Chhatrapati Shahu Ji Maharaj University, Kanpur

Answer Script Details
Barcode 4852223

Roll No. 23081000411
Total Mark 45/75.00

Exam BACHELOR OF SCIENCE_DEC-2023
Subject B030101T - DIFFERENTIAL CALCULUS AND INTEGRAI

Question wise Mark Summary

Q.No	Mark	Q.No	Mark	Q.No	Mark	Q.No	Mark
1A	0/6						
1B	4/6						
1C	4/6						
1D	4/6						
1E	0/6						
1F	0/6						
1G	0/6						
1H	4/6						
1I	0/6						
2	8/12						
3	0/12						
4	0/12						
5	5/12						
6	8/12						
7	0/12						
8	8/12						
9	0/12						

Chhatrapati Shahu Ji Maharaj University
Kanpur, Uttar Pradesh

PART-II

Date of Exam: 10/01/24 Shift: I Room No.: 31
Paper Code: B030101T Subject: mathematics Year/Sem: I
Name of Candidate: Sneha Shukla
Roll No. 2 3 0 8 1 0 0 0 4 1 1

Signature of Candidate: Sneha
Signature of Invigilator
COE Facsimile

MARKS OBTAINED

Q.	1	2	3	4	5	6	7	8	9	10
(a)										
(b)										
(c)										
(d)										
(e)										
(f)										
(g)										
(h)										
(i)										
(j)										
Total										

Total Marks in Figures | Max. Marks
Total Marks in Words

B 0 3 0 1 0 1 T
Paper Code

Signature of Evaluator

Course: B.Sc.
Session: 23-24 Year/Semester: I
Subject Name: mathematics
Medium: English ● Hindi ○

Paper Code: B 0 3 0 1 0 1 T

Exam Date: 1 0 0 1 2 0 2 4

Name of Candidate: S N E H A S H U K L A

Father's Name: A T U L K U M A R

महाविद्यालय का कोड
College Code: A U 0 3

परीक्षा केन्द्र का कोड
Exam Centre Code: A U 0 3

परीक्षा का प्रकार
Type of Exam
संस्थागत Regular ● भूतपूर्व छात्र Ex-Student ○
व्यक्तिगत Private ○ बैक पेपर परीक्षा Back Paper Exam ○

ANSWER BOOKLET NO.
4852223

B 0 3 0 1 0 1 T
Paper Code

नामांकन संख्या
Enrollement Number: C S J M A 2 3 0 0 0 0 0 3 8 5 4

परीक्षार्थी अनुक्रमांक संख्या Candidate's Roll Number: 2 3 0 8 1 0 0 0 4 1 1

पेपर कोड Paper Code: B 0 3 0 1 0 1 T

Signature of Candidate: Sneha

Signature of Invigilator

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नोट- 1. परीक्षार्थी को निर्देशित किया जाता है कि आवरण पन्ने के पृष्ठ भाग पर अंकित सभी निर्देशों को सावधानी पूर्वक पढ़ें।
2. गोलाकार में भरी जाने वाली प्रविष्टियाँ बायीं तरफ से शुरू की जायें। 3. गोलों को काले या नीले बॉलपेन से भरा जायें।

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-I

1. Read the instructions carefully given on the answer script and admit card.
2. Write Date of Exam, Shift, Paper Code & Name of Subject Correctly.
3. Write Name & Roll No. Correctly.
4. Write Semester & Branch Correctly.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-III

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ boxes.
2. Carefully study the example before you start marking.
3. As shown in the example below, blacken the circles completely.

Correct Method	Incorrect Method
① ● ③ ④	✓ ✗ ◉ ⊙

4. Make no Stray marks on this sheet.

5. DO NOT WRITE OR MARK ON THE BAR CODE.

IN ORDER TO AVOD UFM (UNFAIR MEANS) :

1. The Roll No. and Answer Book no. found elsewhere or any other symbol found in the answer book will be treated as unfair means.
2. Any tempering of Bar Code and Booklet no shall be treated as Unfair Means.
3. Do Not bring the materials like slip of paper/mobile/digital diaries/ study material/ revision notes in examination hall. Possession of the mobiles/ digital diaries/electronic/digital/ watch and any other electronic gadget except memory less scientific calculator shall be considered as UFM case.
4. Do not keep or paste currency note in answer script it shall be consider as UFM.

