right)^{n+0} + \frac{(-1)^1}{1!(n+1)!}\left(\frac{x}{2}\right)^{n+2}$$

$$+ \frac{(-1)^2}{2!(n+2)!}\left(\frac{x}{2}\right)^{n+2\cdot 2} + \cdots\Big]$$

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7

$$J_n(x) = \left(\frac{x}{2}\right)^n \left[\frac{1}{n!} - \frac{1}{(n+1)n!}\left(\frac{x}{2}\right)^2 + \frac{1}{2!(n+2)(n+1)n!}(x/2)^4 + \frac{1}{3!(n+3)(n+2)(n+1)n!}\left(\frac{x}{2}\right)^6 + \cdots \right]$$

$\therefore (n+1)! = (n+1)n!$

$$J_n(x) = \left(\frac{x}{2}\right)^n \frac{1}{n!}\left[1 - \frac{1}{(n+1)}\left(\frac{x}{2}\right)^2 + \frac{1}{2!(n+2)(n+1)}\left(\frac{x}{2}\right)^4 + \cdots \right]$$

now put $n = \pm 1/2$ Take +ve sign

$$= \sqrt{\frac{x}{2}} \frac{1}{1/2!}\left[1 - \frac{1}{3/2}\left(\frac{x}{2}\right)^2 + \frac{1}{2!(3/2)(5/2)}\left(\frac{x}{2}\right)^4 + \cdots \right]$$

$\therefore (n+r)! = \Gamma n+r+1$

$n\Gamma n = \Gamma n+1$

$\Gamma 1/2 = \sqrt{\pi}$

$$= \sqrt{\frac{x}{2}} \frac{1}{1/2\sqrt{\pi}}\left[1 - \frac{1}{3!}\left(\frac{x}{2}\right)^2 + \frac{1}{5!}\left(\frac{x}{2}\right)^4 + \cdots \right]$$

multiple and divided by x on both sides we get

$$= \sqrt{\frac{4x}{2\pi}} \cdot \frac{1}{x}\left[x - \frac{x^3}{3!} + \frac{1}{5!}x^5 - \cdots \right]$$

$$\boxed{J_{1/2}(x) = \sqrt{\frac{2}{x\pi}} \sin x} \quad \text{—(1) Ans.}$$

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as well as Take -ve sign

$$\boxed{J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x} \quad -②$$

Ans.

Answer no. 1(G)

given $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

To proof: (a) $\sigma_i^2 = I$

Let $\sigma_i = \sigma_x$

$\sigma_x \sigma_x = \sigma_i \sigma_i = \sigma_i^2 = I$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0+1\times1 & 0+1\times0 \\ 0+0\times1 & 1\times1+0\times0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\sigma_i^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I}$$

$\sigma_i^2 = I$ Hence proved.

To proof: (b) $\sigma_i \sigma_j = i\sigma_k$

Let $i = x$, $y = j$, $z = k$

then $\sigma_i \sigma_j = i\sigma_k$

$$\sigma_x \sigma_y = i\sigma_z$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0+i & 0+0 \\ 1\times 0+0 & 1\times(-i)+0 \end{pmatrix} = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

or

$$\boxed{i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$ Hence proved

Hence prove that

$$\sigma_i \sigma_j = i\sigma_k$$

Answer no. 1 (H)

Let a consider operator $\hat{H}$ is operate on state $|\phi\rangle$ then $\hat{H}$ is Hermitian operator if it is satisfies some conditions

Do Not Write anything in this Portion

Paper Code
8010701T

It has eigen value let be λ then
eigen value equation is

$$\boxed{\hat{H}|\phi\rangle = \lambda|\phi\rangle}$$

Case-I

If $\hat{H}$ Hermitian operator has one eigen state then

$$\boxed{\int_{-\infty}^{\infty} \phi \hat{H} \phi \, d\tau = \int \phi^* \hat{H}^* \phi \, d\tau}$$

must be satisfied.
where ϕ and ϕ are normalised

$$\therefore \int \phi \phi^* d\tau = 1$$

case-II

If Hermitian operator has two eigen state $|\psi_1\rangle$ and $|\psi_2\rangle$ then it must satisfied.

