Chhatrapati Shahu Ji Maharaj University, Kanpur

Answer Script Details
Barcode 5552429

Roll No. 24077000697
Total Mark 49/75.00

Exam MASTER OF SCIENCE _ODD EXAM-DEC-24
Subject B010704T - QUANTUM MECHANICS - I

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	4/5	8	NA/15				
1B	3/5	9A	NA/7				
1C	4/5	9B	NA/7				
1D	3/5						
1E	4/5						
1F	3/5						
1G	3/5						
1H	3/5						
1I	4/5						
2	NA/15						
3	11/15						
4	NA/15						
5A	NA/7						
5B	NA/7						
6	NA/15						
7A	4/7						
7B	3/7						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

Date of Exam: 25/01/25 Shift: I Room No. 24
Paper Code: B010704T Subject: QUANTUM MECHANICS Year/Sem: I/I
Name of Candidate: FARHEEN RAHMAN
Roll No. 2 4 0 7 7 0 0 0 6 9 7

Signature of Candidate: Farheen Rahman
Signature of Invigilator
COE Facsimile

PART-II

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures — Max. Marks
Total Marks in Words

B 0 1 0 7 0 4 T
Paper Code

Signature of Evaluator

Course: M.Sc
Session: 2024-25 Year/Semester: I/I
Subject Name: QUANTUM MECHANICS
Medium: English ● Hindi ○

Paper Code: B 0 1 0 7 0 4 T

Exam Date: 2 5 0 1 2 0 2 5

Name of Candidate: F A R H E E N R A H M A N

Father's Name: N A B I B U R R A H M A N

महाविद्यालय का कोड
College Code: K N 0 4

परीक्षा केंद्र का कोड
Exam Centre Code: K N 0 4

परीक्षा का प्रकार
Type of Exam

संस्थागत Regular ● पूर्व छात्र Ex-Student ○
व्यक्तिगत Private ○ बैक पेपर परीक्षा Back Paper Exam ○

ANSWER BOOKLET NO.

5552429

B 0 1 0 7 0 4 T
Paper Code

PART-IV

नामांकन संख्या
Enrollment Number: C S J M A 2 4 0 0 0 1 3 0 9 5 4

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number: 2 4 0 7 7 0 0 0 6 9 7

पेपर कोड Paper Code: B 0 1 0 7 0 4 T

Signature of Candidate: Farheen Rahman

Signature of Invigilator

C S Facsimile

COE Facsimile

नोट- 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पन्ने के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें। 2. कॉपी में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below, blacken the circles completely.

Correct Method	Incorrect Method
① ● ③ ④	✓ ⊗ ◉ ❹

4. Make no Stray marks on this sheet.

5. DO NOT WRITE OR MARK ON THE BAR CODE.

IN ORDER TO AVOD UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/electronic/digital/ watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु :

1. उत्तर पुस्तिका के निर्धारित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखे तथा कोई भी चिन्ह न बनाये क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ छाड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, डिजिटल वॉच, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लेस साइंटिफिक केल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

≑ ≑ ≑

परीक्षार्थियों को दिशा निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशों को ध्यान से पढ़ें।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठों पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र ID सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसकी परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेंसिल का प्रयोग न करें।
10. बी कॉपी या अतिरिक्त शाक नहीं दिया जायेगा।

INSTRUCTION TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper, Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-24) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subje Name, and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no . corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in status as Carry Over. Those appearing as Ex- Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9

Note- If your Roll No. is of 10 digits. Please leave first three columns .

≑ ≑ ≑

Paper Code

B 0 1 0 7 0 4 T

Section-A

Short Answer type Questions

Answer: 1(a)

Schrondinger Picture → In old assumption particle associated single wave train which is impossible. Schrondinger solved this problem by him statement that, " A material particle consist a wave packed not a single wave."

Shrondinger shows his representation for wave function of a system is called "schrondinger picture".

Property of Schrondinger Picture

He gives the following four properties -

1) **State vector are evolve time** :- In Quantum state vector $|\psi\rangle$ has evolves the time t of the system of particle

and $|\psi(t)\rangle \longrightarrow$ evolves time

so, $\hat{A}|\psi(t)\rangle = \lambda|\psi(t)\rangle$ —①

which gives a complex or real multiple with the state vector $|\psi\rangle$.