अनुचित साधन से बचने हेतु :

1. उत्तर पुस्तिका के निर्देशित स्थान को छोड़कर अनुक्रमांक एवं उत्तरपुस्तिका का क्रमांक कहीं और न लिखे तथा कोई भी चिन्ह न बनायें क्योंकि यह अनुचित साधन प्रयोग की परिधि में आता है।
2. उत्तर पुस्तिका के बारकोड अथवा उत्तर पुस्तिका संख्या पर छेड़ छाड़ करने पर अनुचित साधन प्रयोग माना जायेगा।
3. परीक्षा कक्ष में निम्न वस्तुएं साथ न लायें, जैसे लिखे हुए कागज के टुकड़े, मोबाईल, डिजिटल डायरी, डिजिटल वॉच, कॉपी, पुस्तक यह सभी वस्तुएं जो अनुचित साधन के अन्तर्गत आती है। केवल संबंधित प्रश्नपत्र में ही मेमोरी लेस साइंटिफिक कैल्कुलेटर ले जाने की अनुमन्यता होगी।
4. उत्तर पुस्तिकाओं में रुपये न रखें न ही उत्तर पुस्तिका में चिपकायें। ऐसा करना अनुचित साधन प्रयोग की परिधि में आता है।

परीक्षार्थियों के लिए निर्देश

1. प्रवेश पत्र एवं उत्तर पुस्तिका पर दिये गये निर्देशो को ध्यान से पढ़े।
2. कवर पृष्ठ के दूसरी तरफ कुछ न लिखें।
3. उत्तर पुस्तिका के पृष्ठो पर दोनों तरफ लिखें।
4. प्रश्न पत्र पर अपने अनुक्रमांक के अतिरिक्त कुछ न लिखें।
5. प्रश्न पत्र कोड एवं प्रश्न पत्र ID सावधानी पूर्वक लिखें।
6. अपनी स्थिति स्पष्ट लिखें।
7. उत्तर पुस्तिका के पृष्ठों की संख्या देखें। अगर उत्तर पुस्तिका में पृष्ठ (1-24) से कम है या फटे हुए है, तो परीक्षा शुरू होने के पूर्व दूसरी उत्तर पुस्तिका ले लें।
8. प्रश्नपत्र को देख, यदि प्रश्नपत्र के विषय कोड, विषय का नाम तथा प्रश्न में कोई त्रुटि है तो उसके परीक्षा शुरू होने के 30 मिनट के अन्दर कक्ष निरीक्षक को तत्काल सूचित करें, उसके बाद विश्वविद्यालय द्वारा कोई कार्यवाही नहीं की जायेगी।
9. प्रश्नों के उत्तर लिखने के लिये पेंसिल का प्रयोग न करें।
10. बी कॉपी या अतिरिक्त ग्राफ नहीं दिया जायेगा।

INSTRUCTION TO THE CANDIDATE

1. Read the instructions carefully given on the Question Paper, Admit Card & Answer Script.
2. Do not write anything on back side of the cover page.
3. Write on both sides of pages of answer book.
4. Do not write anything on question paper except Roll Number.
5. Write Paper Code & Question Paper Id carefully.
6. CHECK the number of pages (1-24) or any other kind of damage in your answer script, if found than change the answer script immediately before the commencement of examination.
7. CHECK the Question Paper for any kind of discrepancy e.g. Subject Code, Subject Name, and Question of the Question Paper during first THIRTY MINUTES of the commencement of the exam, so that it can be corrected in TIME. After that no corrections shall be entertained by the university.
8. Do not use pencil for answering the question.
9. Write status correctly e.g. those appearing in carry over papers should fill in status as Carry Over. Those appearing as Ex- Students should fill in status as ex.
10. No supplementary answer book & graph paper will be provided.