$$\int_{-\infty} \quad \hat{H}|\phi_1\rangle = \lambda|\phi_1\rangle$$

$$\hat{H}|\phi_2\rangle = \lambda|\phi_2\rangle$$

since

$$\boxed{\int_{-\infty}^{\infty} \phi_1 \hat{H} \phi_2 \, d\tau = \int_{-\infty}^{\infty} \hat{H}^* \phi_1^* \phi_2 \, d\tau}$$

must be satisfied by Hermitian operator.

for Skew Hermitian-

$$\boxed{\int_{-\infty}^{\infty} \psi \hat{H}^{*} \psi \, d\tau = -\int_{-\infty}^{\infty} \psi^{*} H^{*} \psi \, d\tau}$$

and
for two state

$$\boxed{\int_{-\infty}^{\infty} \psi_1 \hat{H} \psi_2 \, d\tau = -\int_{-\infty}^{\infty} \psi_1^{*} H^{*} \psi_2 \, d\tau}$$

These two condition must be satisfied by Skew Hermitian operator.

Answer - I

we know mixed Tensor $\bar{A}_i^j$ has 2 rank

$$\bar{A}_i^j = \frac{\partial \bar{x}^j}{\partial x^p} \frac{\partial x^q}{\partial \bar{x}^i} A_q^p$$

we know $\delta_i^j = \frac{\partial x_i}{\partial x_j}$

$$= \frac{\partial x_i}{\partial x_k} \frac{\partial x_k}{\partial x_j}$$

$$= \frac{\partial x_i}{\partial x_k} \frac{\partial x_k}{\partial x_l} \frac{\partial x_l}{\partial x_k}$$

$$\therefore \delta_m^l = \frac{\partial x_l}{\partial x_l}$$

$$= \frac{\partial x_i}{\partial x_k} \delta_m^l \frac{\partial x_l}{\partial x_k}$$

$$\boxed{\delta^i_j = \frac{\partial \bar{x}^i}{\partial x^p} \frac{\partial x^q}{\partial \bar{x}^j} A^p_q}$$ proved

which is same as mixed tensor

so, kronecker delta function is mixed Tensor of rank 2.

Answer (F)

Linear vector space → Let the V be the set of vector whose component are vector and F be a field whose component are scalar then V defined on F such that

V(F) then it is called linear vector space.

properties → i) $V_1 + V_2 = V_3 \in V$

ii) $V_1 + (V_2 + V_3) = (V_1 + V_2) + V_3$

iii) $V + (-V) = 0$

identity $V_1 + 0 = V_1$

Schwartz Inequality → for two vectors α and β in inner product.

$$\boxed{||\alpha|| \, ||\beta|| \geq |\langle \alpha | \beta \rangle|}$$

Paper Code B0107017

Section - B

Solution of Legendre's differential equation.

It has differential equation has the form

$$\boxed{(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0} \quad -①$$

has power series solution is

$$\boxed{y = \sum_{r=0}^{\infty} a_r x^{k+r}} \quad -②$$

On differentiation.

$$\frac{dy}{dx} = a_r x^{k+r-1}(k+r) \quad -③$$

$$\frac{d^2y}{dx^2} = (k+r)\, a_r x^{k+r-1}(k+r-1) \quad -④$$

put in eq.①

$$(1-x^2)\left[(k+r)(k+r-1)a_r x^{k+r-1}\right] - 2x\left[(k+r)a_r x^{k+r-1}\right] + n(n+1)a_r x^{k+r} = 0$$

or,

$$(k+r)\left[(1-x^2)(k+r-1) - 2x\right] a_r x^{k+r-1} + n(n+1)a_r x^{k+r} = 0$$

or,

$$(k+r)(k+r-1)a_r x^{k+r-1} - [(k+r)(k+r-1)a_r x^{k+r+1} - 2(k+r)a_r x^{k+r} + n(n+1)a_r x^{k+r} = 0$$