Paper Code

B0107047

2.) Operator has time independent.→

Schrondinger, operator has time independent means there are no involvement of time. operator is totally independent on time.

we can write. $\frac{\partial}{\partial t} \to 0$
when A is operator
of eigen value equation ①

3) Usefulness in Quantom mechanics→

The schrondinger picture is first picture in quantom mechanics which involve the time in wave function ψ.

4) Matrix Representation→ From this schrondinger picture is helpful in solving the waves function and quantum mechanics problem solve by the matrix representation.

Which is first introduce by Schrondinger and he proposed the Schrondinger equation in respect to the time.

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

which is time dependent and operator is time independent.

Do Not Write anything in this Portion

Answer : (b)

Linear Vector space → Let V be a set of vectors and field F be a set of scalars then 'V' is said to be linear vector space over the field F if V(F) satisfies properties-

1) Associative property → $a+v = a + v = v+a$
$a \in V$

2) Closure property → $u+v = c \in V$

3) Commutative property → $u+(v+w) = (u+v)+w$
$u, v, w \in V$

4) Inverse → $(u+(-u)) = 0$ $u \in V$

5) Identity → $u+(0) = u$ $u \in V$

If V(F) satisfied these property then we called V(F) is linear vector space.

Linear Operator → Linear operator is defined as, " If a operator $\hat{A}$ is said to be linear operator when an eigen state apply on this and operator $\hat{A}$ follows commutation property, is called linear operator

Let operator is $\hat{A}$ operate with two quantities α and β then we get

Do Not Write anything in this Portion

Paper Code
B 0 1 0 7 0 4 T

$$\boxed{\hat{A}(\alpha+\beta) = \alpha\hat{A} + \beta\hat{A}}$$

or.

$$\boxed{\hat{A}(\alpha+\beta)|\psi\rangle = \alpha\hat{A}|\psi\rangle + \beta\hat{A}|\psi\rangle}$$

Answer -(C)

Postulate of Quantum mechanics

1) Wavefunction → Any state of system is described by a wave function which gives the all infomation about the state of system of particle

$$\boxed{\psi = \psi_0 e^{-i\omega t}}$$

2) Operator → An operator is a rule by which we change one eigen state vector to another state vector.

$$\boxed{\hat{A}|\psi\rangle = |\phi\rangle}$$

where

$\hat{A}$ = operator

$|\psi\rangle$ = eigen state vector

$|\phi\rangle$ = new state

If the operator is hermitian then must be. It has real eigen value that's why we use hermitian Operator.

Paper Code B0107040

i.e. $\boxed{\hat{H}|\psi\rangle = \lambda|\psi\rangle}$

λ = real eigen value of corresponding state with the Hermitian operator

3) Expectation value → It is essential to find expectation value of of operator because wave function ψ is probabilistic interpretation.
It weighted by the propability
i.e. for dynamical quantity f which is function of r.

$$\hat{f} = \int P f \, d\tau$$

or

$$\boxed{\langle f(r)\rangle = \frac{\int_{-\infty}^{\infty} \psi^*(r,t) f(r) \psi(r,t) d\tau}{\int_{-\infty}^{\infty} \psi^* \psi \, d\tau}}$$

4) Shrondinger picture → Schrondinger was the first which derived the equation with relates to the time

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

These are the basic postulates of the Quantum mechanics.

Do Not Write anything in this Portion

Paper Code B 0 1 0 7 0 4 T

Answer -(e)

Dirac notation → Dirac gives the following two vectors such as ket and bra for the system of particle with the notation.

Dirac Bra notation → Let vector $\hat{A}$ and $\hat{B}$

$$\hat{A} = |a_1 e_1\rangle + |a_2 e_2\rangle + \cdots |a_n e_n\rangle \quad -①$$

$$\hat{B} = b_1 e_1\rangle + b_2 e_2\rangle + \cdots b_n e_n\rangle \quad -②$$

In brac notation shows by $\langle\alpha|$ and kets are like this $|\alpha\rangle$

bra belongs to Helbert space

$$\boxed{\langle \hat{A} | \hat{B} \rangle = \sum_i a_i^* \ldots} \quad -③$$

bra are rumese by row vector

$$\langle \hat{A} | = [a_1^* e_1 \quad a_2^* e_2 \; a_3^* e \cdots a_n^* e]$$

$$\langle B | = [b_1^* e_1 \quad b_2^* e_2 \quad b_3^* e_3 \cdots b_n^* e_3]$$

Dirac' ket notatation → ket notation basically belong to linear vector space.