INSTRUCTION TO THE CANDIDATE FOR FILLING PART-IV

1. Use blue or black ball point pen for writing alphabets & numerals in ☐ Boxes.
2. Use blue or black ball point pen for filling the circles.

	1	8	1	5	4	3	2	1	6	9
0	0	0	0	0	0	0	0	0	0	0
1	●	1	●	1	1	1	1	●	1	1
2	2	2	2	2	2	2	●	2	2	2
3	3	3	3	3	3	●	3	3	3	3
4	4	4	4	4	●	4	4	4	4	4
5	5	5	5	●	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	●	6
7	7	7	7	7	7	7	7	7	7	7
8	8	●	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	●

Note- If your Roll No. is of 10 digits. Please leave first three columns .

Section A

Answer - 1 (B)

$\sum_{n>2}^{\infty} \frac{1}{n(\log n)^P}$ Test the convergence of the series

by Cauchy condensation formula

$$\lim_{n\to\infty} m^n f(m^n) = m^n \frac{1}{m^n (\log m^n)^P}$$

$$= \frac{1}{(\log m^n)^P}$$

$$= \frac{1}{n^P (\log m)^P}$$

by Auxiliary series

if $P>1$, series is convergent

if $P<1$, series is divergent

$P=1$, series is divergent

by Ratio and Rabe's test

this series is divergent because $n<1$

so $\frac{1}{n(\log n)^P}$ is divergent and if $n>1$,

$\frac{1}{n(\log n)^P}$ is convergent

Do Not Write anything in this Portion

Answer - 1 (c)

Evaluate

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$$

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right), \ (\infty-\infty)$$

$$\lim_{x\to 0}\left(\frac{\sin^2 x - x^2}{x^2\sin^2 x}\right), \ \left(\frac{0}{0}\right)$$

by L'Hospital rule

$$\lim_{x\to 0}\frac{2\sin x\cos x - 2x}{x^2\cdot 2\sin x\cos x + \sin^2 x\cdot 2x} \cdot \left(\frac{0}{0}\right)$$

again. L'Hospital rule

$$\lim_{x\to 0}\frac{2[\sin x(-\sin x)+\cos x\cdot\cos x]-2}{x^2\cdot 2(\sin x(-\sin x)+\cos x\cdot\cos x)+2\sin x\cos x\, 2x + \sin^2 x\cdot 2 + 2x\cdot 2\sin^2 x\cos x}$$

put $x=0$

$$\frac{2[0+1]-2}{0+0}$$

$$= \frac{2-2}{0} \cdot \left(\frac{0}{0}\right)$$

again L'Hospital rule

$$\lim_{x\to 0}\frac{2[\cos^2 x-\sin^2 x]-2}{x^2\cdot 2(\cos^2 x-\sin^2 x)+4x\sin x\cos x+2\sin^2 x + 4x\sin^2 x\cos x}$$

again L' Hospital rule

$$\lim_{x\to 0} \frac{8[-\sin 2x] - 0}{-2\sin 2x \cdot 2x + 4x(\sin^2 x - \cos^2 x) + 8\sin x\cos x + 8\sin x + \ldots(\sin^3 x - \cos x^2 \cdot 2\sin^2 x) + 8\sin^2 x\cos x^{\cos x}} \quad \left(\frac{0}{0}\right)$$

again L' Hospital rule

$$\lim_{x\to 0} \frac{-8\cos 2x}{-8\cos 2x}$$

put $x=0$

$= \frac{-8}{}$ ... $-1/3$ Ans

Answer-1(E)