Paper Code B010701T

equating the coefficient of x^k

we get

$$\boxed{K = n} \quad -⑥$$

$$K = -(n+1) \quad -⑦$$

again equating
the 2nd lowest value of x^{k-1}
we get

$$\boxed{a_1 = 0} \quad -⑧$$

put $k = n$ in equation ⑤ we get
the
Recurrence relation.

$$\boxed{a_r = -\frac{a_{r-2}(K+r)(K+r-1)}{(K+r-n)(K-n+r)}} \quad -⑨$$

from this equation we can calculate
$a_0, a_1, a_2, a_3 \dots$ so on.

we know $a_1 = 0$ so

$\boxed{a_3 = 0}$ must be zero

$a_5 = 0$ also,

put $r = 2$ in equation ⑨

$$a_2 = -\frac{a_0(K-n+r)}{(n+1)}$$

Case - I now put $K = n$ in series solution

$$y = \sum_{r=0}^{\infty} a_r x^{K+r}$$

$$y = \sum_{r=0}^{\infty} a_r x^{n+r} \quad \text{expand it}$$

$$y = a_0 x^n + a_1 x^{1+n} + a_2 x^{n+2} + \cdots$$

$$y = x^n \left[a_0 + a_1 x^1 + a_2 x^2 + \cdots \right]$$

On putting the values of $a_1 = 0$
$a_2, a_3, a_4 \ldots$
we get

we take $a_0 = \frac{1.3.5 \ldots (2n-1)}{n!}$

$$y = \frac{1.3.5 \ldots (2n-1)}{n!} \left[x^n - \frac{n(n-1)x^{n-2}}{2!(2n-1)} + \ldots \frac{n(n-1)(n-2)x^{n-4}}{(2n-3)} + \cdots \right]$$

$$\boxed{P_n(x) = \frac{1.3.5 \ldots (2n-1)}{n!} \left[x^n - \frac{n(n-1)x^{n-2}}{2!(2n-1)} + \cdots \right]}$$

This is the Legendre's differential equation of 1st kind and order n.

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Case - II put $k = -(n+1)$ we get the $Q_n(x)$

General solution →

$$\boxed{y = C_1 P_n(x) + C_2 Q_n(x)}$$

where $P_n(x)$ = Legendre differential equation of 1st kind

$Q_n(x)$ = legendre differential equation of 2nd kind.

and C_1 and C_2 are the constant with respectively to $P_n(x)$ and $Q_n(x)$

Generating function → Generating function of legendre differential equation is

$$\boxed{(1-2xt+t^2)^{-1/2} = \sum_{r=0}^{\infty} t^n P_n(x)}$$

Series solution of legendre →

$$y = a_0 x^n + a_1 x^{n+1} + a_2 x^{n+2} + \cdots a_n x^{n+2r}$$

$$y = \sum_{r=0}^{\infty} a_r x^{n+2r}$$

This is the power series solution of legendre differential equation

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Section - C

Laguerre polynomials → Laguerre polynomial has the differential equation of the form-

$$\boxed{x^2\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0}$$

or

$$x^2y'' + (1-x)y' + ny = 0$$

It is represented by $L_n(x)$

Generating function → Laguerre polynomial has generating function as the form

$$\boxed{L_n(x) = e^{+x}\left(\frac{d}{dx}\right)^n x^n e^{-x}}$$

To show:

$$e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty} t^n J_n(x)$$

we know

$$e^x = 1 + x + x^2 + x^3 + \cdots$$

$$e^{-x} = 1 - x + x^2 - x^3 + \cdots$$

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$$e^{\frac{x}{2}(t-\frac{1}{t})} = e^{\frac{x}{2t}} \cdot e^{-\frac{x}{2t}}$$