Paper Code B 0 1 0 7 0 4 7

and it shows by column matrix

$$|A\rangle = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} \quad \text{and} \quad |B\rangle = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

~~for~~ from eq. ③

bra-ket $\langle \hat{A} | \beta \rangle = \sum_i a_i^* b_i$

$$\langle A | B \rangle = [a_1^* \quad a_2^* \; \ldots \; a_n^*] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

This representation is known as Dirac notation or bra-ket for corresponding vector $\hat{A}$ & $\hat{B}$

Answer: (f)

Hermitian Operator has real value

Let $\hat{H}$ be hermitian operator and
$\hat{A}$ is also hermitian operator
ψ is the eigen state
λ is eigen value

then
eigen value equation is

$$\hat{A} | \psi \rangle = \lambda | \psi \rangle \quad -①$$

Paper Code: B0107047

Take complex conjugate of eq^n ①

$$\hat{A}|\psi^*\rangle = \lambda^*|\psi^*\rangle \quad -②$$

we know

the condition of Hermitian operator

$$\int_{-\infty}^{\infty} \psi \hat{A} \psi \, d\tau = \int_{-\infty}^{\infty} \hat{A}^* \psi^* \psi \, d\tau \quad -③$$

must satisfied by operator $\hat{A}$

using eq^n ① and ②

we get

$$A|\psi\rangle = \lambda|\psi\rangle$$
$$A|\psi\rangle^* = \lambda^*|\psi\rangle^*$$

put in eq. ③

$$\int_{-\infty}^{\infty} \psi(\lambda|\psi\rangle) d\tau = \int_{-\infty}^{\infty} (\lambda^* |\psi^*\rangle)\psi \, d\tau$$

or

$$\int_{-\infty}^{\infty} [\lambda|\psi\rangle - \lambda^*|\psi^*\rangle] \psi \, d\tau = 0$$

or

$$\int_{-\infty}^{\infty} \lambda - \lambda^* \int \psi^* \psi \, d\tau = 0$$

since,

$$\int \psi^* \psi \, d\tau \neq 0$$

$$\lambda - \lambda^* = 0$$

$$\boxed{\lambda = \lambda^*}$$ proved

This only possible when eigen value is real so
Hermitian operator has real eigen value is proved.

Answer: (g)

Ladder operator → The Ladder operator is defined as. Linear momentum operator L is ladder operator but this is unique by its property.
Ladder operator increases or decreases the value of function.
They are two type.

a) Lowering ladder operator → It is represented by $\hat{L}_-$ and It has value is

$$\boxed{\hat{L}_- = \hat{L}_x - i\hat{L}_y}$$

b) Raising ladder operator → It is represented by $\hat{L}_+$
and value is →

$$\boxed{\hat{L}_+ = \hat{L}_x + i\hat{L}_y}$$

Commutation relation with $\hat{L}_z$ →

$$[L_z, L_+] = [xP_y - yP_x, L_x + iL_y]$$

Paper Code B 0 1 0 7 0 4 T

Do Not Write anything in this Portion

$= [xP_y, L_x + xP_y, L_y] - [yP_x, L_x] - [yP_x, iL_y]$

$= x[L_x P_x] + [x, P_y]L_x + x[P_y L_y] + [xL_y]P_y$
$\downarrow 0$ $\downarrow 0$

$- [y, P_x]L_x - [y, L_x]P_x$

$- [y, P_x]iL_y - [y, iL_y]P_x$

$\therefore [AB, C] = A[B, C] - [A, C]B$

$= i\hbar z L_x + i\hbar z P_y - i\hbar z L_x - iL_y\hbar$
$i\hbar L_z P_y + 0 - P_x L$

$= \hbar L_z$

Answer (h)

To Show momentum operator $= \frac{\hbar}{i}\frac{\partial}{\partial x}$

momentum $P = m \times v$ —(1)