$y^3 + x^2y + 2xy^2 - y + 1 = 0$, find the asymptotes

put $x=1$, $y=m$

$m^3 + m + 2m^2 - m + 1 = 0$

$\phi_3(m) = m^3 + m + 2m^2$

$\phi_2(m) = 0$

$\phi(m) = -m$

highest degree of m

$m^3 + 2m^2 + m = 0$

$m(m^2 + 2m + 1) = 0$

$m = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m+2)^2 = 0$, $m = -2$

$(m+2)(m+2) = 0$

Do Not Write anything in this Portion

Paper Code: 0030 1017

$m=0, \quad m=-2$

$c = \dfrac{-\phi_{n-1}(m)}{\phi'_n(m)}$

put $n=3$

$= -\dfrac{-\phi_2(m)}{\phi'_3(m)} = 0$

$c=0$

$y = mx + c$

$y = -2x + 0$

$\boxed{2x + y = 0}$

put $m=0$

$c = \dfrac{-\phi_{n-1}(0)}{\phi'_{n-1}(0)} = 0$

$y = mx + c$

$\boxed{y = 0}$

cal ~~high~~ ~~degree of~~ y

$\dfrac{~~2xy^2 - y~~}{y} = 0 \qquad n=2$

$= \dfrac{c^2}{\lfloor 2}\phi''_{n+1} + c\phi'_n + \phi_{(n-1)} = 0$

$= \dfrac{c^2}{\lfloor 2}(6m+4) + 0 + (-m) = 0 \qquad | \; \phi''_3 = 6m+4$

$\dfrac{c^2}{1}(6\times(-2)+4) + 2 = 0$

$c^2(-8) + 2 = 0 \qquad | \; c^2 = \dfrac{-1}{4}, \quad c = \dfrac{-1}{2}$

$8c^2 + 2 = 0$

$\boxed{y = -2x - \frac{1}{2}}$

Paper Code B 0 3 0 1 0 1 T

5

Answer - 1 (H)

$\vec{r} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$

Show that

$$\int_1^2 \vec{r} \times \frac{d^2r}{dt^2}\, dt = -14\hat{i} + 75\hat{j} - 15\hat{k}$$

$$\frac{dr}{dt} = 10t\hat{i} + \hat{j} - 3t^2\hat{k}$$

$$\frac{d^2r}{dt^2} = 10\hat{i} - 6t\hat{k}$$

$$\left[\vec{r} \times \frac{d^2r}{dt^2}\right] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5t^2 & t & -t^3 \\ 10 & 0 & -6t \end{vmatrix}$$

$$= \hat{i}(-6t^2+0) - \hat{j}(-30t^3+10t^3) + \hat{k}(0-10t)$$

$$= -6t^2\hat{i} - 40t^3\hat{j} - 10t\hat{k}$$

$$\int_1^2 \vec{r} \times \frac{d^2r}{dt^2}\, dt = \int_1^2 (-6t^2\hat{i} + 20t^3\hat{j} - 10t\hat{k}) \cdot dt$$

$$= \left[-\frac{6t^3}{3}\hat{i} + \frac{20t^4}{4}\hat{j} - \frac{10t^2}{2}\hat{k}\right]_1^2$$

$$= -2(8-1)\hat{i} + 5(16-1)\hat{j} - 5(4-1)\hat{k}$$

$$= \boxed{-14\hat{i} + 75\hat{j} - 15\hat{k}}$$

Do Not Write anything in this Portion

Paper Code: B030J0JT

Section (B)

Part I.

Answer - 2(a)

$x_n = \left(1+\frac{1}{n}\right)^n$ is convergent and show that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ lies between 2 & 3

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$= \lim_{n\to\infty}(a_n) = a_n = \left(1+\frac{1}{n}\right)^n = \left(\frac{n+1}{n}\right)^n$

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{1+\frac{1}{n+1}}{1+\frac{1}{n}} = \frac{\frac{n+1+1}{n+1}}{\frac{n+1}{n}}$

$a_{n+1} = \left(\frac{n+2}{n+1}\right)^{n+1} = \frac{(n+2)n}{(n+1)^2}$

$= \frac{n^2\left(1+\frac{2}{n}\right)}{n^2\left(1+\frac{1}{n}\right)}$

$= \frac{1}{1}$

$\lim_{n\to\infty}\frac{\left(\frac{n+2}{n+1}\right)^{n+1}}{\left(\frac{n+1}{n}\right)^n} = \frac{(n+2)^{n+1}\cdot n^n}{(n+1)^{n+1}(n+1)}$