$$e^{\frac{xt}{2t}} = 1 + \left(\frac{xt}{2t}\right) + \left(\frac{xt}{2t}\right)^2 + \cdots$$

$$\left(\frac{xt}{2t}\right)^{n-1} + \left(\frac{xt}{2t}\right)^n \quad -①$$

and

Similarly

$$e^{-\frac{x}{2t}} = 1 + (-1)\left(\frac{x}{2t}\right)^1 + (-1)^2\left(\frac{x}{2t}\right)^2 + \cdots$$

$$+ (-1)^3\left(\frac{x}{2t}\right)^3 + \cdots$$

$$\cdots \left(\frac{x}{2t}\right)^{2n-1} + \left(\frac{x}{2t}\right)^{2n-2} \quad -②$$

multiply eq. ① by equation ②

$$e^{\frac{xt}{2}} \times e^{-\frac{x}{2t}} = \left[1 + \left(\frac{xt}{2}\right) + \left(\frac{xt}{2}\right)^2 + \right.$$

$$\left. + \left(\frac{xt}{2}\right)^3 + \cdots \left(\frac{xt}{2}\right)^{n-1} + \left(\frac{xt}{2}\right)^n\right] \times$$

$$\left[1 - \left(\frac{x}{2t}\right)^1 + \left(\frac{x}{2t}\right)^2 - \left(\frac{x}{2t}\right)^3 + \right.$$

$$\left. \cdots (-1)^{n-1}\left(\frac{x}{2t}\right)^{n-1} + \left(\frac{x}{2t}\right)^n (-1)^n\right]$$

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equating the coefficient of t^n on both experssion 1) expansion of $e^{\frac{xt}{2}}$ and 2) expansion $e^{-\frac{x}{2t}}$

$$e^{\frac{xt}{2}-\frac{x}{2t}} = \frac{1}{n!} - \frac{1}{(n+1)!}\left(\frac{x}{2}\right)^{n+2} + \frac{1}{(n+2)!}\left(\frac{x}{2}\right)^{n+4} + \cdots \Big] \quad (3)$$

we can directly write it as

$\therefore (n+1)! = (n+1)n!$

$$e^{\frac{xt}{2}-\frac{x}{2t}} = \sum_{r=0}^{\infty}\left[\frac{(-1)^0}{0!(n)!} + \frac{(-1)^1}{1!(n+1)!}\left(\frac{x}{2}\right)^{n+2} + \frac{(-1)^2}{2!(n+2)!}\left(\frac{x}{2}\right)^{n+4} + \cdots\right]$$

$$e^{\frac{xt}{2}-\frac{x}{2t}} = \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!(n+r)!}\left(\frac{x}{2}\right)^{n+2r}\right]$$

$$e^{\frac{xt}{2}-\frac{x}{2t}} = \sum_{r=0}^{\infty}\left[\frac{(-1)^r}{r!\,\Gamma(n+r+1)}\left(\frac{x}{2}\right)^{n+2r}\right] \quad (4)$$

$\therefore (n+r)! = \Gamma(n+r+1)$

$$e^{\frac{xt}{2}-\frac{x}{2t}} = t^n J_n(x) \quad (5)$$

where the

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Bessel's function $J_n(x)$ is

$$\boxed{J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,(n+r)!} \left(\frac{x}{2}\right)^{n+2r}}$$

from equation ④
we see that coefficient of t^n as $J_n(x)$

Hence

$$\boxed{e^{\frac{xt}{2} - \frac{x}{2t}} = \sum_{n=-\infty}^{\infty} t^n J_n(x)}$$

proved

Since, Generating function of Bessel's polynomial is equal to the coefficient ~~for~~ of the expansion of $J_n(x)$ Bessel function of first kind.

$$\boxed{e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{n=+\infty} t^n J_n(x)}$$

proved

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