$P = m \times \frac{dx}{dt}$

$\therefore T = \frac{1}{2}mv^2$

$2T = mv^2$

$\frac{2T}{v} = mv$

$\frac{\frac{-\hbar^2}{2m}\nabla^2}{v} = mv$ —(2)

Then from eq. ② kinetic energy

$$\text{momentum} = -\frac{\hbar^2}{m}\frac{m}{\hbar}\frac{\partial}{\partial x}\, i$$

$$= -i\hbar\frac{\partial}{\partial x} \quad \text{or}$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\boxed{\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}} \quad \text{proved.}$$

Answer (i)

Symmetric wave functions Pauli introduce the symmetry of the system related to spin by his pauli matrix

Boson → Boson has symmetric wave function and integral spin $s = n\hbar \quad (n = 0, 2, 3 \cdots)$

Boson follow 'Bose Einstein statistic'.

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}[\uparrow\downarrow + \downarrow\uparrow]}$$

This is symmetric wave function for Boson

for $1s^2$ has $n=1$
$J=0$
$m=0$
$s=1/2$

ex → Photon
H-atom

Do Not Write anything in this Portion

Paper Code
B	0	1	0	7	0	4	T

Antisymmetric wave function→

Fermions has antin symmetric wave function which follow Fermi Dirac statistics and it has half and odd $(n+\frac{1}{2})$ integer spin

$$|\psi\rangle = \frac{1}{\sqrt{2}}[\uparrow\downarrow - \downarrow\uparrow]$$

Antisymmetric wave function for Fermions

ex→ electron, proton.

Answer (d)

Expectation value → According to Born, " wave function ψ has probabalistic interpre- -tation so it is essential to calculate the expectation value of dynamical quantity for the system."

Statement, " expectation value of dynamical quantity is the average value weighted by the system."

f is dynamical quantity

then $\bar{f} = \int P f \, d\tau$

or f is function of r

$$\boxed{\langle \bar{f}(r) \rangle = \frac{\int_{-\infty}^{\infty} \psi^*(r,t) \, f(r) \, \psi(r,t) \, d\tau}{\int_{-\infty}^{\infty} \psi^* \psi \, d\tau}}$$

for position

$$\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \psi^* \, x \, \psi \, d\tau$$

for momentum

$$\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi \, d\tau$$

Paper Code B 0 1 0 7 0 4 T

Section - B

Ehrenfest's Theorem

Ehrenfest Theorem is the bridge between classical (Newton law) mechanics and Quantum mechanics.

Statement's, " Schrondinger equation of motion (quantum mechanics) leads to the Classical Newton's law of motion"

Schrondinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

Newton's second law of motion

$$F_x = \frac{dP_x}{dt}$$

From this we find two relation related for these mechanics

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p_x\rangle}{m} \quad -①$$

$$\frac{d\langle P_x\rangle}{dt} = -\nabla V \quad -②$$

Paper Code B0107047T

proof : $\frac{d\langle x\rangle}{dt} = \frac{\langle P_x\rangle}{m}$

expectation value of x

$$\langle x\rangle = \int_{-\infty}^{\infty} \psi^* x \psi \, d\tau \quad -①$$

differentiate w.r.t. t, we get

$$\frac{d\langle x\rangle}{dt} = \frac{\partial}{\partial t}\int_{-\infty}^{\infty} \psi^* x \psi \, d\tau$$

$$\frac{d\langle x\rangle}{dt} = x\int_{-\infty}^{\infty} \frac{\partial}{\partial t} \psi^* \psi \, d\tau \quad -②$$

$[\because d\,\mathrm{I}\,\mathrm{II} = \mathrm{I}\, d\,\mathrm{II} + \mathrm{II}\, d\,\mathrm{I}]$

$$\frac{d\langle x\rangle}{dt} = x\int_{-\infty}^{\infty}\left[\psi^* \frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi^*}{\partial t}\right] d\tau \quad -③$$

from $\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial \psi}$ $-④$

$$\frac{-\hbar^2}{2m}\nabla^2\psi^* + V\psi^* = -i\hbar\frac{\partial \psi^*}{\partial t} \quad -⑤$$

eqn ④ and ⑤ put in ③
we get

$$\frac{d\langle x\rangle}{dt} = -\frac{\hbar}{2im}\int x \frac{d}{dx}\left[\psi^*\frac{\partial \psi}{\partial x} - \psi\frac{\partial \psi^*}{\partial x}\right] d\tau \quad -⑥$$