$= \frac{(n+2)^n(n+2)n^n}{(n+1)^n(n+1)^2}$

$n>1$ $= \frac{n^n\left(1+\frac{2}{n}\right)^n\left(1+\frac{1}{2}\right)\cdot n^n}{n^n\left(1+\frac{1}{n}\right)n\left(1+\frac{2}{n}\right)^2}$

So series is convergent

e

because e lies between 2 and 3

$2 < e < 3$

$x_n = \left(1+\frac{1}{n}\right)^n$ is convergent and e lies between 2 and 3

Answer - 2 (b)

Absolutly Convergent :-

If f(x) is convergent its |f(x)| is also convergent then this series is also absolutely convergent.

Ex. $e^x \sin \frac{1}{x^2}\, dx$

$\phi(x) = e^x$ is bounded and monotonic

$f(x) = \sin\frac{1}{x^2} < \frac{1}{x^2}$

$n = 2 \quad n > 1$

so f(x) is convergent

and $|f(x)| = \left|\sin\frac{1}{x^2}\right| < \left|\frac{1}{x^2}\right|$

$n > 1$

|f(x)| is also ~~conv~~ absolutely convergent

Conditionally Convergent :-

If f(x) is convergent but its mode |f(x)| is divergent so the convergent is called conditionally convergent

Q. - $\int_0^\infty \frac{\sin x}{x}$

$\int_0^\infty \frac{\sin x}{x}$, $\phi(x) = \sin x$ is bounded because its improper integral and monotonic but its $\left|\frac{1}{x}\right|$, $n<1$ so it is divergent

Part - II

Answer- 5 (a)

The improper integral $\int_0^\infty \frac{\sin x}{x} dx$ is convergent but not absolute convergent

$\int_0^\infty \frac{\sin x}{x} dx$

It is improper integral, limit infinte integrand unbounded

by Abel's test

$\phi(x) = \sin x$,

$\sin x$ is bounded and monotonic

and $f(x) = \frac{1}{x}$.

by Auxiliary series

$n<1$, series is convergent

But by Absolute test

$\left|\frac{1}{x}\right| = \frac{1}{x^2}$, $n>1$ so

$|f(x)|$ is divergent

so $\int_0^\infty \frac{\sin x}{x} dx$ is convergent but not absolute convergent.

Answer - 1 (b)

$$\int_0^{\pi/2} \cos^m \sin^n \theta \, d\theta = \frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\Gamma\frac{m+n+2}{2}}$$

$$m = \int_0^{\pi/2} \cos^q \theta \sin^p \theta \, d\theta = \frac{\Gamma\frac{p+1}{2}\Gamma\frac{q+1}{2}}{2\Gamma\frac{p+q+2}{2}}$$

$$B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

let $x = \sin^2\theta$ $y = \cos^2\theta$

$dx = 2\sin\theta\cos\theta \, d\theta$

$$\int_0^{\pi/2} \sin^{2m-2}\theta \cdot (1-\sin^2\theta)^{n-1} \cdot 2\sin\theta\cos\theta \, d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-2}\theta \cdot \sin\theta \cdot \cos^{2n-2}\theta \cdot \cos\theta \, d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2m-1}\theta \, d\theta$$