Do Not Write anything in this Portion

Paper Code
B010704T

on integration

$\therefore \int f_1 f_2 = f_1 \int f_2 dx - \int [df_1 \int f_2 dx] dx$

we get

$$= \frac{\hbar}{2m} \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} d\tau - \hbar \int_{-\infty}^{\infty} \psi \frac{\partial \psi^*}{\partial x} d\tau \quad -(7)$$

we know

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \qquad \therefore \langle p \rangle = \int_{-\infty}^{\infty} \psi^* \left(\frac{-i\hbar \partial \psi}{\partial x} \right) d\tau$$

$$p\psi = -i\hbar \frac{\partial \psi}{\partial x} \quad -(8)$$

and

$$p\psi^* = +i\hbar \frac{\partial \psi^*}{\partial x} \quad -(9)$$

put (8) and (9) in (7), we get

$$\frac{d\langle x \rangle}{dt} = \frac{\hbar}{2m} \left[\frac{\langle p_x \rangle}{-i\hbar} - \frac{\langle p_x \rangle}{i\hbar} \right]$$

$$\frac{d\langle x \rangle}{dt} = \frac{-2 \langle p_x \rangle}{i^2 2m}$$

$$\boxed{\frac{d\langle x \rangle}{dt} = \frac{\langle p_x \rangle}{m}}$$

hence proved (1)

Paper Code B01 07 04 T

proof :② $\frac{d\langle P_x \rangle}{dt} = -\nabla V$ —①

Let particle of mass m moving in space with velocity V_x by force F_x then ~~Fx~~ then expection value of momentum is

$$\langle P_x \rangle = \int_{-\infty}^{\infty} \psi^* \hat{P} \psi \, d\tau \quad —②$$

$$\langle P_x \rangle = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial \psi}{\partial x}\right) d\tau$$

differentiate w.r.t. t, we get

$$\frac{d\langle P_x \rangle}{dt} = -\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \psi^* (i\hbar) \frac{\partial \psi}{\partial x} \, d\tau \quad —③$$

$$= -i\hbar \int \frac{\partial}{\partial t} \psi^* \frac{\partial \psi}{\partial x} \, d\tau$$

$$= -i\hbar \left[\int \psi^* \frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial \psi}{\partial x} \frac{\partial \psi^*}{\partial t} \, d\tau \right.$$

$$= -i\hbar \int \psi^* \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right) d\tau - \int \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} (-i\hbar) d\tau \quad —④$$

putting

$$\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{i\hbar\, 2m} \nabla^2 \psi + \frac{V\psi}{i\hbar} \quad —⑤$$

$$\frac{\partial \psi^*}{\partial t} = \frac{-\hbar}{-i2m} \nabla^2 \psi^* + \frac{V\psi^*}{-i\hbar} \quad —⑥$$

put ⑤ and ⑥ in eq. ④ , we get

$$\frac{d\langle P_x\rangle}{dt} = \psi^* \int \left(\frac{\partial V}{\partial x} \psi - \psi \frac{\partial \psi}{\partial x} \right) d\tau \quad -⑦$$

we know that $\int \psi^* \psi \, d\tau = 1$
for momalised

since,

$$\frac{d\langle P_x\rangle}{dt} = \int \psi^* \psi (- dV) d\tau \quad -⑧$$

$$\boxed{\frac{d\langle P_x\rangle}{dt} = -\nabla V} \quad -⑨$$

tence proved.

we say that from equation

$$\frac{d\langle x\rangle}{dt} = \frac{\langle P_x\rangle}{m} \quad \text{and}$$

$$\frac{d\langle P_x\rangle}{dt} = -\nabla V$$

classical and quantom gives same vresult for large system of pouticle

Paper Code B0107047

Section: C

(b) $[J^2, J_x] = [J_x^2 + J_y^2 + J_z^2, J_x]$

$= J_x[J_x, J_x] + J_y[J_y, J_x] +$

$J_z[J_z, J_x]$

$\because [AB, C] = A(B,C) + (A,C)B$

$= J_x(J_x, J_x] + [J_x, J_x]J_x +$

$J_y[J_y, J_x] + [J_y, J_y]J_x +$

$[J_z, J_z]J_x + J_z[J_z, J_x]$

$\therefore [J_z, J_z] = 0$

So,

$= 0 + 0 + 0 + 0 + 0 + 0$

$[J^2, J_x] = 0$

Similarly

$[J^2, J_y] = 0$

$[J^2, J_z] = 0$ hence prove.

Do Not Write anything in this Portion

Paper Code B 0 1 0 7 0 4 T

Section -C

Long Questions

Clebsch- Gorden Coefficient

Clebsch- gorden coefficient are base of quantom mechanics. which deffine a.

Let we have two non interacting system having angular momonatom J_1 with its m_1

$$J_1 \rightarrow m_1\, m_1$$

$$J_2 \rightarrow m_2\, m_2$$

for paurticle excited state then J_1 becomes

posibble value of $J_1 \rightarrow -J_1$ to $+J_1$

posible value of $J_2 \rightarrow -J_2$ to $+J_2$

for m

$$J_1 = -J_1 \text{ to } +J_1$$

$$m_1, m_1 \rightarrow -m_1 \text{ to } +m_1$$

and for J_2

Paper Code 8010 7047

21

$J_2 = -J_2$ to J_2

$m_1, m_2 = -m_1$ to $+m_2$

if particle has J we can find value of m * so we can write it as

$$J_1|m_1 m_2\rangle = J_1(J_1+1)|m_1 m_2\rangle *$$

$$J_2|m_1 m_2\rangle = J_2(J_2+1)|m_1 m_2$$

$J_{Jmm_1m_2}$ we can write for couple state.

Clebsch - Gordan coefficient has two write for

coupled kets $|m_1 m_2\rangle$ also called known ket

or

uncoupled kets $|J, M\rangle$ also called unknow ket

we can write it

$$\boxed{|J\ m\rangle = \sum_{m_1 m_2} \langle m_1 m_2|JM\rangle |m_1 m_2\rangle}$$

or.

$$\boxed{|J\ m\rangle = \sum_{m_1 m_2} C_{Jmm_1m_2} |m_1 m_2\rangle}$$

This is required clebsch gorden coefficient.

Do Not Write anything in this Portion

Paper Code 8 0 1 0 7 0 4 T

example → Let $j' = 1/2$, $j = 1/2$

uncouple kets are

$|m_1 m_2\rangle = |1/2, 1/2\rangle \quad |1/2, -1/2\rangle \quad |-1/2, 1/2\rangle$
$|-1/2, -1/2\rangle$

$m = m_1 + m_2$

$|m_1 m_2\rangle = 1 \quad 0 \quad 0 \quad -1$

maximum $m = 1$
minimum $m = -1$

possible value of J

$J = 1/2 = J_1 + J_2 = 1 \Rightarrow m = 1, 0, -1$

$J = 1/2 = |J_1 - J_2| = 0 \Rightarrow m = 0$

so,
Coupled ket are possible as

$|J, m_1\rangle = |1, 0\rangle \quad |1, -1\rangle \quad |1, 1\rangle$
$|0, 0\rangle$

we won't wheau compihalim
for this by help of

Paper Code 80107047

Clebsch Gordan coefficies

$$|Jm\rangle = \sum_{Jm} \langle m_1 m_2 | Jm\rangle |m_1 m_2$$

$$\begin{bmatrix} |1,1\rangle \\ |1,0\rangle \\ |1,-1\rangle \\ |0,0\rangle \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \begin{bmatrix} |1/2, 1/2\rangle \\ |1/2, -1/2\rangle \\ |-1/2, 1/2\rangle \\ |-1/2, -1/2\rangle \end{bmatrix}$$

unless $m = m_1 + m_2$ all coffient are zero

and $a_1 = d_4 = 1$

$b_2 = 1/\sqrt{3}$, $b_3 = 2/\sqrt{3}$

$c_2 = 2/\sqrt{3}$, $c_3 = -1/\sqrt{3}$

so we can write it

$$\text{Required matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} & 0 \\ 0 & \frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad |-1/2, 1/2$$

for $j_1 = 1/2$ $j_2 = 1/2$
which show with CG coefficient

Do Not Write anything in this Portion

Paper Code

B	0	1	0	7	0	4	T