= put $m = 1/2$

$$= 2\int_0^{\pi/2} \sin^0\theta \cos^{2n-1}\theta \, d\theta = B(m,n)$$

$$2\int_0^{\pi/2} \cos^{2n-1}\theta \, d\theta = \frac{\Gamma m \Gamma n}{\Gamma m+n} = \frac{\Gamma [illegible] \Gamma n}{\Gamma m+n}$$

$$= \frac{\Gamma\frac{1}{2} \Gamma n}{\Gamma n+\frac{1}{2}}$$

Do Not Write anything in this Portion

Paper Code: 8 0 3 0 1 0 1 T

put $m = n$

$$2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2m-1}\theta \, d\theta = \frac{\Gamma m \Gamma m}{\Gamma 2m}$$

$$= \int_0^{\pi/2} \sin^{2m-1} 2\theta \, d\theta$$

let $2\theta = t$

$d\theta = \frac{1}{2} dt$ $= \frac{\Gamma m \Gamma m}{\Gamma 2m}$

$$\frac{1}{2}\int_0^{\pi} \sin^{2m-1} t \, d$$

by property IV

$$= \int_0^{\pi/2} \sin^{2m-1} t \, dt$$

let by property I

$$\int_0^{\pi/2} \sin^{2m-1}\theta \, d\theta$$

$$2I = \int_0^{\pi/2} \sin^{2m-1}\theta \ldots \theta \, d\theta = \frac{\Gamma m \Gamma m}{\Gamma 2m}$$

= put $2m-1 = p$, $2n-1 = q$

$m = \frac{p+1}{2}$ $n = \frac{q+1}{2}$

$$2\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2} \Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}}$$

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2} \Gamma\frac{q+1}{2}}{2\Gamma\frac{p+q+2}{2}}$$

for all value of $p > -1$, $q > -1$

Paper Code
B 0 3 0 1 0 J T

Section 'c'

Part 'I'

Ans - 6 (a)

State and Prove Lagrange's mean Value theorem

Statement :-

If f(x) is the function

(i) f(x) is continuous for [a, b]
f(x) is differentiable for open enterval (a, b).
Then, this is any one unique

f'

$$\frac{f(b) - f(a)}{b-a} = f'(c)$$

Proof :-

let
$\phi(x) = Kx + f$
let there are two point (a, b)
for (a), by Rolle's theorem -
$\phi(a) = \phi(b)$

$K(a) + f(a) = K(b) + f(b)$
$K(b) - K(a) = f(b) - f(a)$
$K(b - a) = f(b) - f(a)$
$K = -\left[\frac{f(b) - f(a)}{b-a}\right]$

$\because \quad f'(c) > 0$

$K > 0$

~~K~~ = ~~f(c(~~

$$f'(c) = \frac{f(b)-f(a)}{(b-a)}$$

langrange meanValue theorem

$$= \boxed{\frac{f(b)-f(a)}{b-a} = f'(c)}$$

Geometrical Interpretation

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

This theorem is only satisfied by close and open interval [a,b] (a,b) given in the function $f(x)$

Answer 6 (b)

$$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$$

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + 2n^2 y_n = 0$$

$$\frac{y}{b} = \cos \log\left(\frac{x}{n}\right)^n$$

$$y = b \cos \log\left(\frac{x}{n}\right)^n$$

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$$\frac{dy}{dx} = y_1 = -b\sin n \log\left(\frac{x}{n}\right) \cdot \frac{1}{n}$$

$$y_2 = x^2 b \cos n \frac{1}{n^2}$$

$$x^2 y_2 + x y_1 + y = 0$$

$$x^2 y_{n+2} + n y_{n+1} 2x + {}^nC_2 y_n 2 + x y_{n+1} + n y_n + y_n = 0$$

$$x^2 y_{n+2} + n y_{n+1}(2x) + \frac{n(n-1)}{2} y_n 2 + x y_{n+1} + n y_n + y_n = 0$$

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + [(n^2-n) + n^2 + n] y_n = 0$$

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + 2n^2 y_n = 0$$

$$\boxed{x^2 y_{n+2} + (2n+1) x y_{n+1} + n y_n = 0}$$

Part II

Answer- Q (a)

$r = a(1+\cos\theta)$ revolves about the initial line. Find the volume of the solid generated

$r = a(1+\cos\theta)$

$$V = \frac{2}{3}\int_0^{\pi/2} r^3 \cdot \cos\theta d\theta$$

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$$= \frac{2}{3}\int_0^{\pi/2} r^3 \cos\theta \, d\theta$$

$$= \frac{2}{3}\int_0^{\pi/2} a^3 (1+\cos\theta)^3 \cdot \cos\theta \, d\theta$$

$$= \frac{2}{3}\int_0^{\pi/2} a^3 (1 + 2\cos^2\tfrac{\theta}{2} - 1)^3 \cdot \cos\theta \, d\theta$$

$$= \frac{2}{3}\int_0^{\pi/2} a^3 \cdot 8\cos^6\tfrac{\theta}{2} \cdot \cos\theta \, d\theta$$

$$2\times 8\times \frac{2}{3} a^3 \int_0^{\pi/2} \cos^6\tfrac{\theta}{2} \cdot \sin\tfrac{\theta}{2} \cdot \cos\tfrac{\theta}{2} \, d\theta$$

$$= \frac{32a^3}{3}\int_0^{\pi/2} \sin\tfrac{\theta}{2} \cos^7\tfrac{\theta}{2} \, d\theta$$

$$= \frac{32a^3}{3} \frac{\Gamma\frac{1+1}{2}\,\Gamma\frac{1+7}{2}}{2\,\Gamma\frac{1+7+2}{2}}$$

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+2}{2}}$$

$$= \frac{32}{3} a^3 \frac{\Gamma 1 \times 3\times 2 \times \pi}{2\times \frac{3}{2}\times\frac{1}{2}\sqrt{\pi}}$$

$$= \frac{1}{8}\times\frac{32}{3} a^3 \times \pi$$

$$= \boxed{\frac{4\pi a^3}{3}} \text{ Unit}^3$$

Answer-(b)

(b) $\iiint x^2\,dx\,dy\,dz$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$

By Dirichilet's theorem

$\frac{x^2}{a^2}=u,\quad \frac{y^2}{b^2}=v,\quad \frac{z^2}{c^2}=w$

$x=au^{1/2},\quad y=bv^{1/2},\quad z=cw^{1/2}$

$dx=\frac{1}{2}u^{\frac{1}{2}-1}a\,du,\quad dy=\frac{b}{2}v^{\frac{1}{2}-1}dv,\quad dz=\frac{c}{2}w^{\frac{1}{2}-1}dw$

$=\iiint u\,a^2\,\frac{a}{2}u^{\frac{1}{2}-1}du\cdot v^{\frac{1}{2}-1}\frac{b}{2}dv\cdot\frac{c}{2}w^{\frac{1}{2}-1}dw$

$=\iiint u\,\frac{a^3}{2}u^{\frac{1}{2}-1}du\cdot\frac{b}{2}dv\,v^{\frac{1}{2}-1}\cdot\frac{c}{2}w^{\frac{1}{2}-1}dw$

$=\frac{a^3bc}{8}\iiint u^{\frac{1}{2}+1-1}v^{\frac{1}{2}-1}w^{\frac{1}{2}-1}du\,dv\,dw$

$=\frac{a^3bc}{8}\iiint u^{\frac{1}{2}+1-1}v^{\frac{1}{2}-1}w^{\frac{1}{2}-1}du\,dv\,dw$

$\frac{a^3bc}{8}=\frac{\Gamma\frac{3}{2}\,\Gamma\frac{1}{2}\,\Gamma\frac{1}{2}}{\Gamma\left(\frac{3}{2}+\frac{1}{2}+\frac{1}{2}+1\right)}$

$\left[\iiint x^{l-1}y^{m-1}z^{n-1}dx\,dy\,dz=\frac{\Gamma l\,\Gamma m\,\Gamma n}{\Gamma(l+m+n+1)}\right]$

$=\frac{a^3bc}{8}\left(\frac{\frac{1}{2}\sqrt{\pi}\cdot\pi}{\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}}\right)$

$=\frac{a^3bc}{8}\times\frac{8\pi}{15}=\frac{8\pi a^3bc}{8\cdot 15}$

$\frac{7}{2}-1=\frac{5}{2}$

Ans $=\frac{\pi a^3bc}{15}$

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